Ray Tracing


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QUARTZ

The Basic Process

- *Create an image plane and viewpoint.*
- *For each pixel trace a ‘ray’ from the eye through a corresponding point in the image plane.*
- *For each ray, return the colour of the object at the hit point.*

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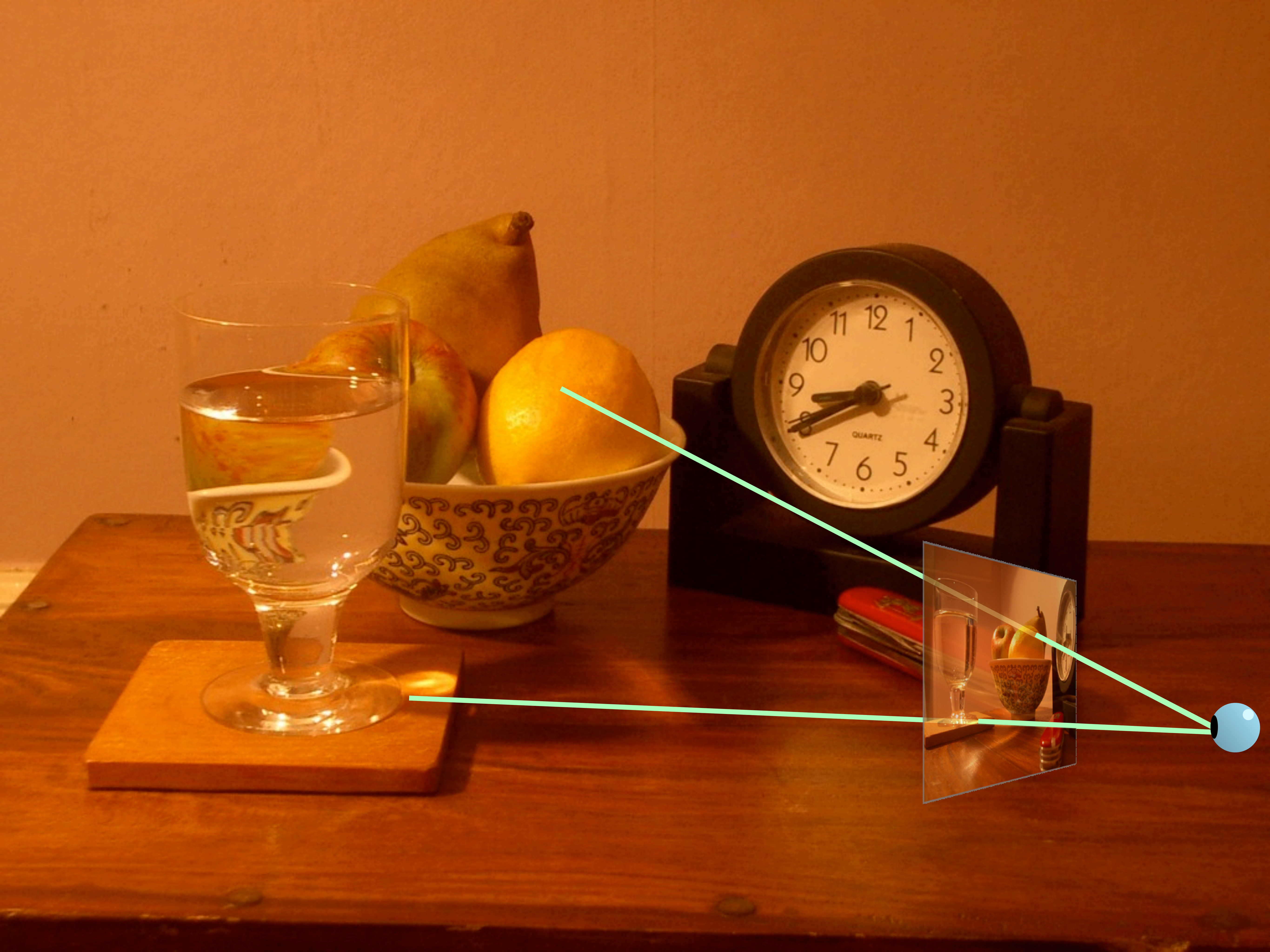
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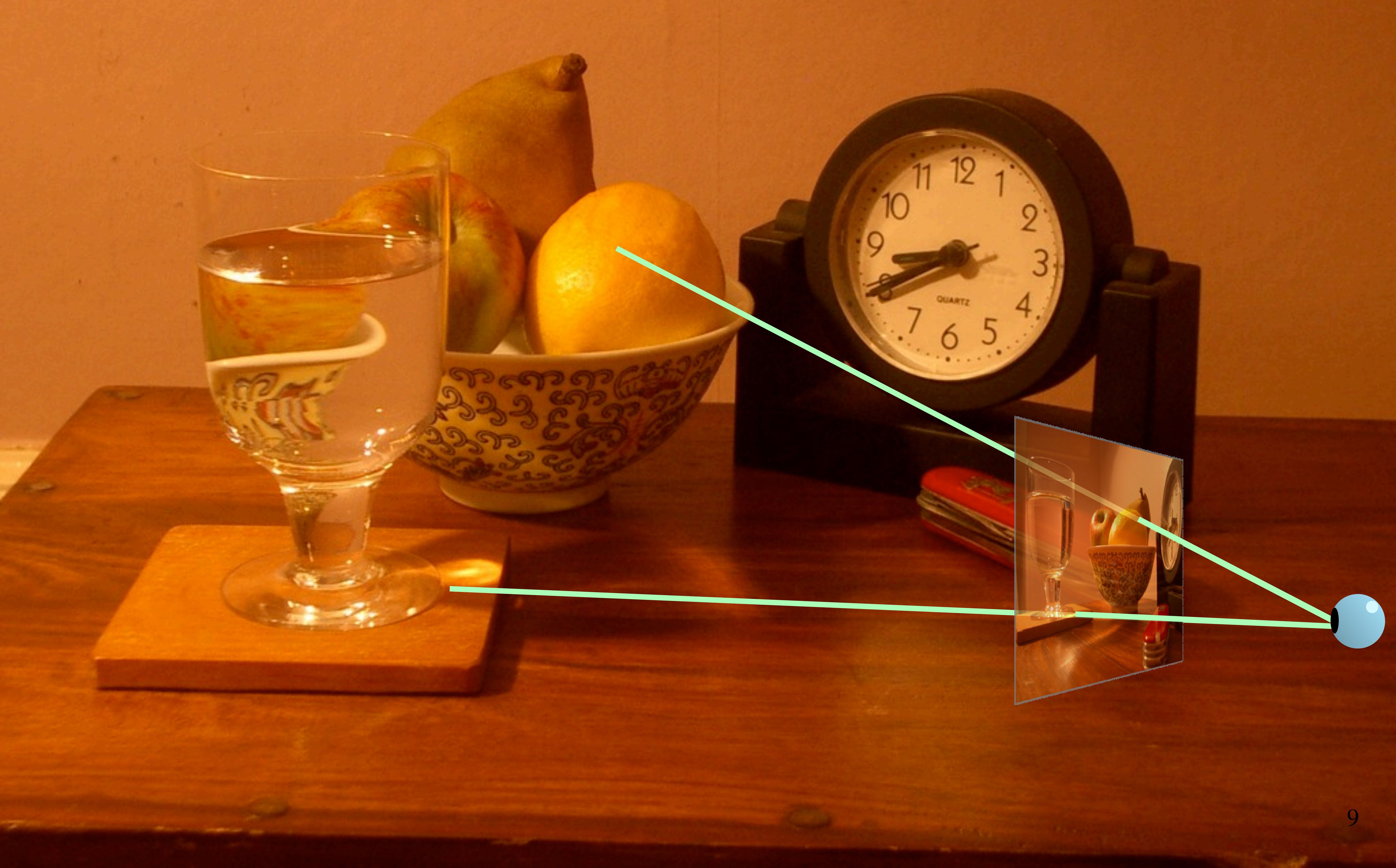
The hit point?



The Hit Point

- *For each ray we have to find what we can see in that direction.*
- *So we need to know which object the ray hits first.*
- *Objects are represented mathematically.*
- *We must find where a ray intersects an object's surface.*

Intersection of line and sphere

- *Why sphere?*
- *How do we represent a line?*
- *How do we represent a sphere?*
- *How do we find the intersection point?*

Sphere is the simplest

Given a point $\mathbf{p} = (x, y, z)$

Its distance from the origin is:

$\mathbf{p}^2 = \mathbf{p} \cdot \mathbf{p} = (x^2 + y^2 + z^2)$

$\mathbf{p}^2 = \mathbf{p} \cdot \mathbf{p} = (x^2 + y^2 + z^2) = 1.0$

describes a sphere of radius 1.0 centred at the origin.

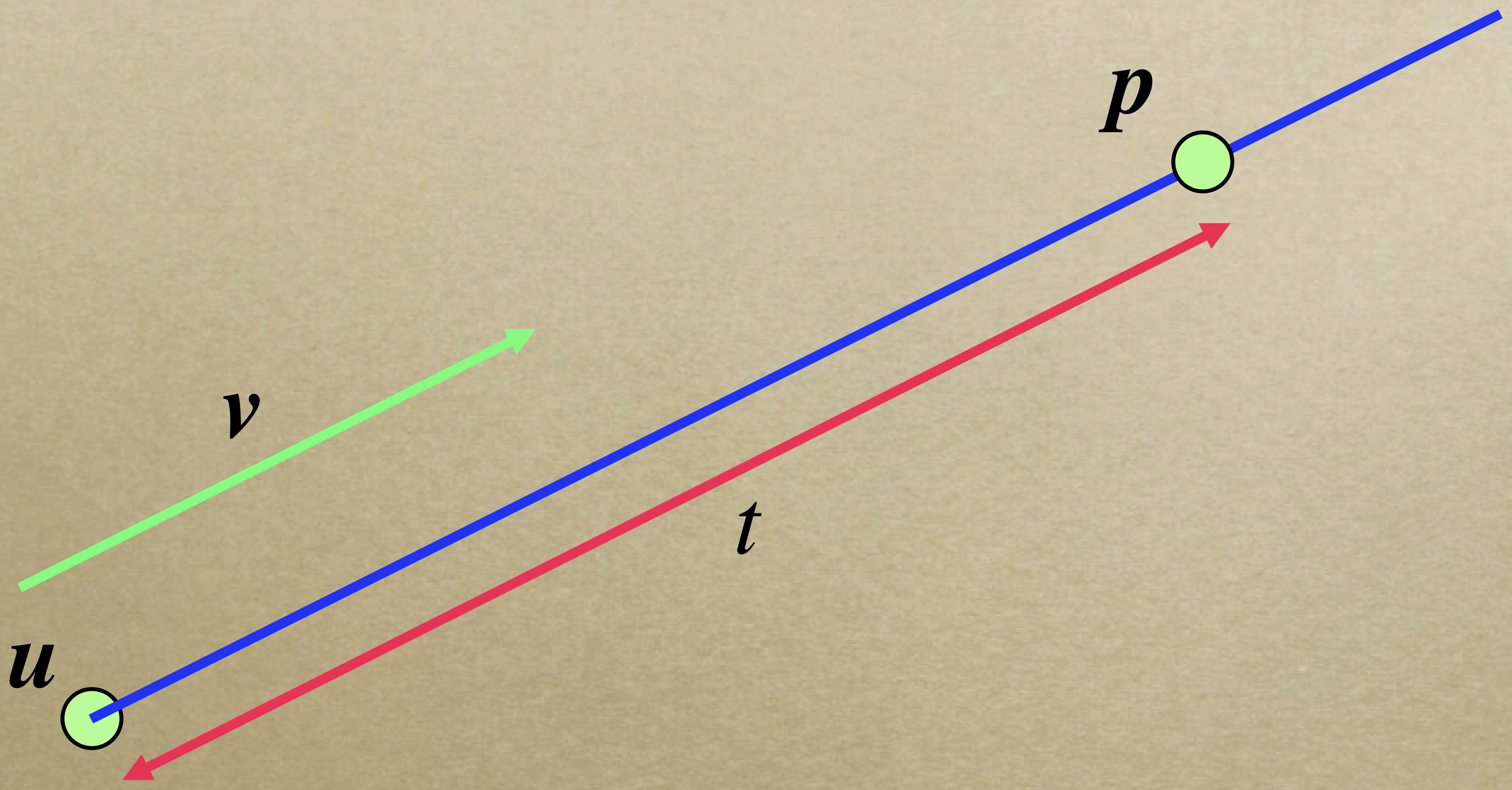
A line ...

We can describe a line using a starting point, $\boldsymbol{u}$ *and direction* $\boldsymbol{v}$*:*

$$\boldsymbol{p} = \boldsymbol{u} + \boldsymbol{v}\, t$$

If $\boldsymbol{v}$ *is a unit vector, we can think of* t *as the distance along the line.*

$$\mathbf{p} = \mathbf{u} + \mathbf{v}\,\mathrm{t}$$



If $\mathbf{p}$ is on the line and the sphere

$$\boldsymbol{p}^2 = \boldsymbol{p}\bullet\boldsymbol{p} = (\boldsymbol{u} + \boldsymbol{v}\, t)^2 = 1.0$$

$$(\boldsymbol{u} + \boldsymbol{v}\, t)\bullet(\boldsymbol{u} + \boldsymbol{v}\, t) = \boldsymbol{v}^2\, t^2 + 2\, \boldsymbol{u}\bullet\boldsymbol{v}\, t + \boldsymbol{u}^2$$

so:

$$\boldsymbol{v}^2\, t^2 + 2\, \boldsymbol{u}\bullet\boldsymbol{v}\, t + \boldsymbol{u}^2 = 1.0$$

This is just an ordinary quadratic equation in t.

Quadratic Solution

$$\boldsymbol{v}^2 t^2 + 2\, \boldsymbol{u} \bullet \boldsymbol{v}\, t + \boldsymbol{u}^2 = 1.0$$

$$A = \boldsymbol{v}^2$$

$$B = 2\, \boldsymbol{u} \bullet \boldsymbol{v}$$

$$C = \boldsymbol{u}^2 - 1.0$$

$$t = (-B \pm \sqrt{(B^2 - 4AC)}) / (2A)$$

Avoid Rounding errors

$if\ B>0,\ t_1 = (-B - \sqrt{(B^2 - 4AC)}) / (2A)$

$else\ t_1 = (-B + \sqrt{(B^2 - 4AC)}) / (2A)$

$t_2 = C/(A\ t_1)$

Explaining the 'odd' t_2 equation

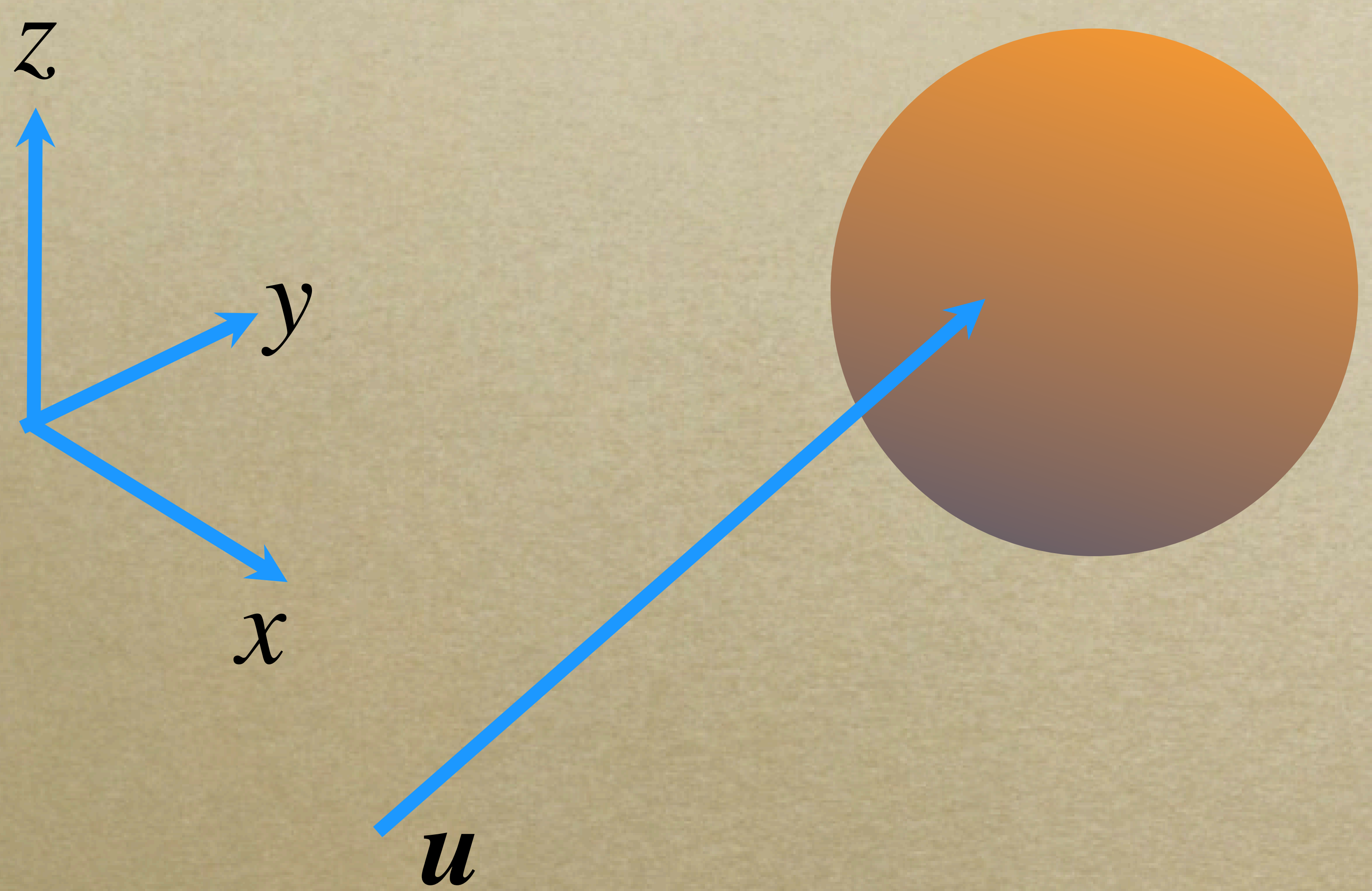
$(t - t_1)(t - t_2) = t^2 - (t_1 + t_2)t + t_1 t_2$

In $At^2 + Bt + C = 0$

$t^2 + (B/A)t + C/A = 0$

and: $t_1 t_2 = C/A$

Sphere not at origin?



Move the ray not the sphere

Sphere at $\boldsymbol{c} = (c_x, c_y, c_z)$

Use $(\boldsymbol{u} - \boldsymbol{c}) + \boldsymbol{v}\, t$

So new $\boldsymbol{u}' = \boldsymbol{u} - \boldsymbol{c} = (u_x, u_y, u_z) - (c_x, c_y, c_z)$

General transformed sphere

Suppose the sphere has been magnified, stretched, rotated and shifted.

That's just one transformation matrix, M

$\boldsymbol{u}' = M^{-1}\,\boldsymbol{u}$

$\boldsymbol{v}' = M^{-1}\,\boldsymbol{v}$

Here is real cunning...

$$\boldsymbol{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \\ 1 \end{bmatrix} \qquad \boldsymbol{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}$$

*M^{-1} **u** includes shift but M^{-1} **v** doesn't*

Next...

Ray tracing triangles