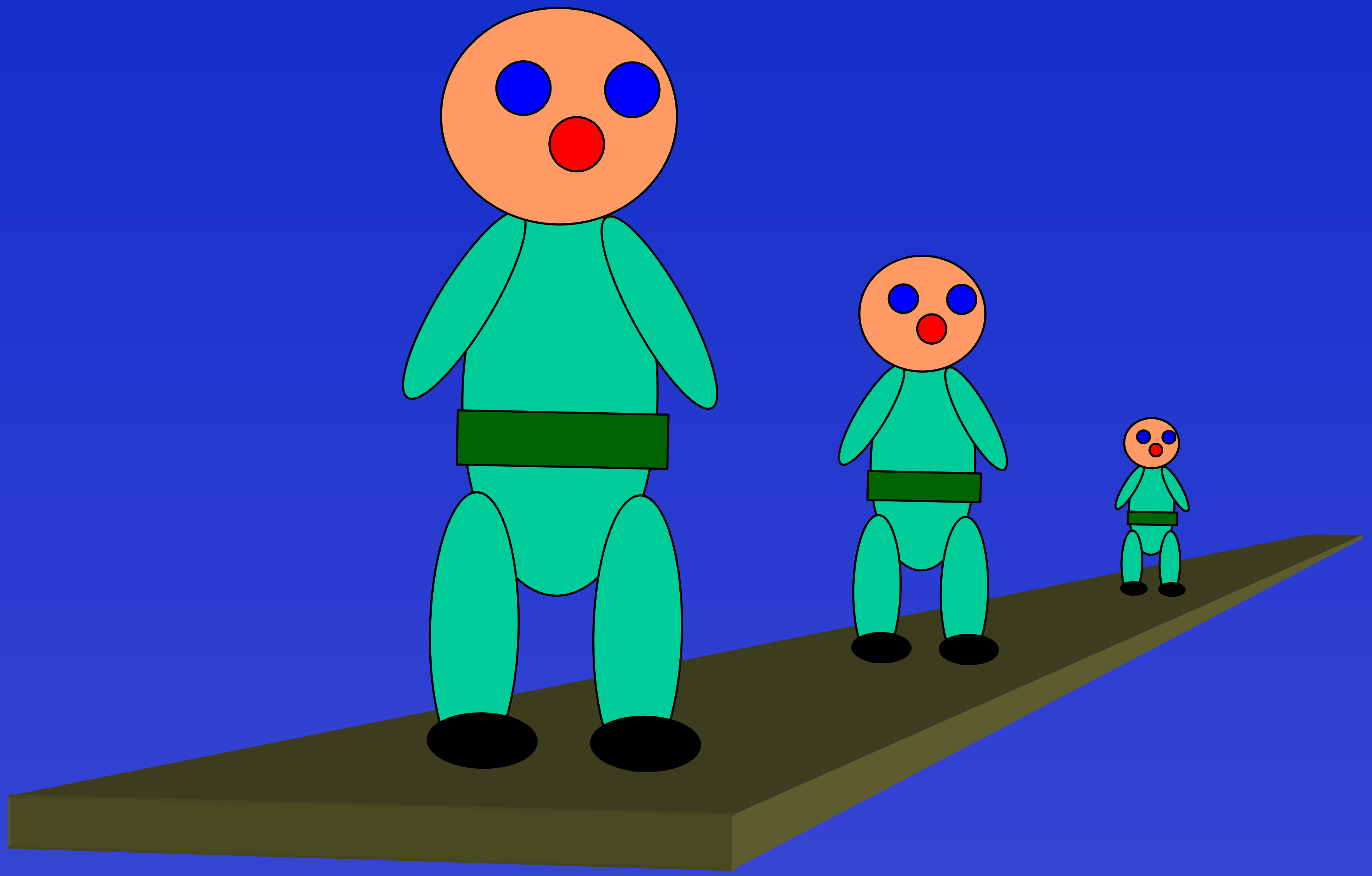
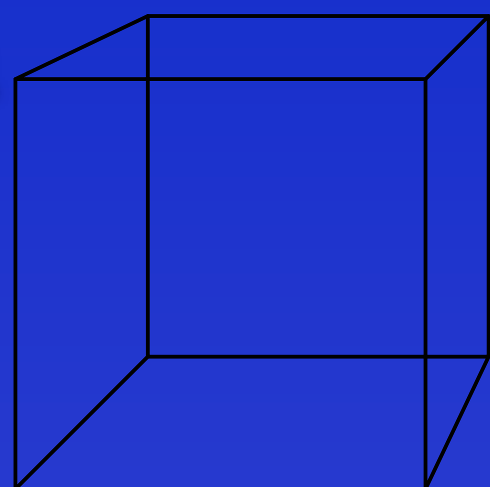
Painless Perspective

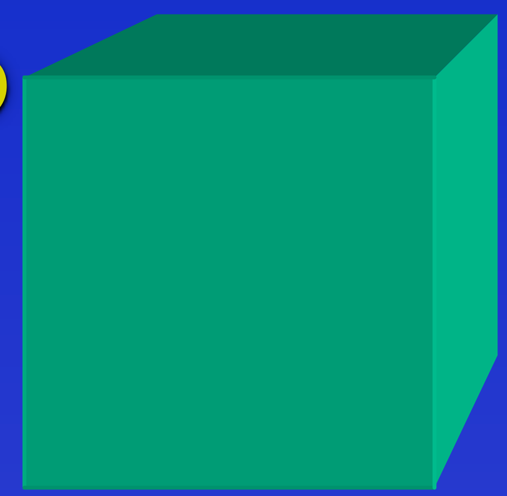


Which are drawn correctly?

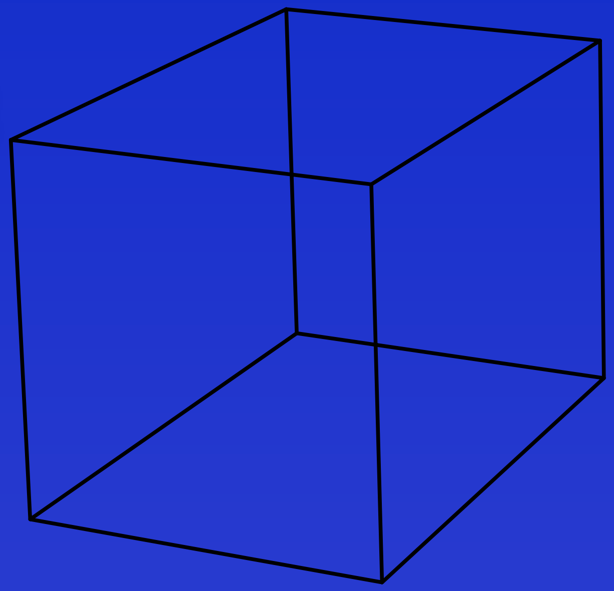
a



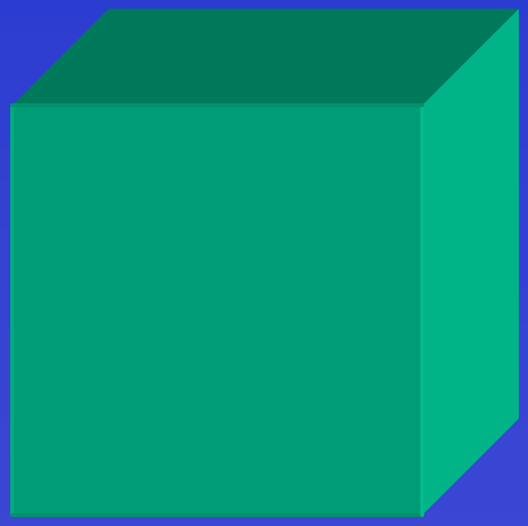
b



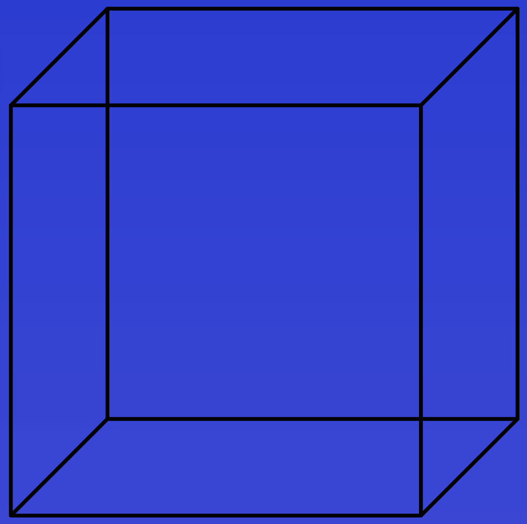
c



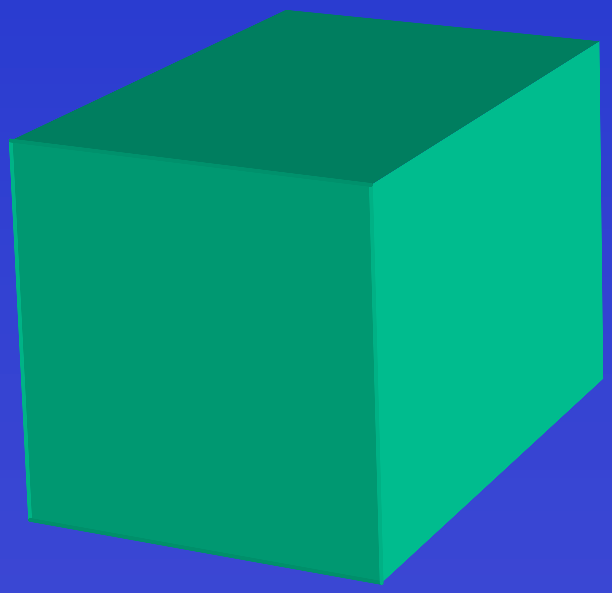
d



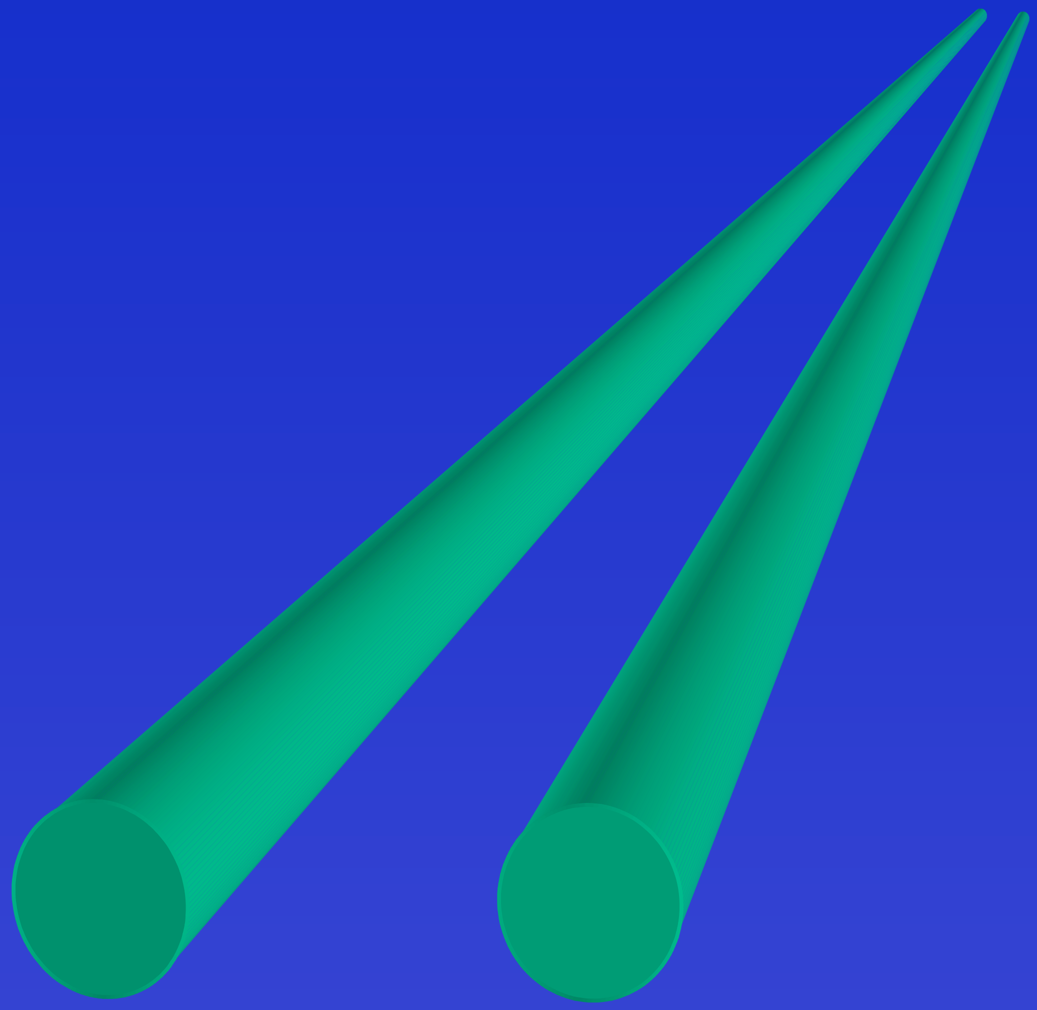
e



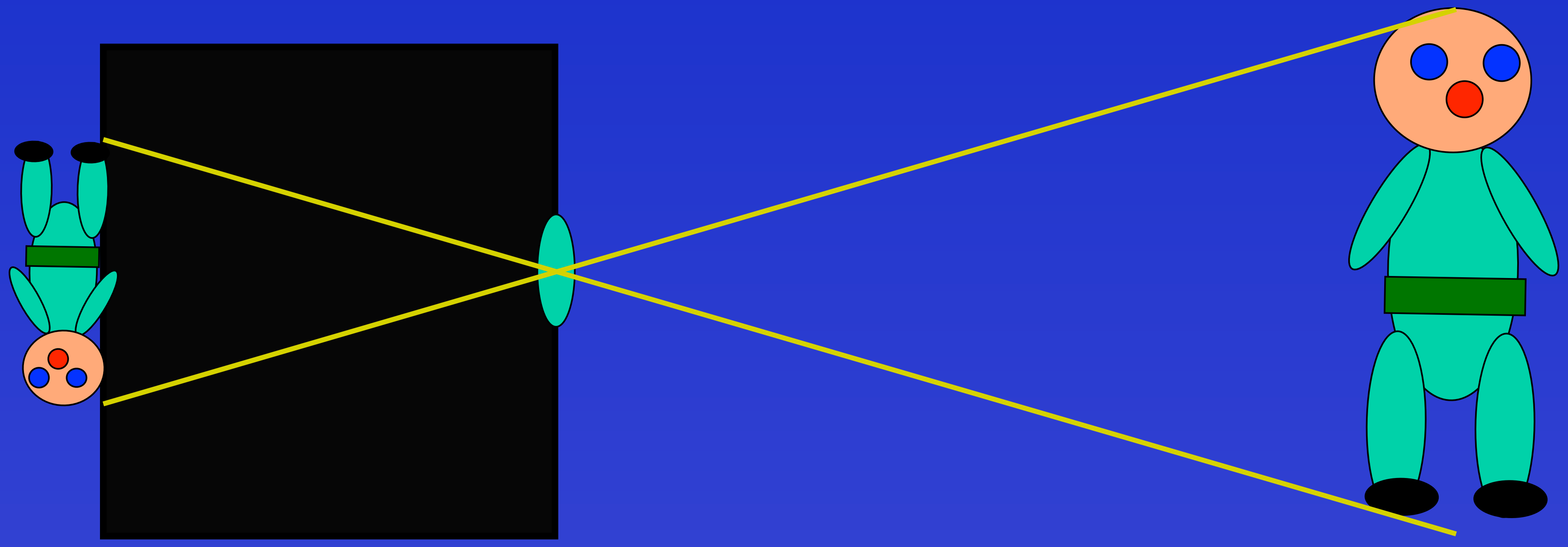
f



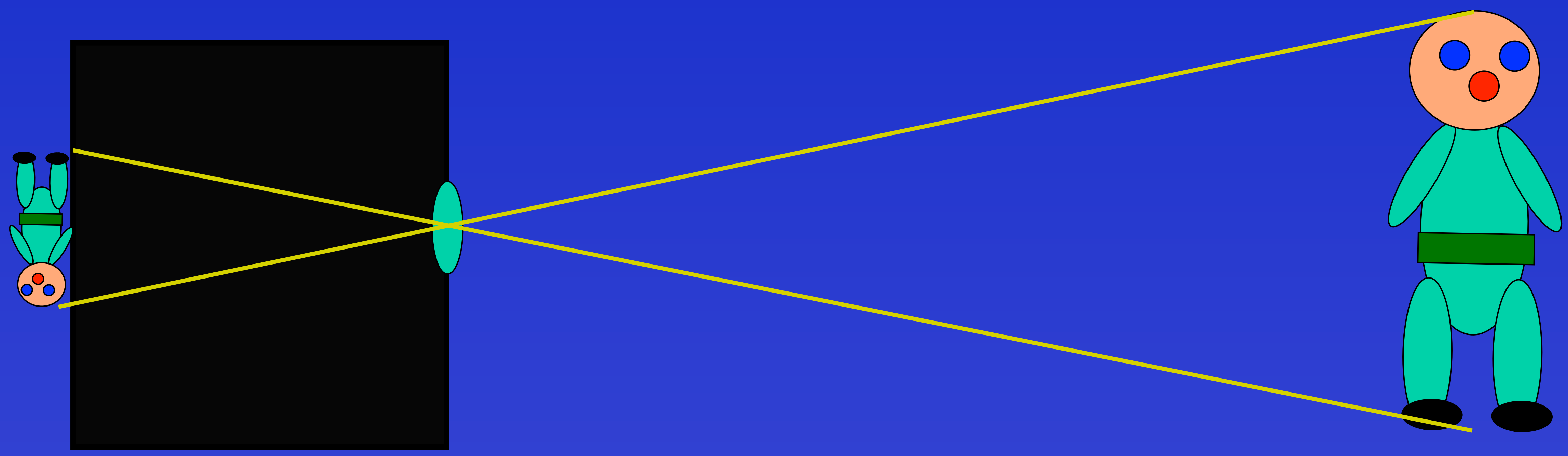
Why do parallel lines seem to converge?



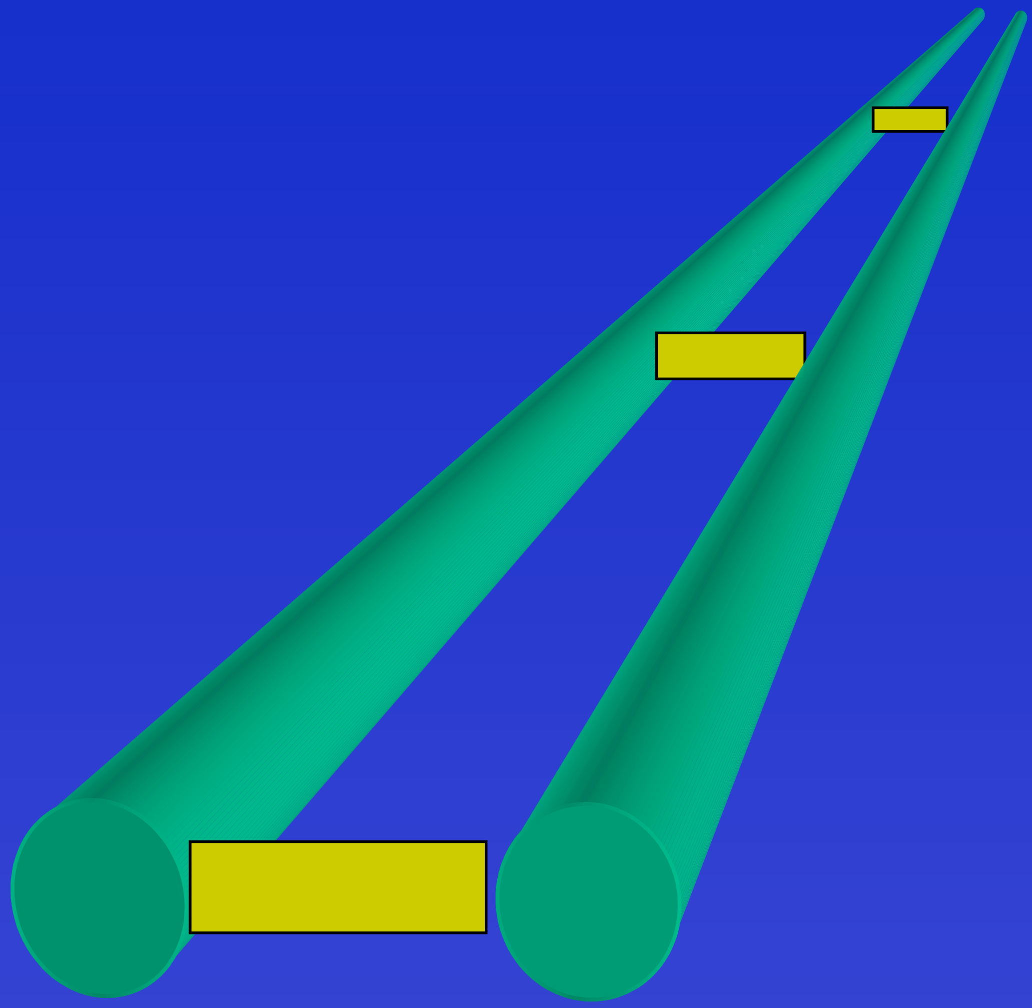
The eye as a camera



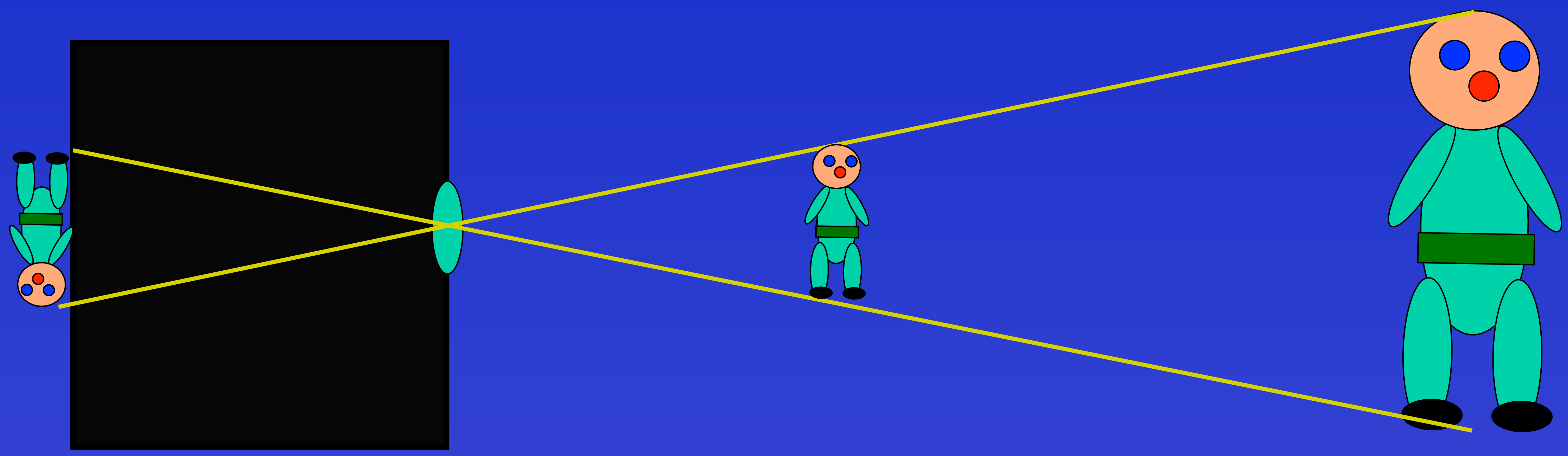
The eye as a camera



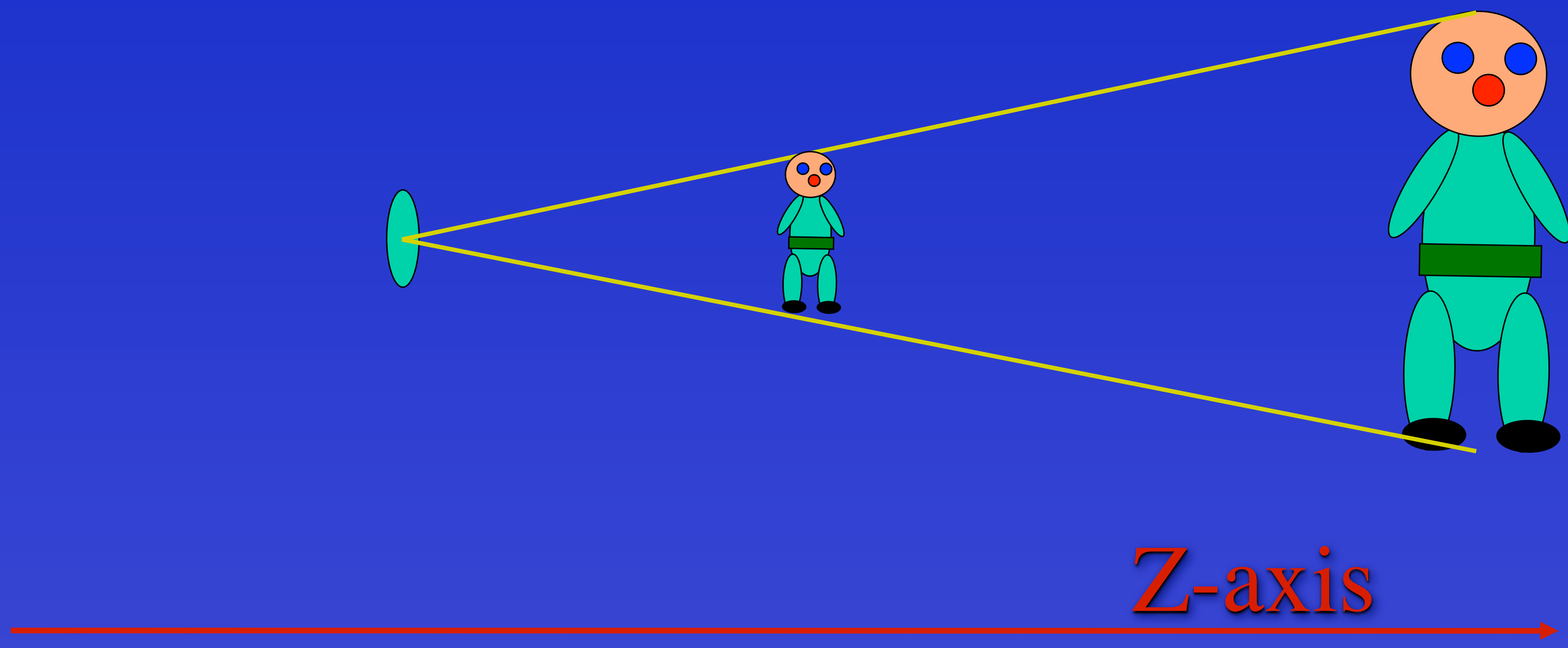
Equal distances appear smaller



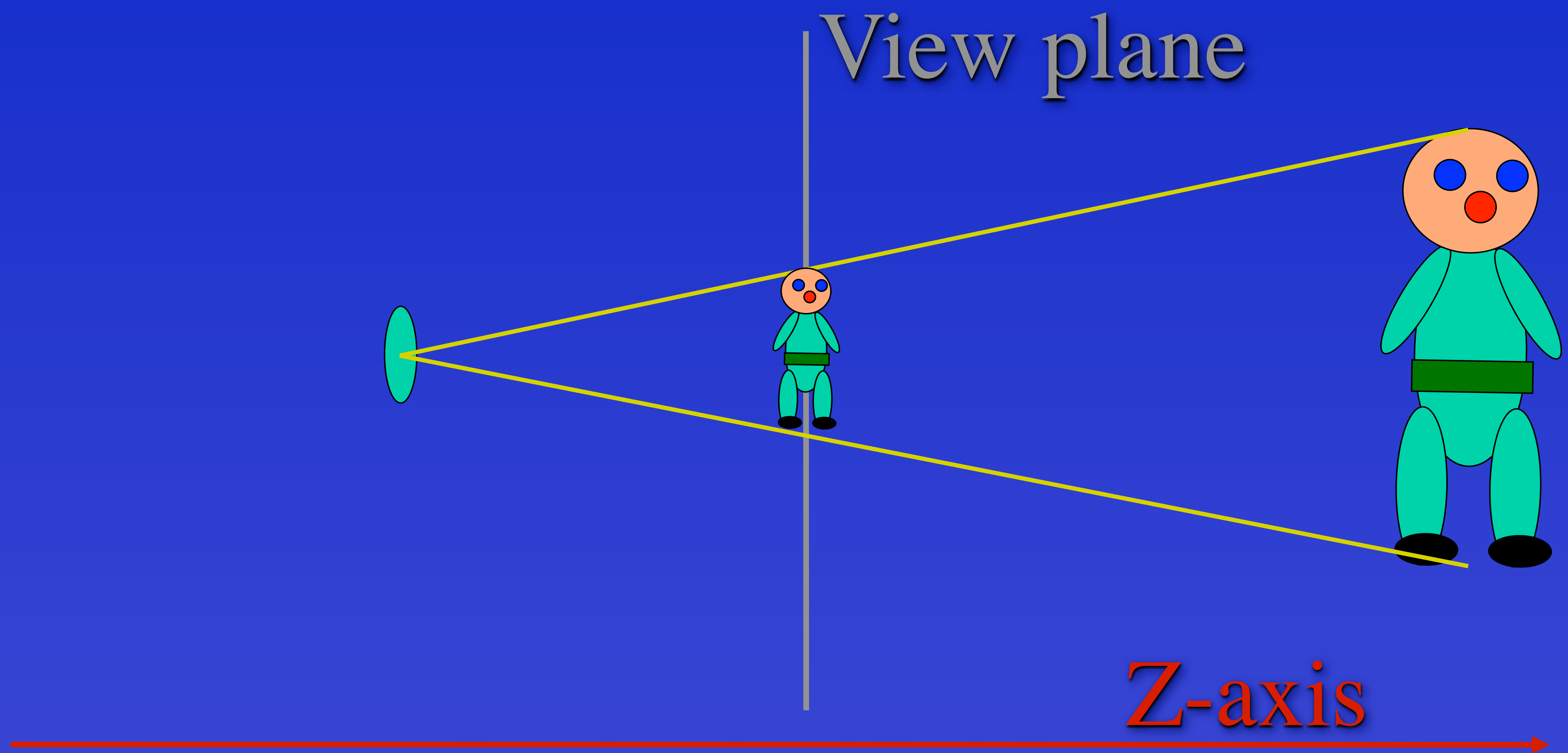
Simplified camera



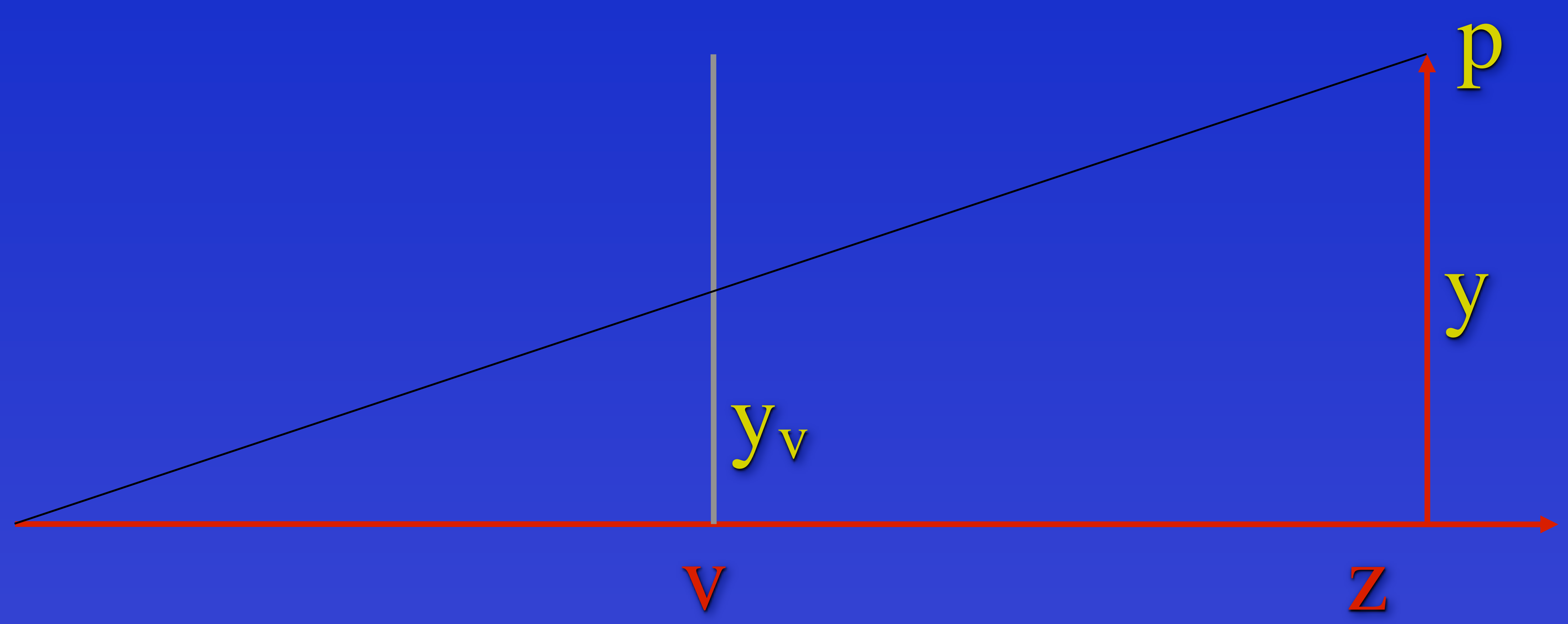
Simplified camera



Simplified camera

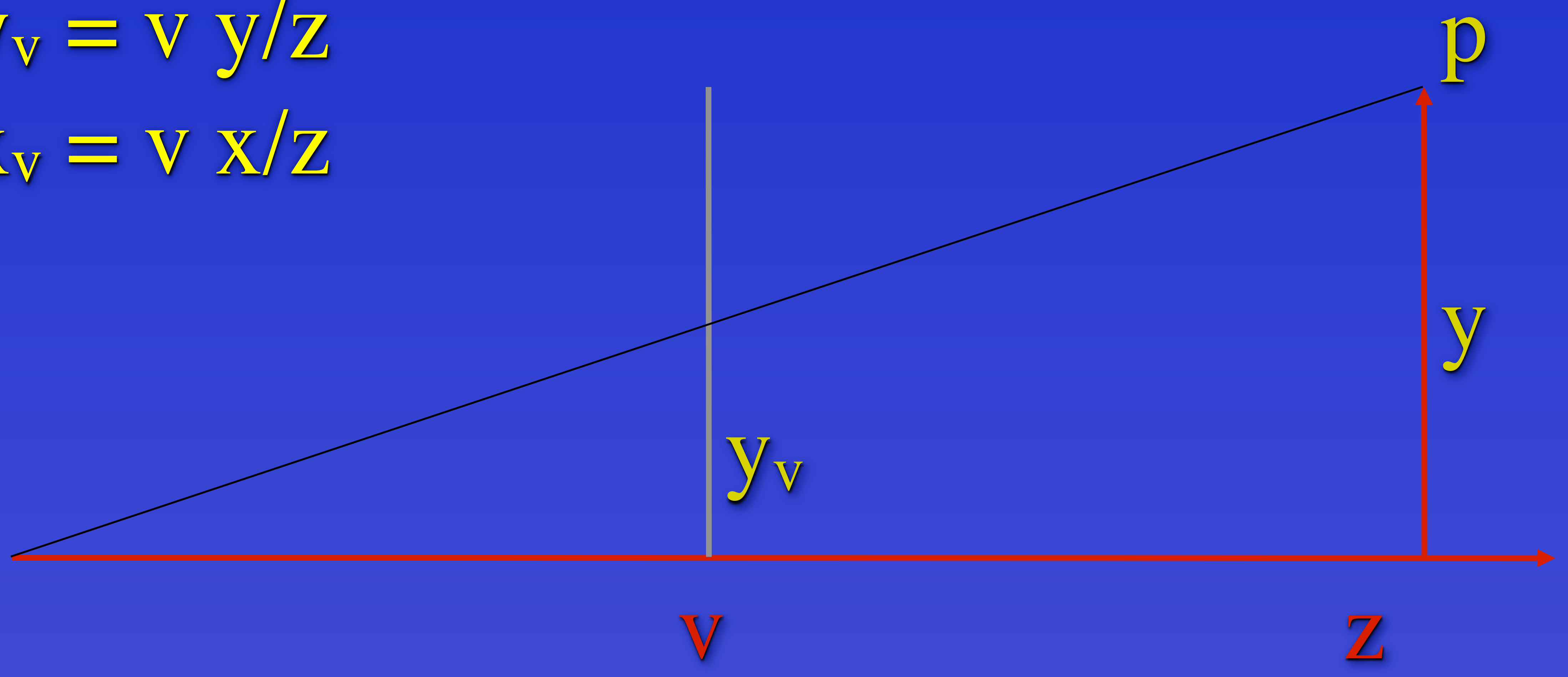


Put the eye at the origin and view down Z-axis find x_v, y_v



Similar triangles

- $y_v / v = y/z$
  $x_v / v = x/z$
- $y_v = v\, y/z$
  $x_v = v\, x/z$



You know how to correct for arbitrary viepoints

- If you are not viewing from the origin, just shift your viewpoint ...
- View from (a, b, c)?
  - Easy:

$$\begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

You know how to move arbitrary axes to the z-axis

- If you are not viewing along the z-axis, just rotate ...

- More complicated, but we did it already
  - Refer back to rotating around an arbitrary axis

Homogeneous coordinates

$$(x, y, z) = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Homogeneous coordinates

$$(x, y, z) = \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

Homogeneous coordinates

$$(x/w, y/w, z/w) = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Perspective matrix

$$\begin{bmatrix} v & 0 & 0 & 0 \\ 0 & v & 0 & 0 \\ 0 & 0 & v & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} xv \\ yv \\ zv \\ z \end{bmatrix} = (xv/z, yv/z, v)$$

- Remember the similar triangle results:
  - $x_v = v\ x/z$
  - $y_v = v\ y/z$

Or in this form...

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & {}^{1}/_{v} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/v \end{bmatrix} = (xv/z, yv/z, v)$$

- Remember: v is the viewplane position

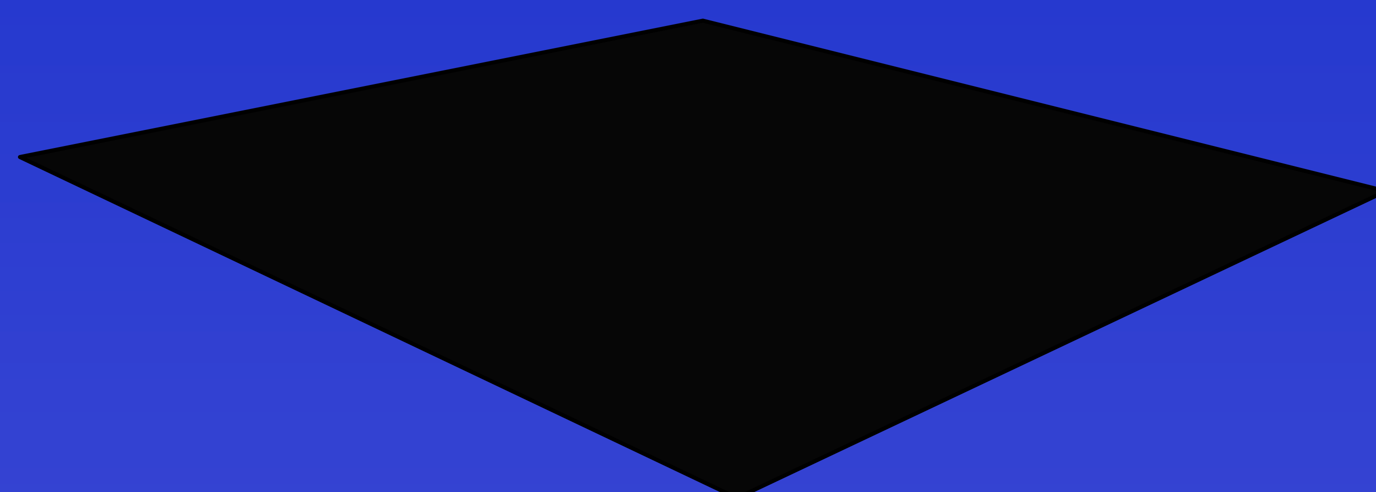
Scaled to make v=1

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z \end{bmatrix} = (x/z, y/z, 1)$$

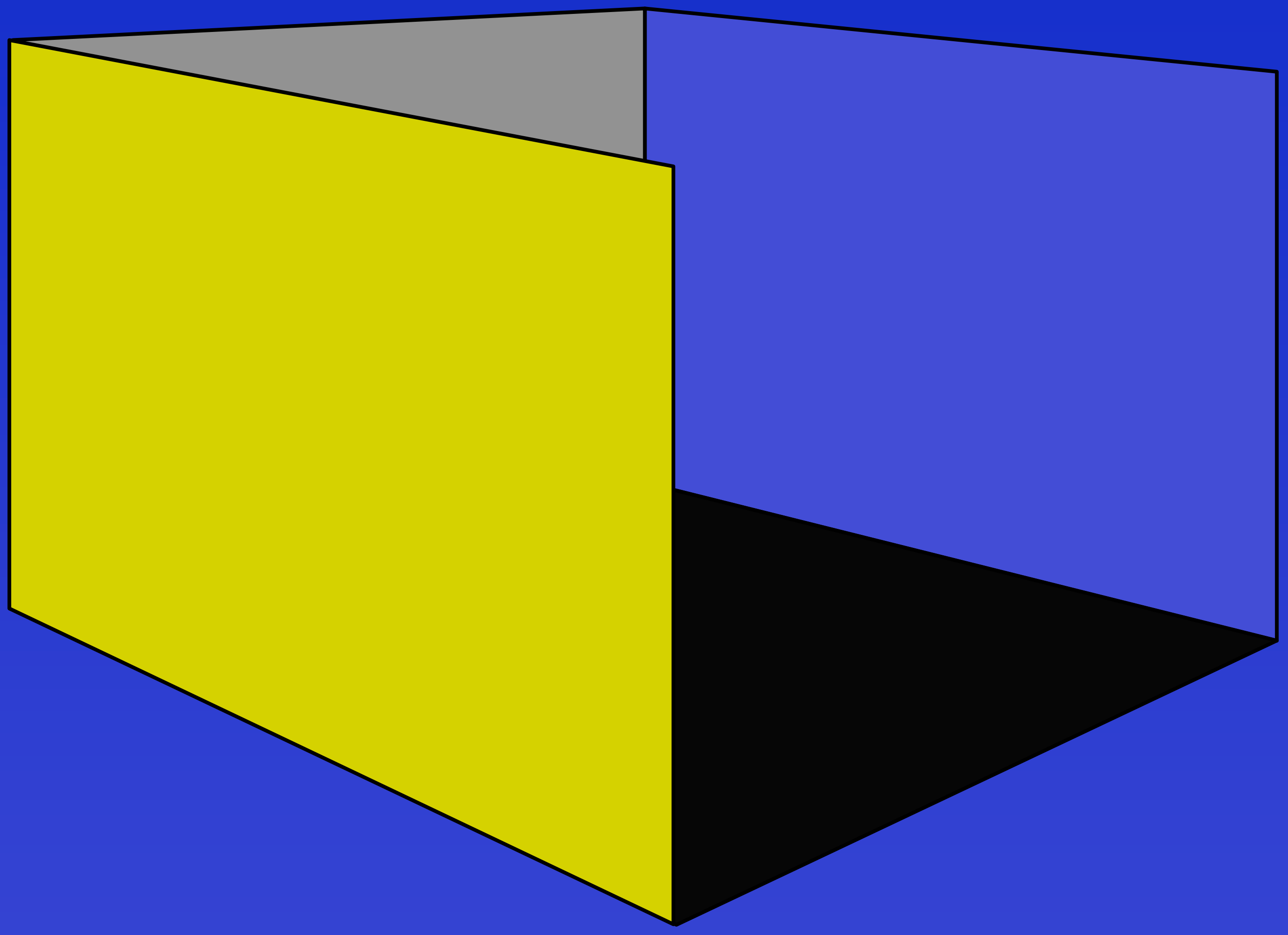
Using just these principles we can draw correct wire frame sketches from 3D models...

...but to get any further we have to deal with the visible surface problem.

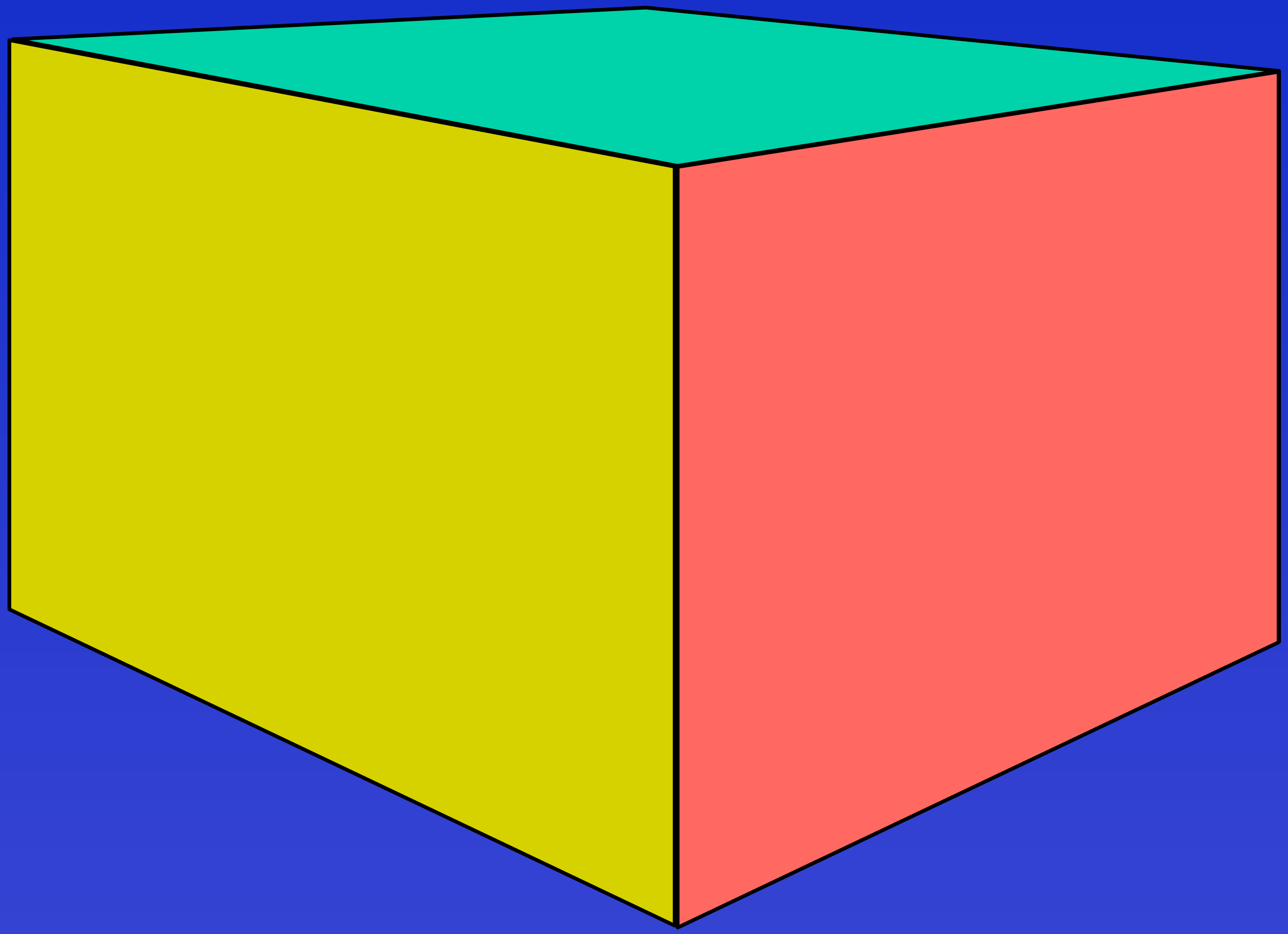
Painter's Algorithm



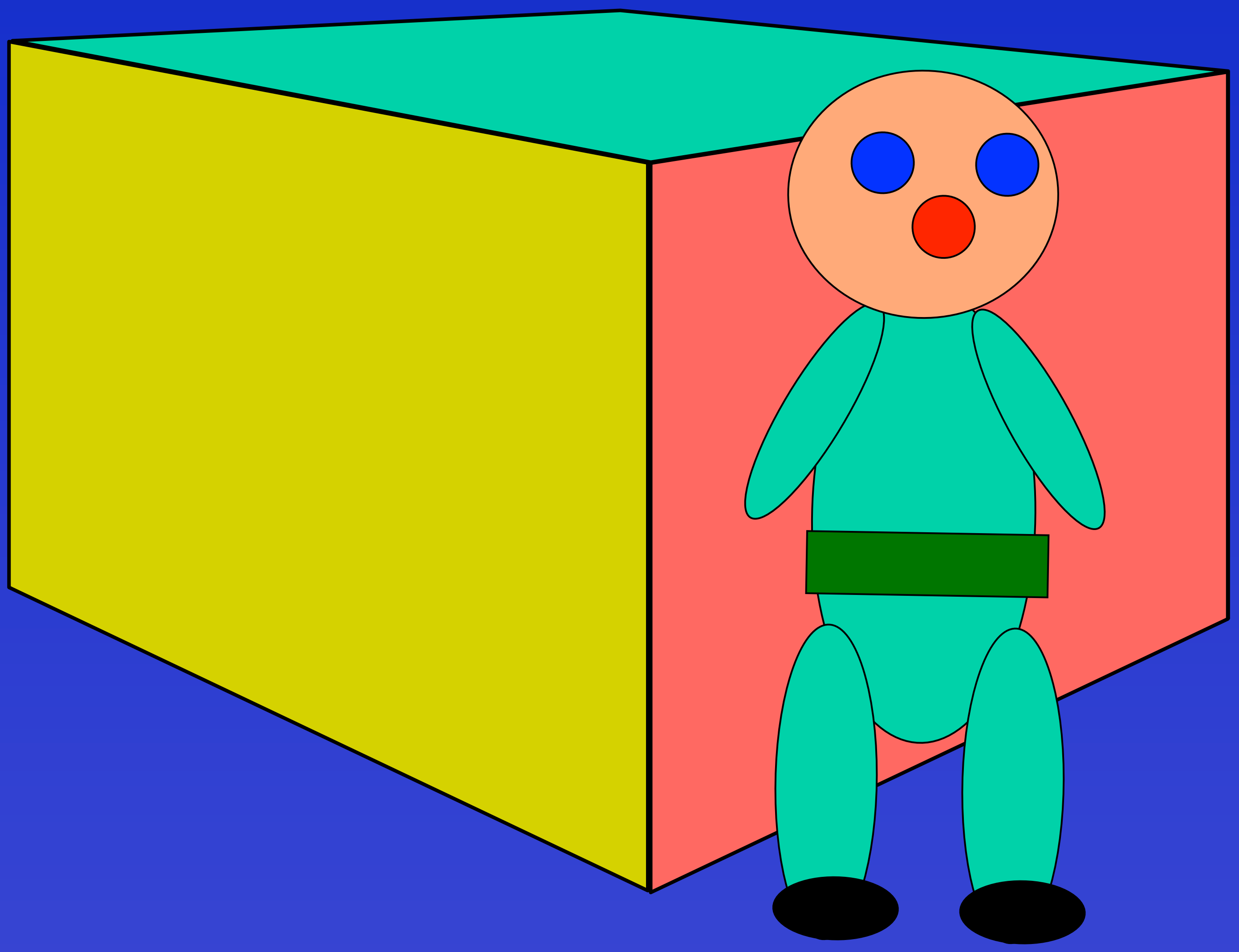
Painter's Algorithm



Painter's Algorithm



Painter's Algorithm



The lost z problem

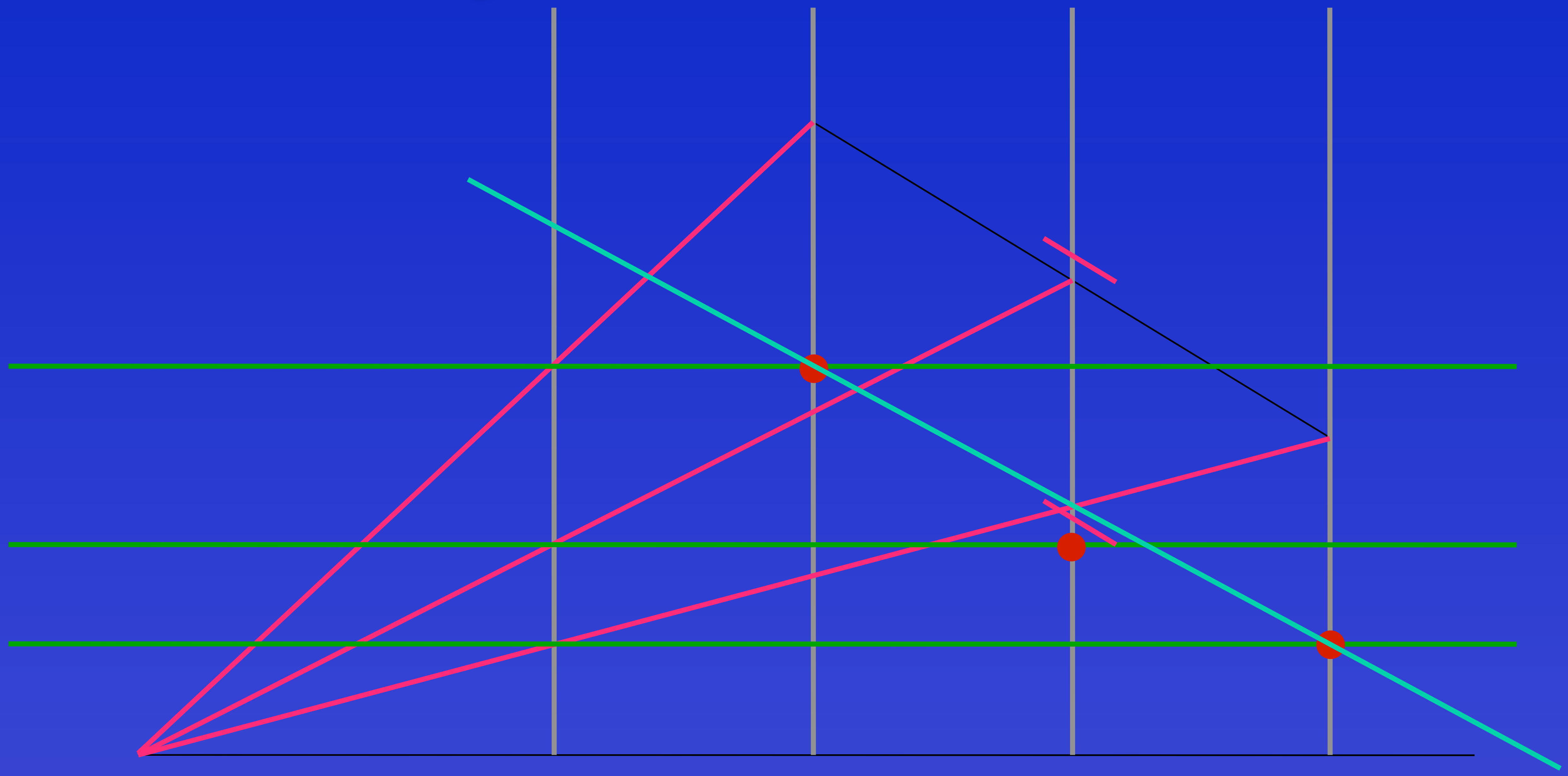
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z \end{bmatrix} = (x/z, y/z, v)$$

- All ‘z’ values end up on the viewplane!
- OK... so keep old ‘z’ value? maybe?

Old Hearn & Baker:

- "... where the original z-coordinate value would be retained in projection coordinates for visible surface and other depth processing."
- This has been fixed in the new edition.

Why it doesn’t work



Moral:
Don’t believe all you read