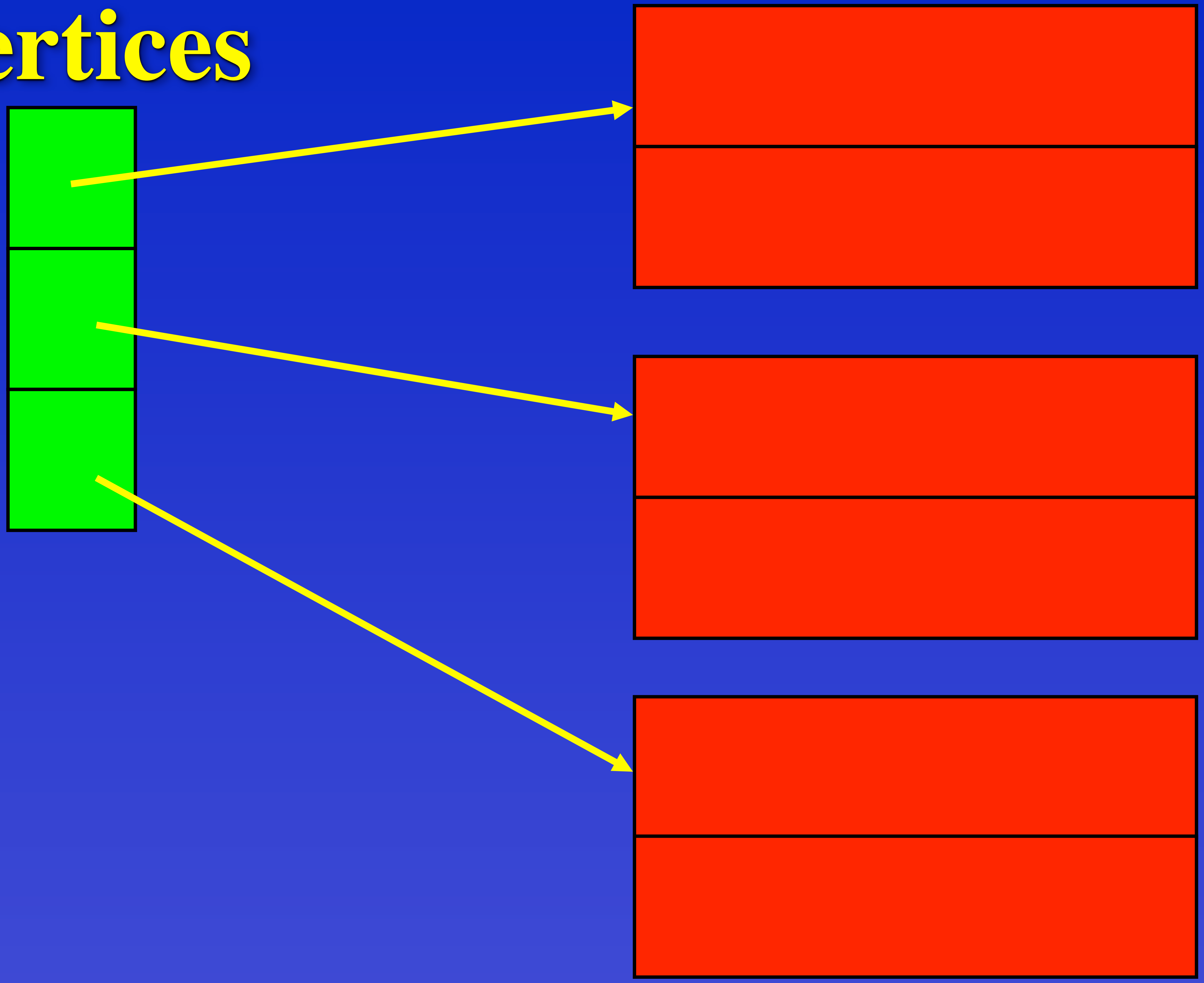
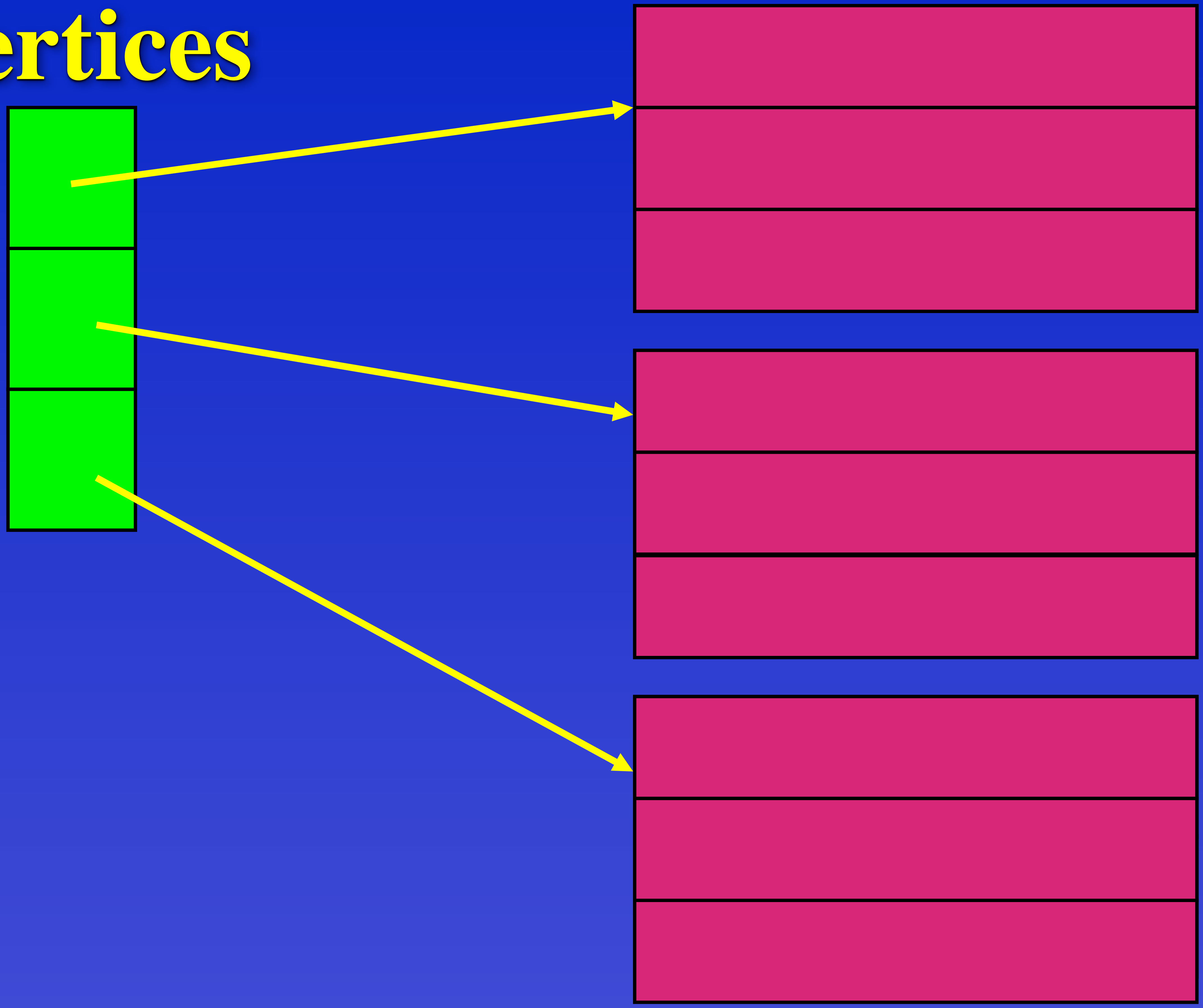
3D Models and Views

- All the principles that apply to 2D points also apply to 3D...
- ...but there are going to be some extras.

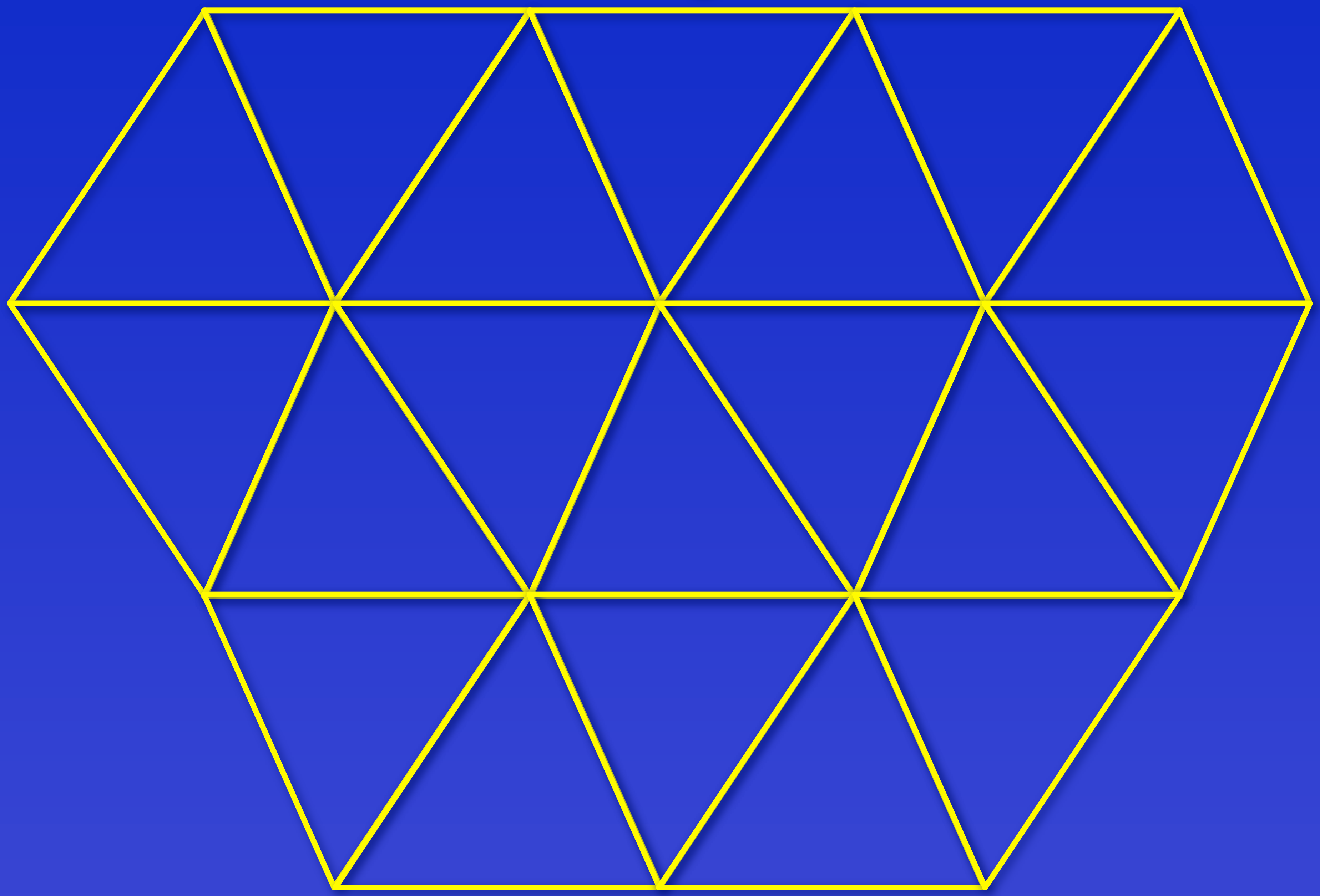
Vertices



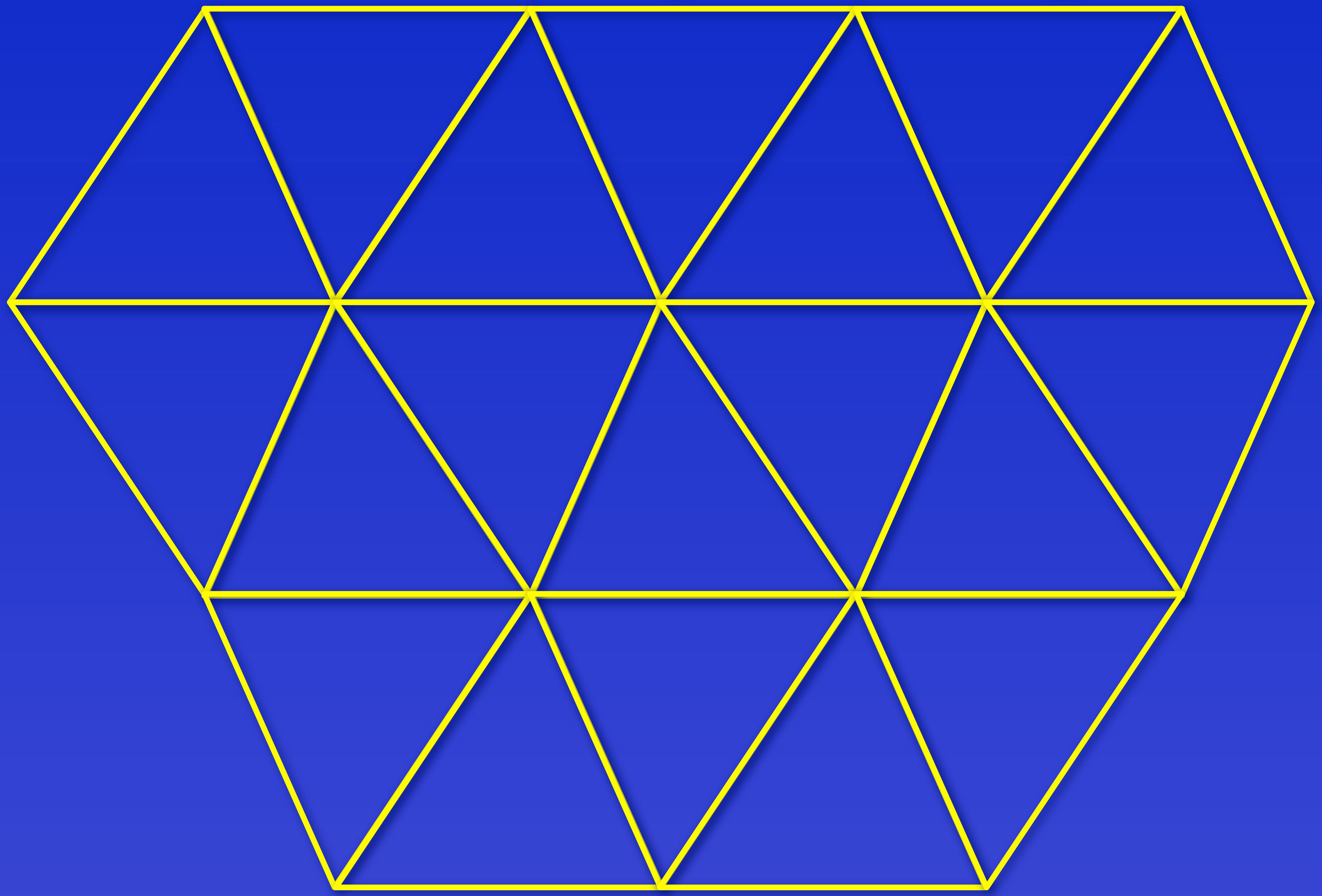
Vertices



How many vertices?



How many triangles?



Points in 3D

(x, y, z) is $\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + a \\ y + b \\ z + c \\ 1 \end{bmatrix}$$

which means (x + a, y + b, z + c)

Association still works

$$(A\ B)\ C = A\ (B\ C)$$

Rotation is more complicated

- $x' = x*\cos(t) - y*\sin(t)$
- $y' = x*\sin(t) + y*\cos(t)$

- This is the basic pattern $x \rightarrow y$

Rotation operates on at least two coordinates

1D not possible		
2D	$x \rightarrow y$	one form
3D	$x \rightarrow y$	z constant
	$y \rightarrow z$	x constant
	$z \rightarrow x$	y constant
4D	$w \rightarrow x$	y, z constant
	…	

$$x' = x*\cos(t) - y*\sin(t)$$
$$y' = x*\sin(t) + y*\cos(t)$$

$$\begin{matrix} \cos & -\sin \\ \sin & \cos \end{matrix}$$

$$x \rightarrow y$$

$$\begin{bmatrix} \cos & -\sin & 0 & 0 \\ \sin & \cos & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

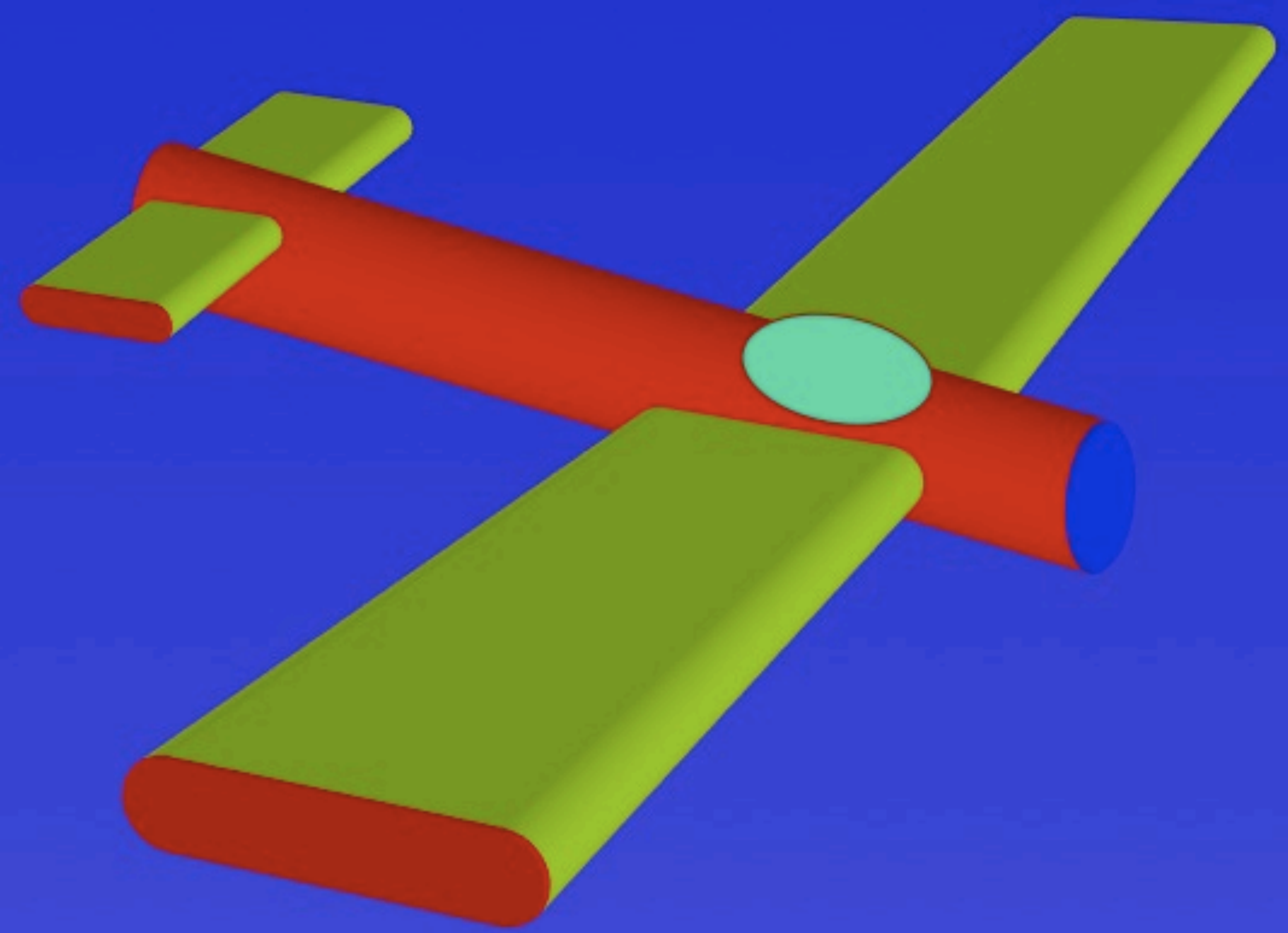
$$y \rightarrow z$$

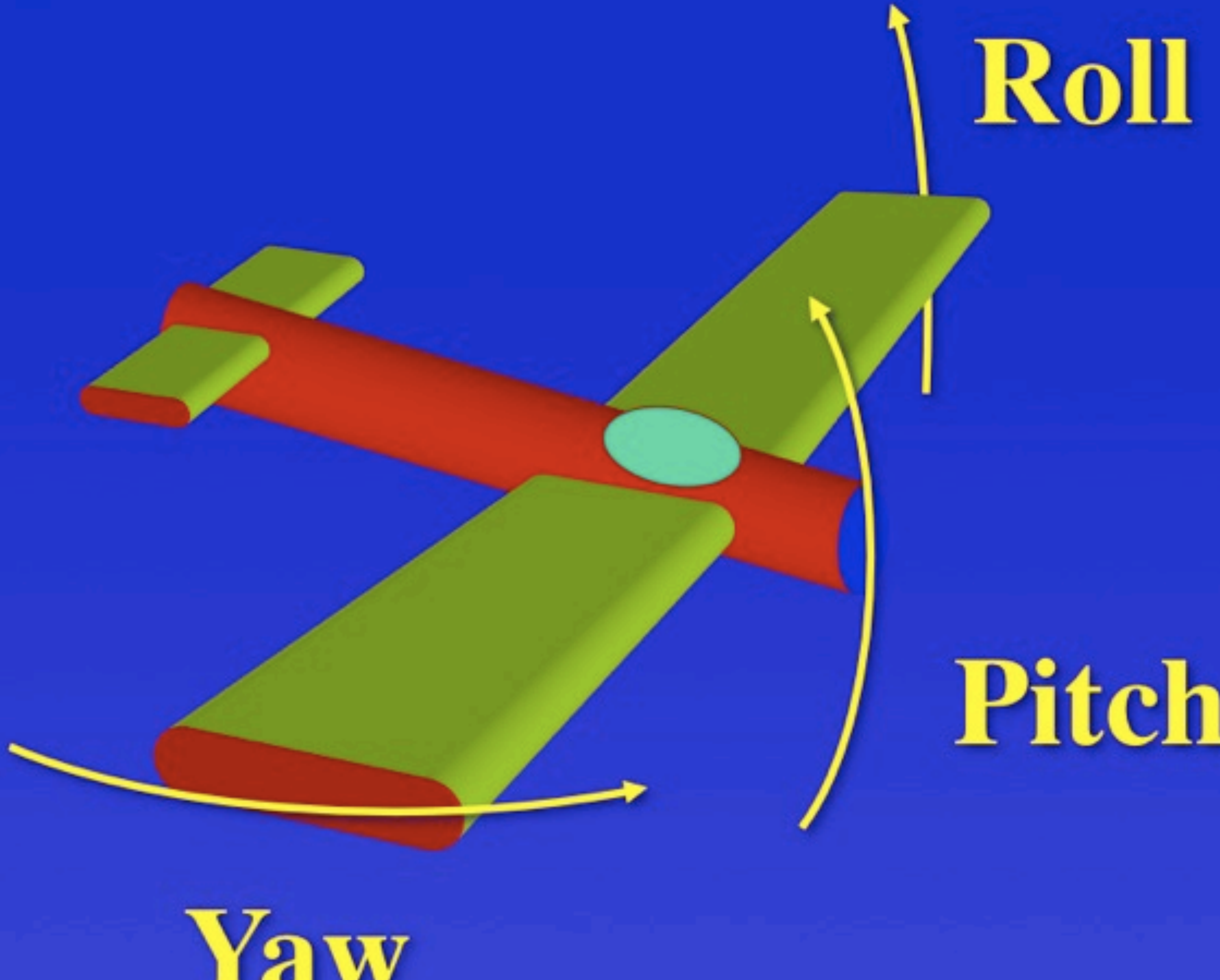
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos & -\sin & 0 \\ 0 & \sin & \cos & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z \rightarrow x \quad \begin{bmatrix} \cos & 0 & \sin & 0 \\ 0 & 1 & 0 & 0 \\ -\sin & 0 & \cos & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos & 0 & -\sin & 0 \\ 0 & 1 & 0 & 0 \\ \sin & 0 & \cos & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad x \rightarrow z$$

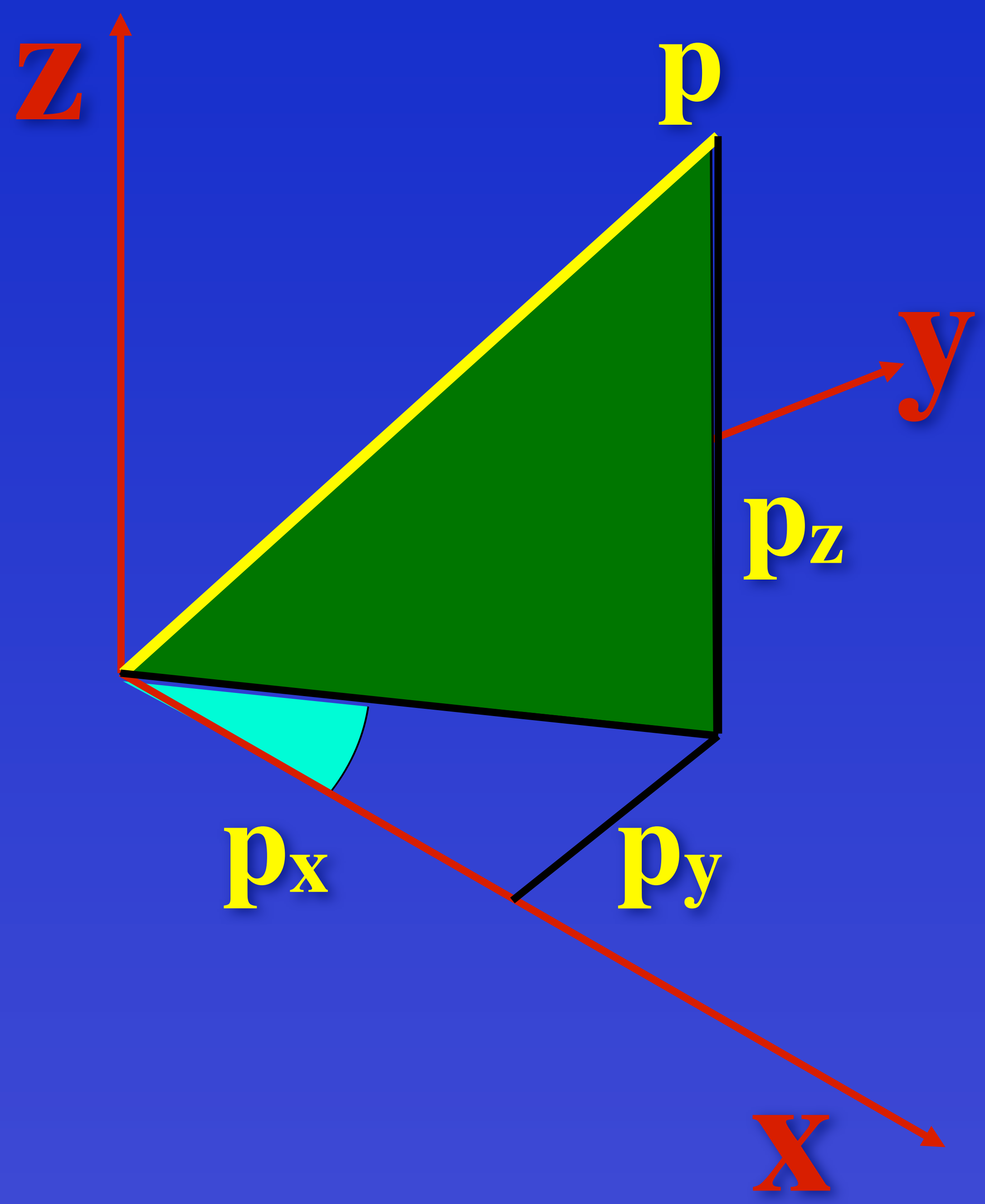
The mechanics of rotation is easy
BUT
Understanding is a little harder

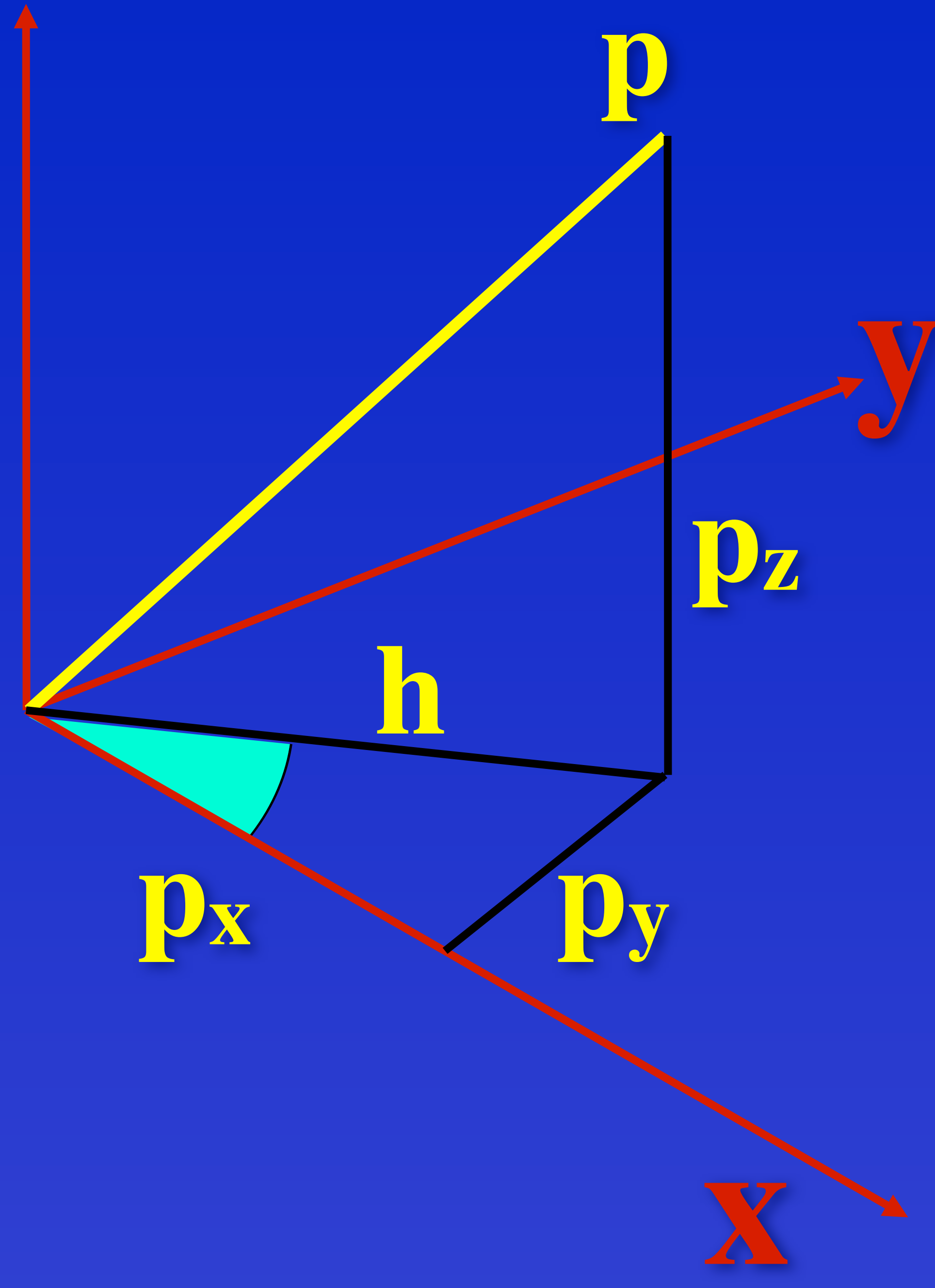



Roll
Pitch
Yaw

Lattitude Longitude

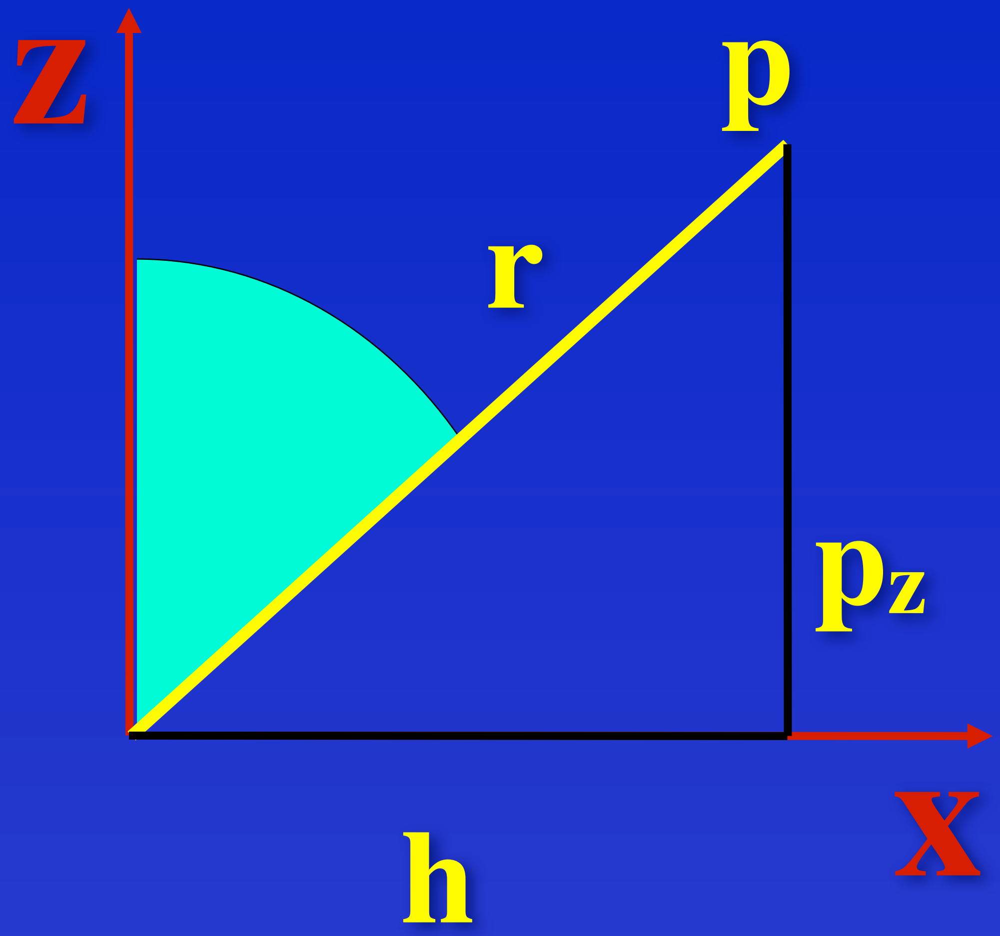
To rotate about any axis





- $h = \sqrt{p_x^2 + p_y^2}$
- $\cos = p_x/h$
- $\sin = p_y/h$

$$A = \begin{bmatrix} p_x/h & p_y/h & 0 & 0 \\ -p_y/h & p_x/h & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$r = \sqrt{p_x^2 + p_y^2 + p_z^2}$$

$$\cos = p_z/r$$

$$\sin = h/r$$

$$B = \begin{bmatrix} p_z/r & 0 & -h/r & 0 \\ 0 & 1 & 0 & 0 \\ h/r & 0 & p_z/r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now rotate about z

$$C = \begin{bmatrix} \cos & -\sin & 0 & 0 \\ \sin & \cos & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Complete rotation is

$A^{-1}B^{-1}CBA$

How do we get A^{-1}?

$$A = \begin{bmatrix} p_x/r & p_y/r & 0 & 0 \\ -p_y/r & p_x/r & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} p_x/r & -p_y/r & 0 & 0 \\ p_y/r & p_x/r & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$