COSC242: Tutorial Week 2
Big-O and Big-Θ
A Few Solutions

1. Suppose $f(n) = 5$ for all n.

   (a) Is $f = O(1)$?

   Model Answer:

   Proof. We need to show that there exists some c and n_0 such that:

   $$f(n) \leq c \cdot 1 \text{ for all } n \geq n_0.$$

   Let's choose $c = 10, n_0 = 3$. Then we have[1]:

   $$\begin{aligned} LHS &= f(n) = 5 \\ RHS &= 10 \cdot 1 = 10 \\ &\geq LHS \end{aligned}$$

   Which is true for all $n \geq 1$ and therefore certainly true for $n \geq 3$. □

   Note that there are an infinite number of combinations of c and n_0 that would also work in this case - you can choose any of them. For example: $(c = 5, n_0 = 1)$, $(c = 1, n_0 = 5)$, $(c = 100000, n_0 = 100000)$.

2. Suppose $f(n) = 3n + 6$ for all n.

   (a) Is $f = O(n)$?

   Model Answer:

   Proof. We need to show that:

   $$3n + 6 \leq c \cdot n \text{ for all } n \geq n_0$$

   Let's choose $c = 9, n_0 = 1$, then we have:

   $$\begin{aligned} LHS &= 3n + 6 \\ &\leq 3n + 6n \text{ for } n \geq 1 \\ &\leq 9n \\ &\leq RHS \end{aligned}$$

   □

[1] LHS: Left Hand Side

(b) Is $f = \Theta(n)$?

(c) Is $f = O(n^2)$?

Model Anser:

Proof. We need to show that:

$$3n + 6 \leq c \cdot n^2 \text{ for all } n \geq n_0$$

Let's choose $c = 9, n_0 = 1$, then we have:

$$\begin{aligned} LHS &= 3n + 6 \\ &\leq 3n + 6n \\ &\leq 9n \\ &\leq 9n^2 \\ &\leq RHS \end{aligned}$$

□

(d) Is $f = \Theta(n^2)$?

3. Suppose $f(n) = 2n^2 + 3$ for all n.

(a) Is $f = O(n^2)$?

Model Answer:

Proof. We need to show that:

$$2n^2 + 3 \leq cn^2 \text{ for all } n \geq n_0$$

Choose $c = 5, n_0 = 1$:

$$\begin{aligned} LHS &= 2n^2 + 3 \\ &\leq 2n^2 + 3n^2 \\ &\leq 5n^2 \\ &\leq RHS \end{aligned}$$

□

(b) Is $f = \Theta(n^2)$?

(c) Is $f = O(n^3)$?

(d) To think about: Is $f = \Theta(n^3)$?