MATROID $\mathcal{M}$ ON E IS SUBSET $\mathcal{I}$ OF 2^E s.t.

1. $\phi \in \mathcal{I}$
2. $J \in \mathcal{I}$, $I \subseteq J \Rightarrow I \in \mathcal{I}$.
3. $I \in \mathcal{I}$, $J \in \mathcal{I}$ $|J| > |I|$
$\Rightarrow \exists\, j \in J - I$ s.t.
$I \cup \{j\} \in \mathcal{I}$.

$\mathcal{I}$ = INDEPENDENT SETS

LABELLED MATRIX OVER FIELD

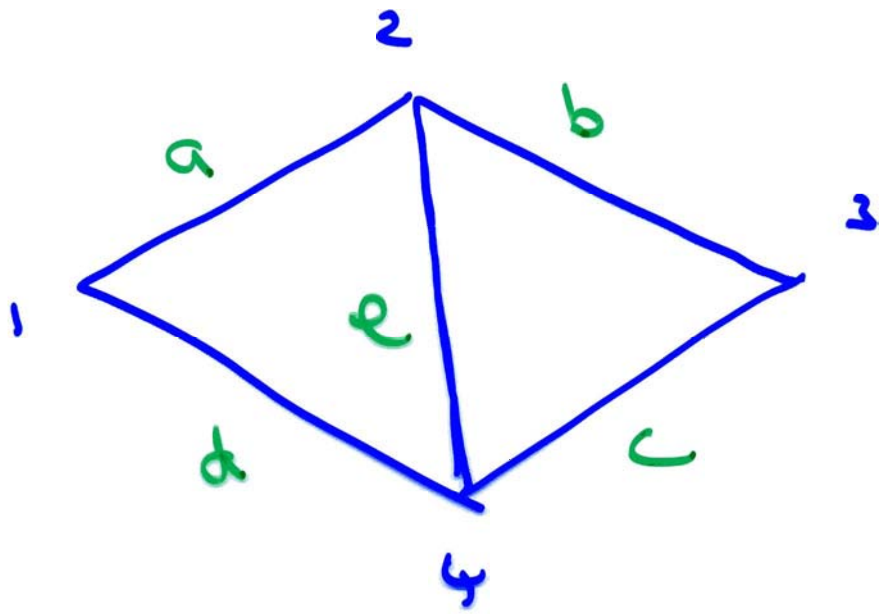
$$
\begin{array}{ccccccc}
a & b & c & d & e & f & g \\
\end{array}
$$

$$
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix}
$$

OVER GF(2)

MATRICES OVER GF(2)

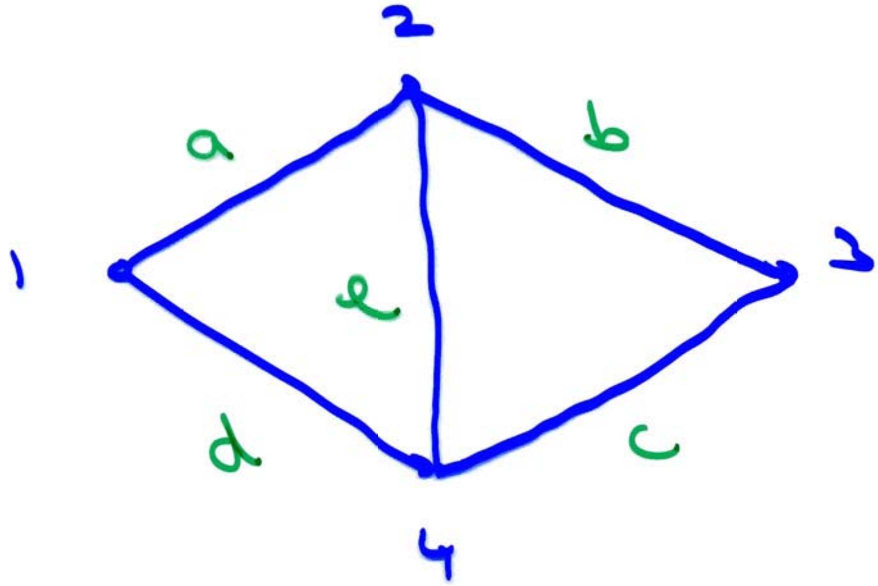
$\Longrightarrow$ **BINARY MATROIDS.**



GRAPHIC MATROIDS

INDEPENDENT SETS ARE EDGE SETS OF FORESTS.

GRAPHIC MATROIDS ARE BINARY.



G

$$
\begin{array}{c c}
 & \begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}
\end{array}
$$

M(G)

MATROID M HAS <u>DUAL</u> M^*.

G PLANAR $\Rightarrow$ $M(G^*) = [M(G)]^*$.

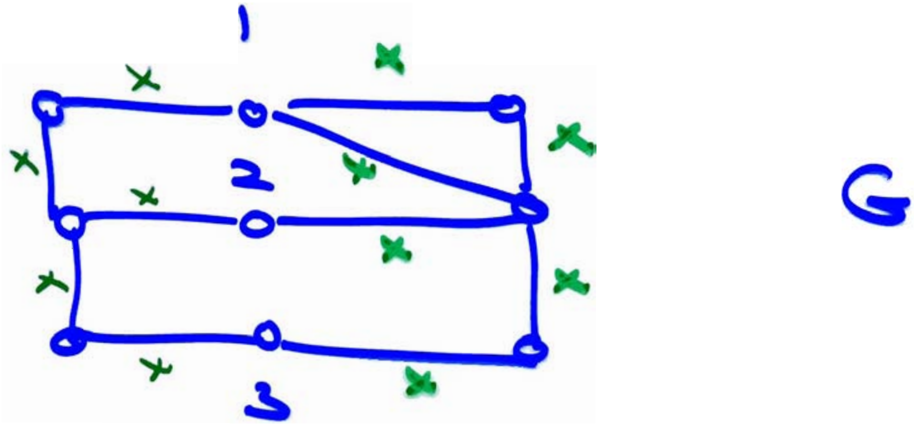
G NON-PLANAR?

M IS <u>COGRAPHIC</u> IF

$M = [M(G)]^*$

FOR SOME GRAPH G.

INTERNAL 4-CONNECTIVITY (IFC)



$A = *$ $B = *$

$\{1,2,3\}$ IS 3-SEPARATION

S.T. $|A|, |B| > 3$

$\Rightarrow$ G NOT IFC.

IFC EXTENDS TO MATROIDS

MATROID COMPLEXITY

M ON E ; $|E| = n$,

HAS UP TO 2^n INDEPENDENT SETS.

HOW TO MEASURE SIZE OF MATROID ?

OFTEN USE ORACLE MODEL OF COMPLEXITY

Example: Recognizing Regular Matroids

Regular Matroids

(Subclass of Binary)

$\Downarrow$

Structural Description

(Seymour – Famous Theorem)

$\Downarrow$

Poly. Time Recognition Algorithm

(Using Independence Oracle)

BINARY MATROIDS WITH NO $M(K_{3,3})$-MINOR

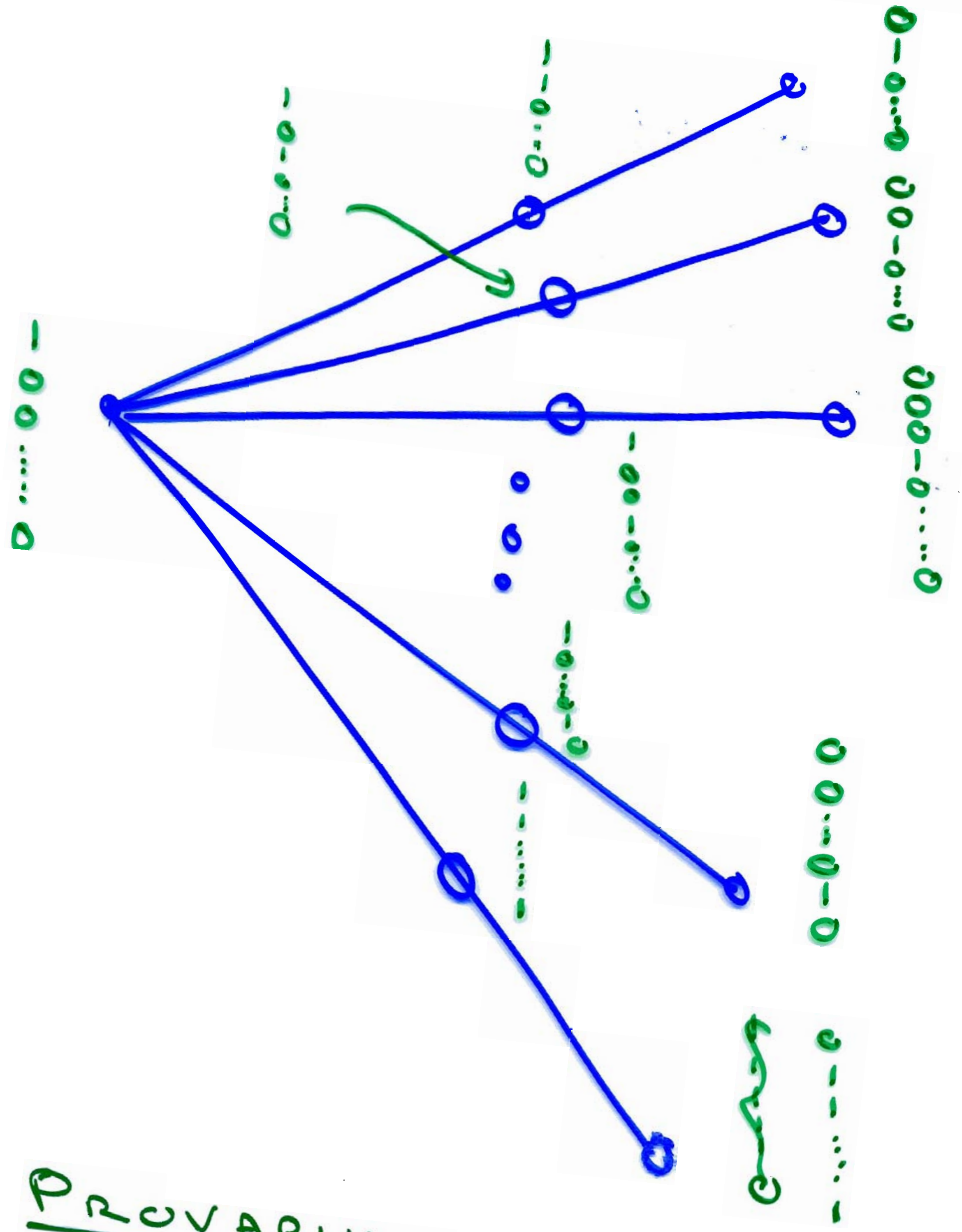
IFC MEMBER OF CLASS IS

EITHER

- ONE OF 17 ? EXCEPTIONAL MATROIDS
- TRIANGULAR OR TRIADIC MÖBIUS MATROID
- COGRAPHIC

IFC MEMBER OF CLASS CAN BE RECOGNIZED IN POLYNOMIAL TIME

WHAT ABOUT NON IFC?



PROVABLY EXPONENTIAL !!

ODD?

USUALLY LOW CONNECTIVITY HELPS.

EG. BOUNDED TREE WIDTH FOR GRAPHS