Ray Tracing


QUARTZ

The Basic Process

- *Create an image plane and viewpoint.*
- *For each pixel trace a ‘ray’ from the eye through a corresponding point in the image plane.*
- *For each ray, return the colour of the object at the hit point.*

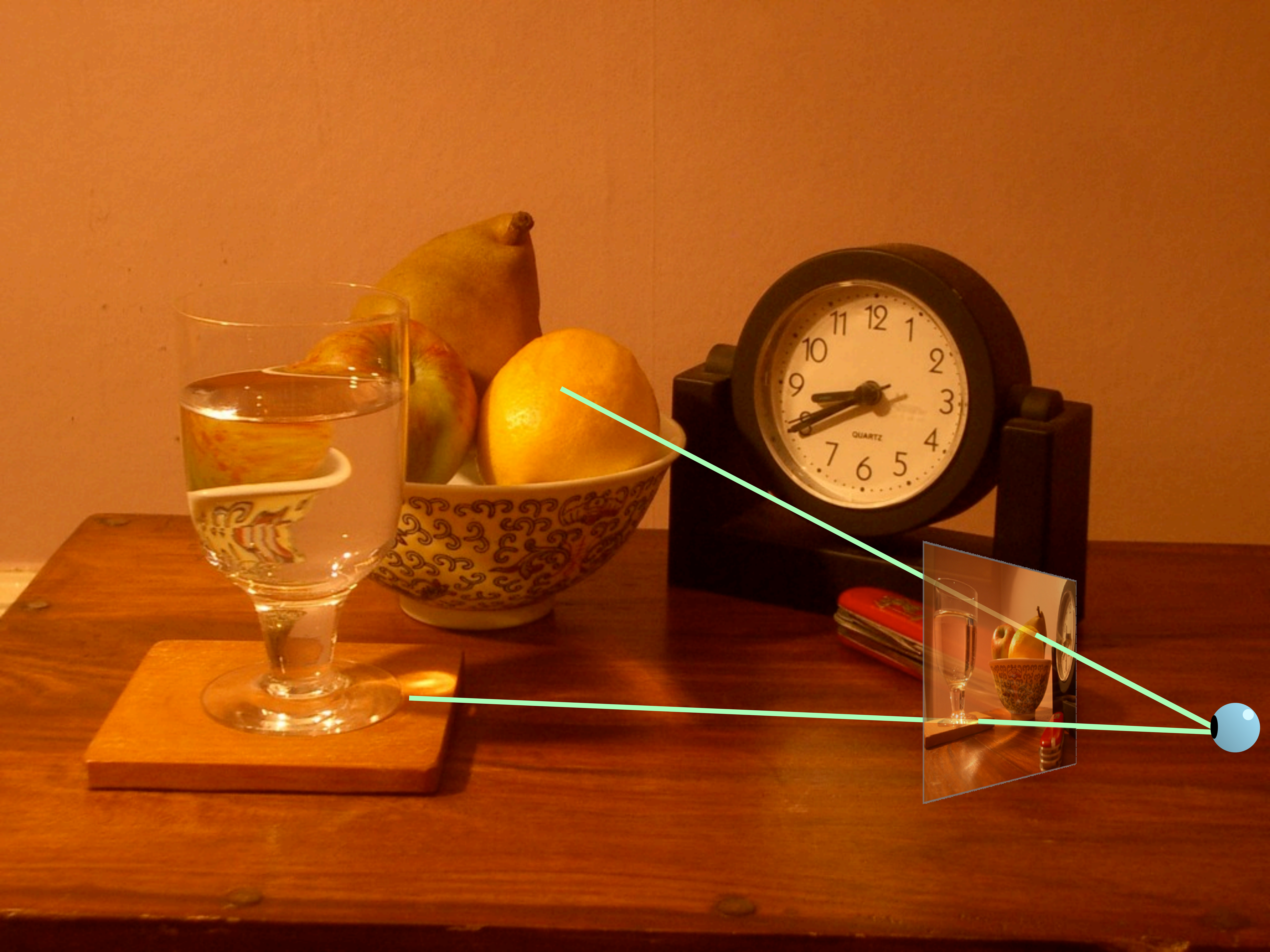
The Basic Process

- *Create an image plane and viewpoint*.
- *For each pixel trace a 'ray' from the eye through a corresponding point in the image plane*.
- *For each ray, return the colour of the object at the hit point*.


QUARTZ

The Basic Process

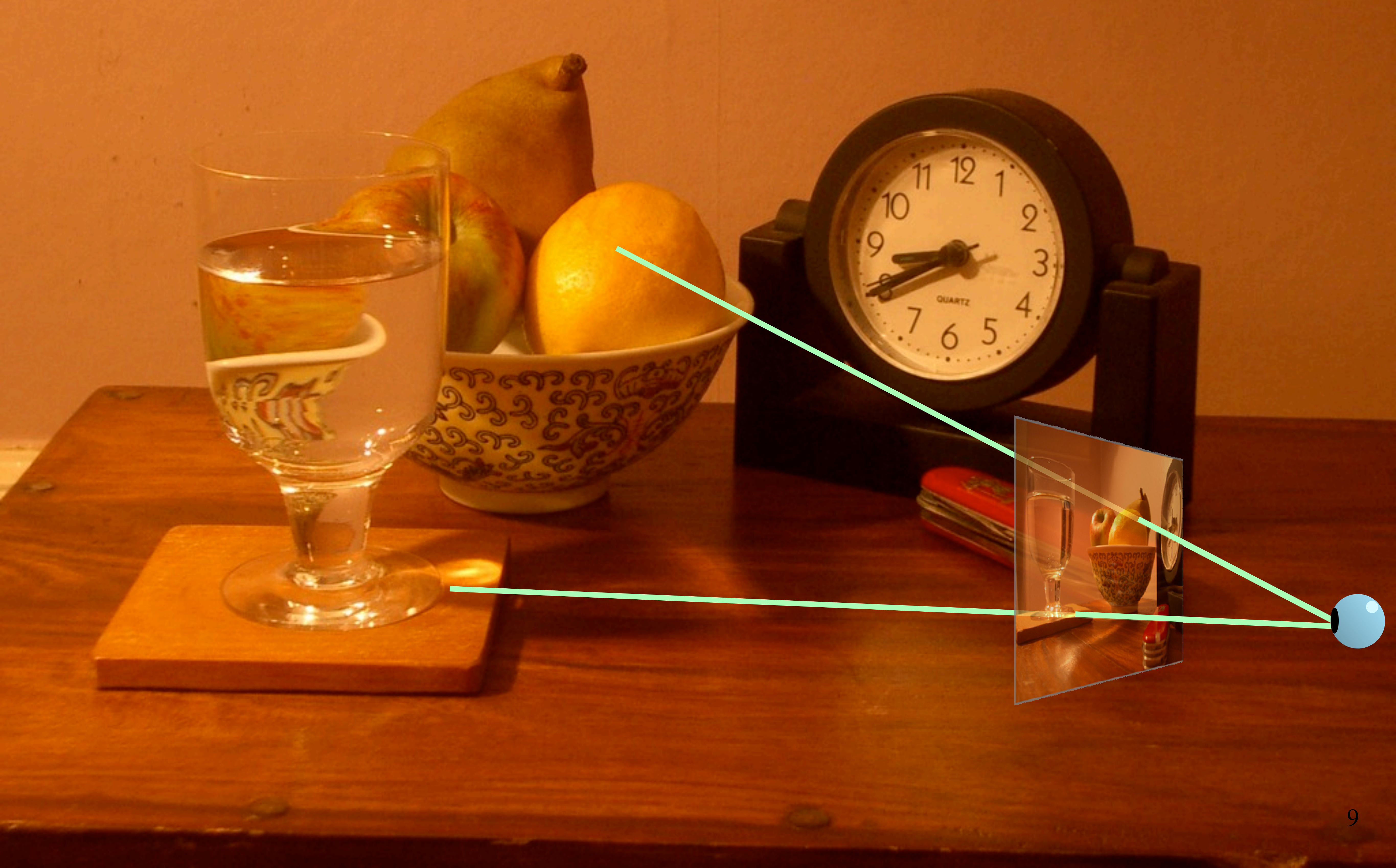
- *Create an image plane and viewpoint.*
- *For each pixel trace a 'ray' from the eye through a corresponding point in the image plane.*
- *For each ray, return the colour of the object at the hit point.*


QUARTZ

The Basic Process

- *Create an image plane and viewpoint.*
- *For each pixel trace a ‘ray’ from the eye through a corresponding point in the image plane.*
- *For each ray, return the colour of the object at the hit point.*

The hit point?



The Hit Point

- *For each ray we have to find what we can see in that direction.*
- *So we need to know which object the ray hits first.*
- *Objects are represented mathematically.*
- *We must find where a ray intersects an object's surface.*

Intersection of line and sphere

- *Why sphere?*
- *How do we represent a line?*
- *How do we represent a sphere?*
- *How do we find the intersection point?*

Sphere is the simplest

Given a point $\mathbf{p} = (x, y, z)$

Its distance from the origin is:

$\mathbf{p}^2 = \mathbf{p} \bullet \mathbf{p} = (x^2 + y^2 + z^2)$

$\mathbf{p}^2 = \mathbf{p} \bullet \mathbf{p} = (x^2 + y^2 + z^2) = 1.0$

describes a sphere of radius 1.0 centred at the origin.

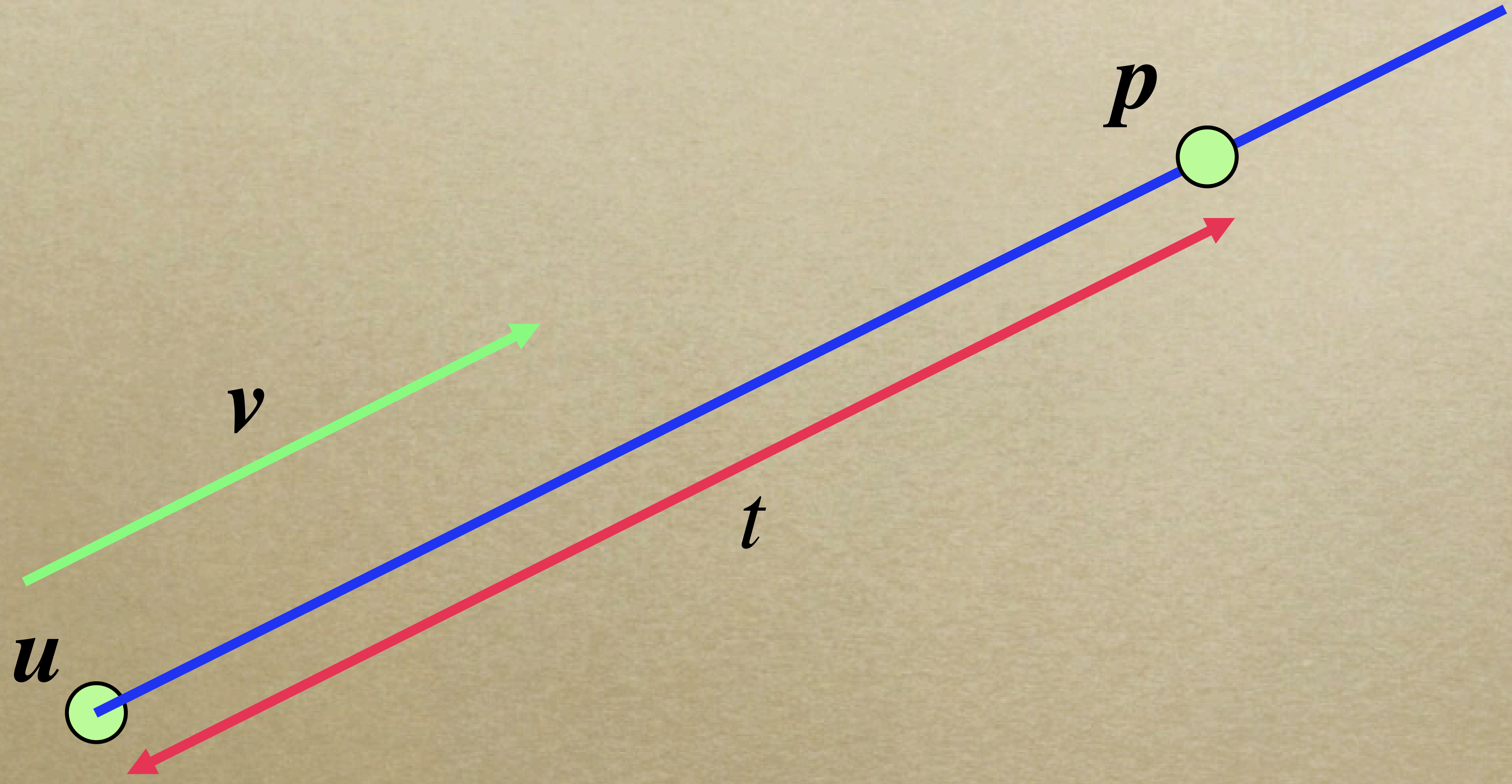
A line ...

We can describe a line using a starting point, $\boldsymbol{u}$ *and direction* $\boldsymbol{v}$*:*

$$\boldsymbol{p} = \boldsymbol{u} + \boldsymbol{v}\, t$$

If $\boldsymbol{v}$ *is a unit vector, we can think of t as the distance along the line.*

$$\mathbf{p} = \mathbf{u} + \mathbf{v}\,t$$



If **p** is on the line and the sphere?

$\boldsymbol{p}^2 = \boldsymbol{p} \cdot \boldsymbol{p} = (\boldsymbol{u} + \boldsymbol{v}\, t)^2 = 1.0$

$(\boldsymbol{u} + \boldsymbol{v}\, t) \cdot (\boldsymbol{u} + \boldsymbol{v}\, t) = \boldsymbol{v}^2\, t^2 + 2\, \boldsymbol{u} \cdot \boldsymbol{v}\, t + \boldsymbol{u}^2$

so:

$\boldsymbol{v}^2\, t^2 + 2\, \boldsymbol{u} \cdot \boldsymbol{v}\, t + \boldsymbol{u}^2 = 1.0$

This is just an ordinary quadratic equation in t.

Quadratic Solution

$\mathbf{v}^2 t^2 + 2\, \mathbf{u} \bullet \mathbf{v}\, t + \mathbf{u}^2 = 1.0$

$A = \mathbf{v}^2$

$B = 2\, \mathbf{u} \bullet \mathbf{v}$

$C = \mathbf{u}^2 - 1.0$

$t = (-B \pm \sqrt{(B^2 - 4AC)}) / (2A)$

Avoid Rounding errors

$if\ B>0,\ t_1 = (-B - \sqrt{(B^2 - 4AC)}) / (2A)$

$else\ t_1 = (-B + \sqrt{(B^2 - 4AC)}) / (2A)$

$t_2 = C/(A\ t_1)$

Explaining the 'odd' t_2 equation

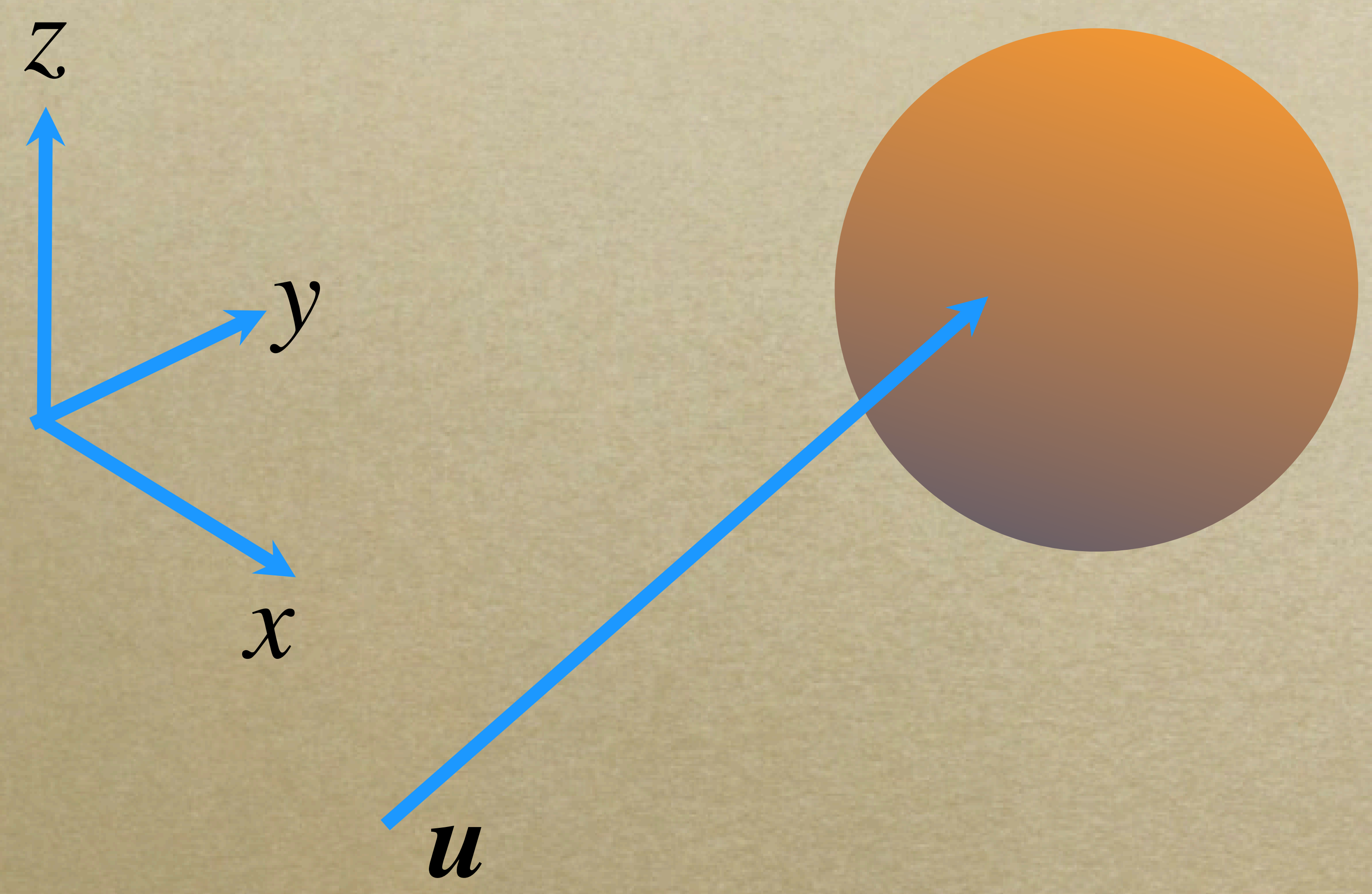
$(t - t_1)(t - t_2) = t^2 - (t_1 + t_2)t + t_1 t_2$

In $At^2 + Bt + C = 0$

$t^2 + (B/A)t + C/A = 0$

and: $t_1 t_2 = C/A$

Sphere not at origin?



Move the ray not the sphere

Sphere at $\mathbf{c} = (c_x, c_y, c_z)$

Use $(\mathbf{u} - \mathbf{c}) + \mathbf{v}\, t$

So new $\mathbf{u}' = \mathbf{u} - \mathbf{c} = (u_x, u_y, u_z) - (c_x, c_y, c_z)$

***Key point**: solving for t in the transformed ray case gives valid t solutions for the original ray equation*

General transformed sphere

Suppose the sphere has been magnified, stretched, rotated and shifted.

That's just one transformation matrix, M

$\boldsymbol{u}' = M^{-1}\,\boldsymbol{u}$

$\boldsymbol{v}' = M^{-1}\,\boldsymbol{v}$

Here is real cunning...

$$\boldsymbol{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \\ 1 \end{bmatrix} \qquad \boldsymbol{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}$$

*M^{-1} **u** includes shift but M^{-1} **v** doesn't*

Next...

Ray tracing triangles