More Ray Tracing: Ray/triangle intersections & basic diffuse illumination

Reminder

- *Create an image plane and viewpoint.*
- *For each pixel trace a 'ray' from the eye through a corresponding point in the image plane.*
- *For each ray, return the colour of the object at the hit point.*

Where are we?

- *Did ray intersection with a sphere*
- *Now do ray intersection with a triangle*

- *Then we'll talk about getting the colour*

Ray triangle intersection

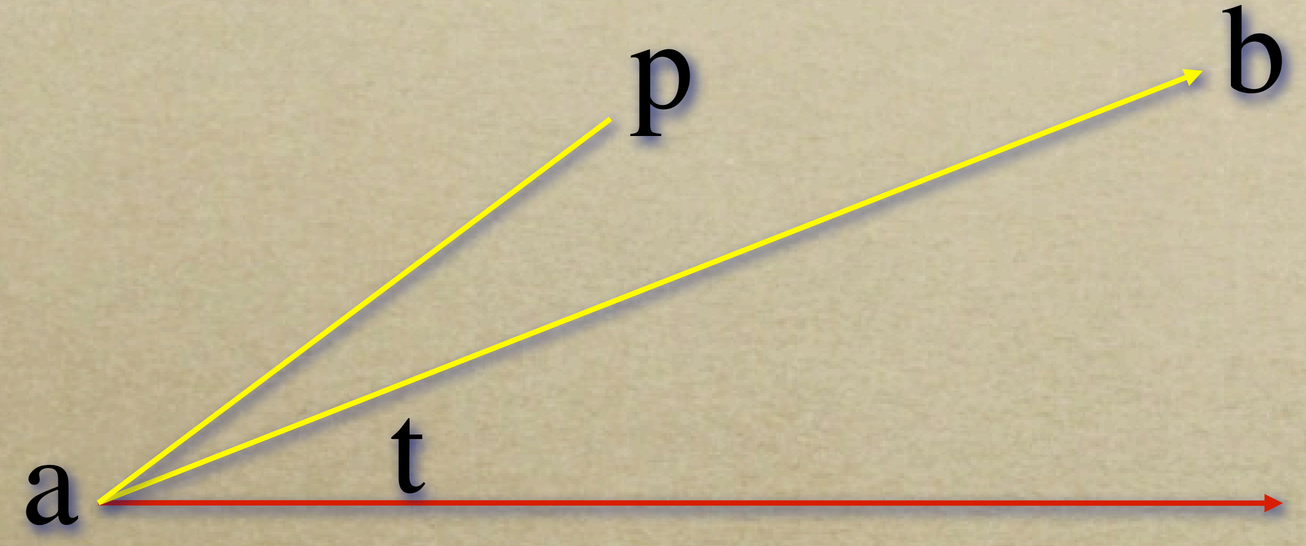
- *A triangle can be represented by three vertices:* $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$
- *The ray is represented as before as a line:* $\boldsymbol{p} = \boldsymbol{u} + \boldsymbol{v}\, t$

Using cross products

Recollect that the cross product of two vectors in 3D is another vector perpendicular to the original two.

The cross product of two parallel vectors is (0, 0, 0)

Remember this figure?



p *is on the left of* ab *if and only if:*

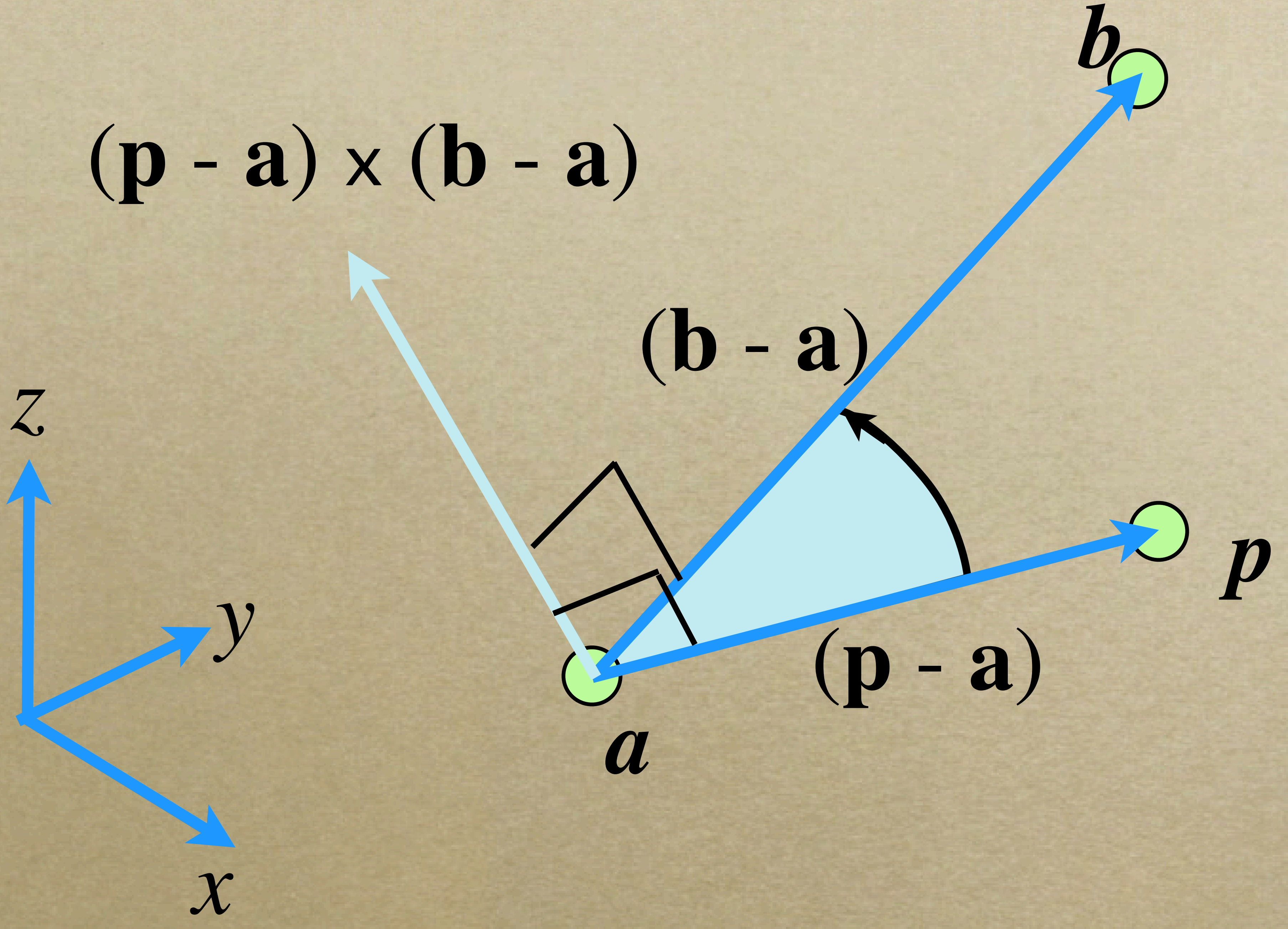
$$-(p_x - a_x)\,(b_y - a_y) + (p_y - a_y)\,(b_x - a_x) > 0$$

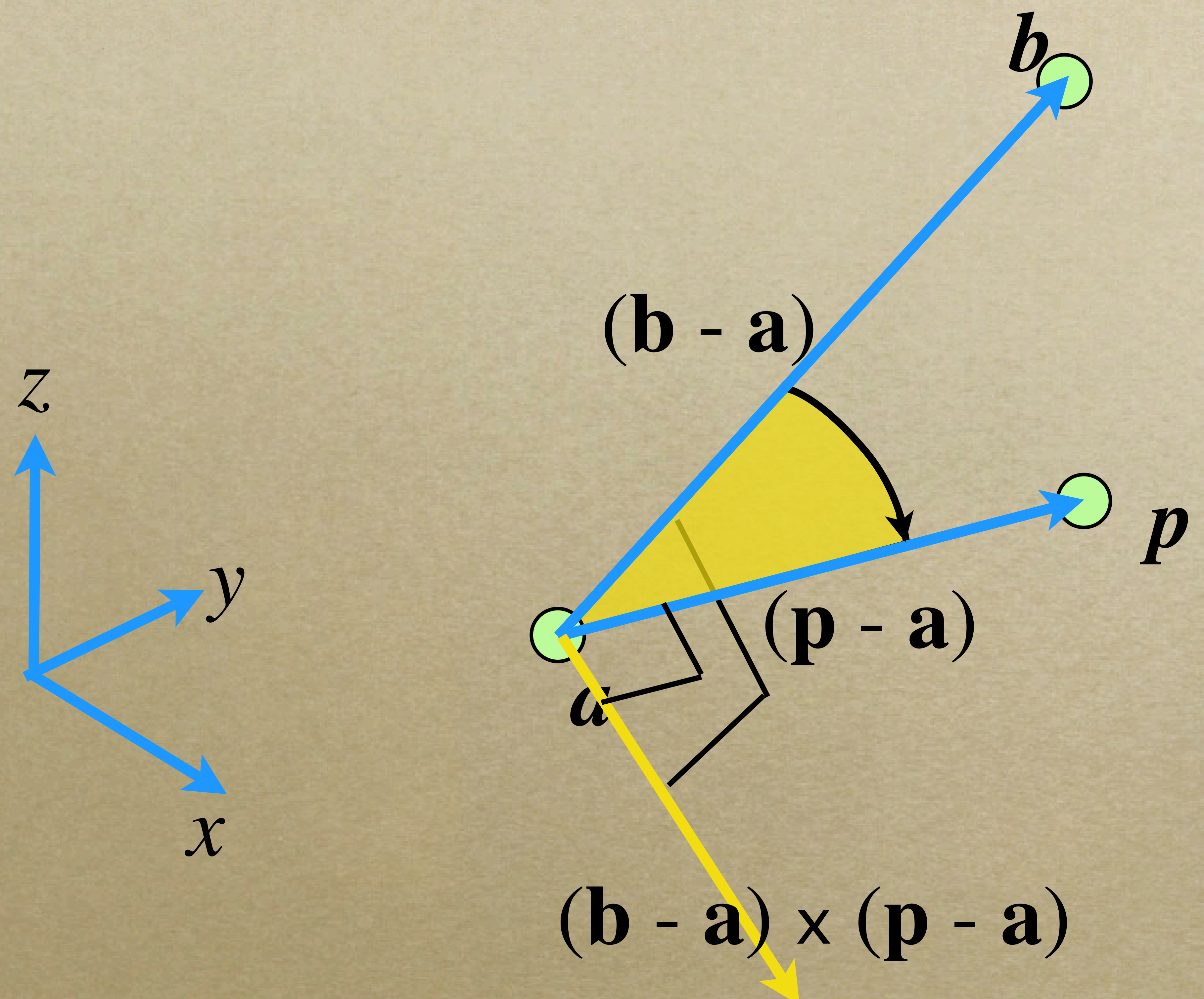
$-(p_x - a_x)(b_y - a_y) + (p_y - a_y)(b_x - a_x)$

is a cross product in 2D:

$(\mathbf{b} - \mathbf{a}) \times (\mathbf{p} - \mathbf{a})$

Now let's look at this in 3D ...


(**p** - **a**) x (**b** - **a**)
b
(**b** - **a**)
z
y
p
(**p** - **a**)
a
x



b
(b - a)
z
p
y
(p - a)
a
x
(b - a) x (p - a)

Now we are ready...

Start with a triangle $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$

Any three points lie in a plane and

$(\boldsymbol{b} - \boldsymbol{a}) \times (\boldsymbol{b} - \boldsymbol{c}) = \boldsymbol{n}$

is perpendicular to the plane and for any point, p, in the plane:

$\boldsymbol{n} \bullet (\boldsymbol{b} - \boldsymbol{p}) = 0$

Just like the sphere?

A line is still $\mathbf{p} = \mathbf{u} + \mathbf{v}\, t$

$\mathbf{n} \bullet (\mathbf{b} - \mathbf{p}) = 0$

and

$\mathbf{n} \bullet (\mathbf{b} - \mathbf{u} - \mathbf{v}\, t) = 0$

$\mathbf{n} \bullet (\mathbf{b} - \mathbf{u}) - \mathbf{n} \bullet \mathbf{v}\, t = 0$

$t = \mathbf{n} \bullet (\mathbf{b} - \mathbf{u}) \,/\, \mathbf{n} \bullet \mathbf{v}$

What does it mean?

$t = \boldsymbol{n} \bullet (\boldsymbol{b} - \boldsymbol{u}) \,/\, \boldsymbol{n} \bullet \boldsymbol{v}$

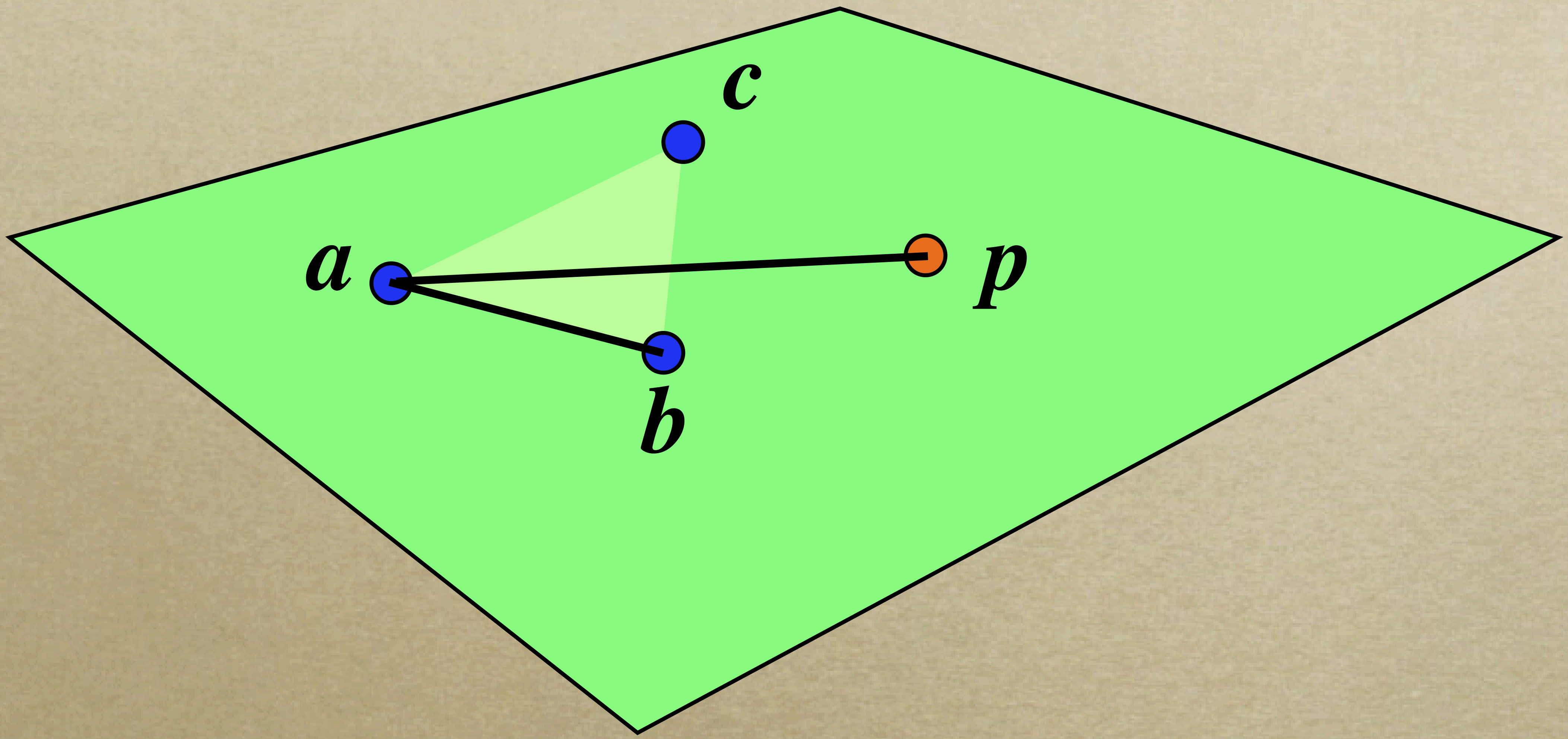
gives us a value for t and

$\boldsymbol{p} = \boldsymbol{u} + \boldsymbol{v}\, t$

is the point where the ray intersects the plane. What if $\boldsymbol{n} \bullet \boldsymbol{v} = 0$ *?*

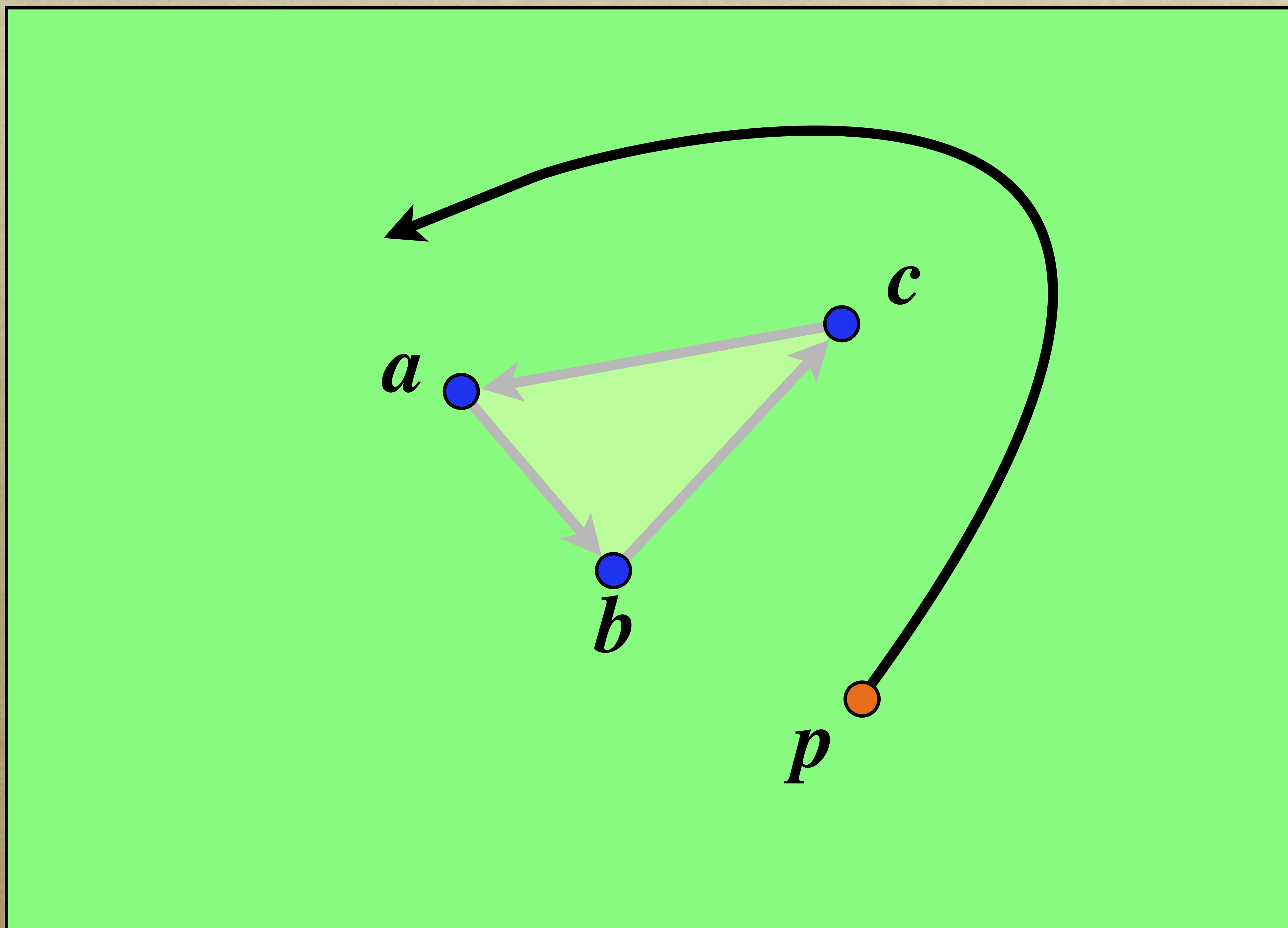
What if $\boldsymbol{b} = \boldsymbol{u}$ *?*

Have we hit the triangle?

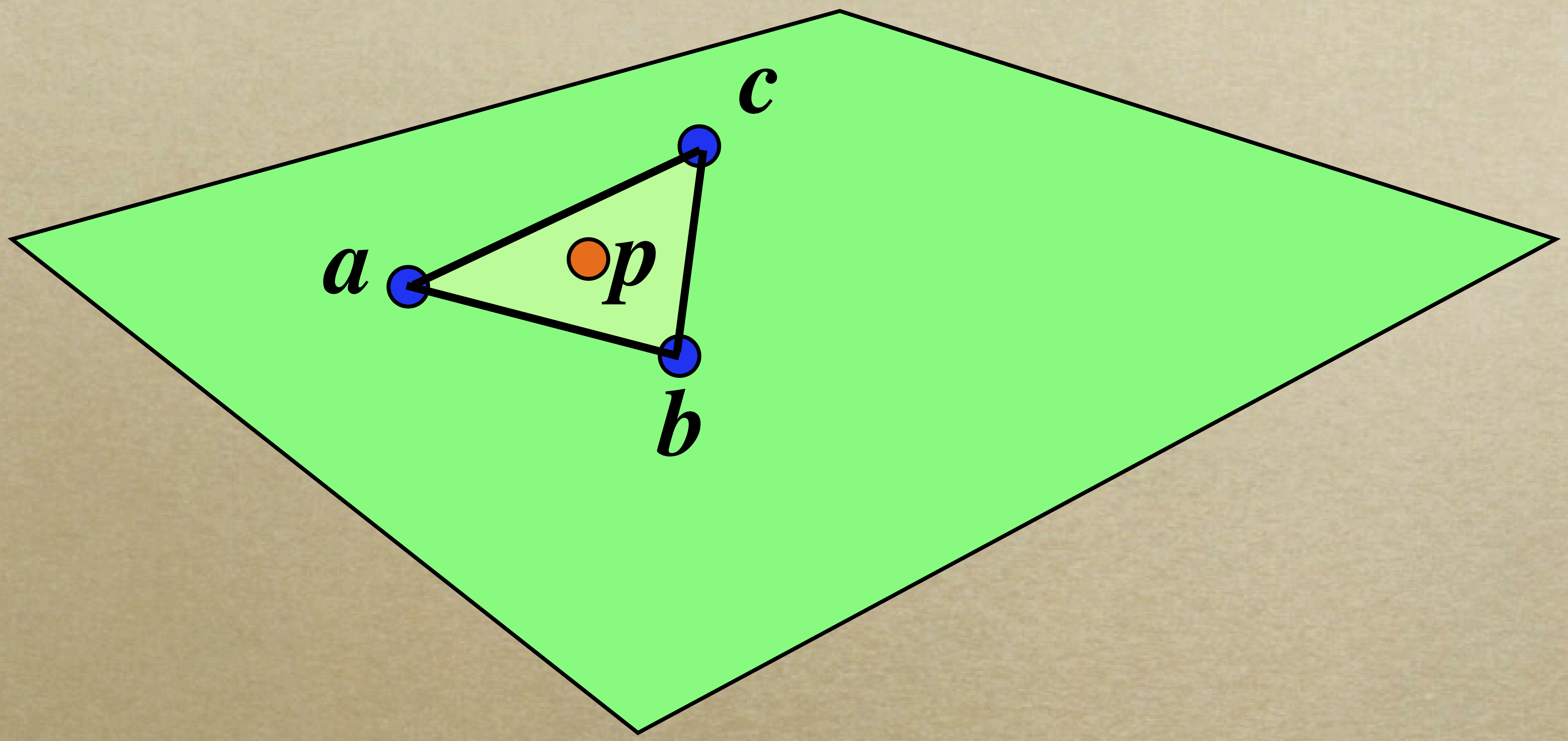


$(\boldsymbol{b} - \boldsymbol{a}) \times (\boldsymbol{p} - \boldsymbol{a})$ *either up or down*

Driving round a paddock



Have we hit the triangle?



all up or all down means ***p*** *is inside*

Inside and outside

$(\boldsymbol{b} - \boldsymbol{a}) \times (\boldsymbol{p} - \boldsymbol{a}), (\boldsymbol{c} - \boldsymbol{b}) \times (\boldsymbol{p} - \boldsymbol{b}), (\boldsymbol{a} - \boldsymbol{c}) \times (\boldsymbol{p} - \boldsymbol{c})$

all up or all down means p is inside a, b, c

$(\boldsymbol{b} - \boldsymbol{a}) \times (\boldsymbol{p} - \boldsymbol{a}) \bullet \boldsymbol{n},$

$(\boldsymbol{c} - \boldsymbol{b}) \times (\boldsymbol{p} - \boldsymbol{b}) \bullet \boldsymbol{n},$

$(\boldsymbol{a} - \boldsymbol{c}) \times (\boldsymbol{p} - \boldsymbol{c}) \bullet \boldsymbol{n}$

all >0 or all <0

Harder than a sphere?

- *Yes, and it takes longer to compute.*
- *Once you can intersect triangles, you can ray trace thousands of published models or build your own with Blender etc.*
- *There are other kinds of model too but now we move on...*

The Basic Process

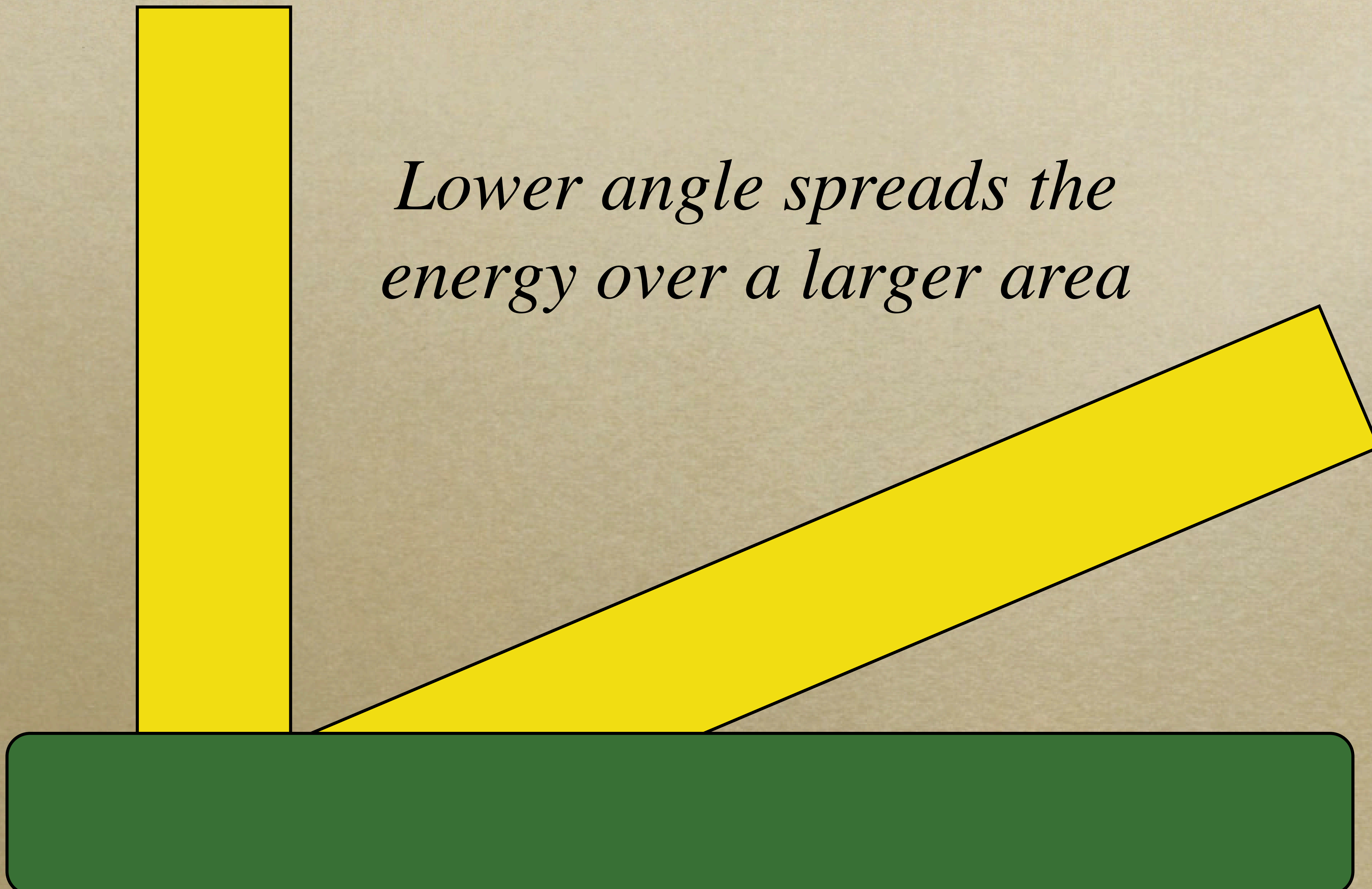
- *Create an image plane and viewpoint.*
- *For each pixel trace a ‘ray’ from the eye through a corresponding point in the image plane.*
- *For each ray, return the colour of the object at the hit point.*

What colour do I see?

- *What do you see in the dark?*
- *What does a yellow object look like in red light?*
- *What determines how bright something looks?*
- *What is the brightest colour?*
- *Why is it that winter is colder than summer?*


QUARTZ

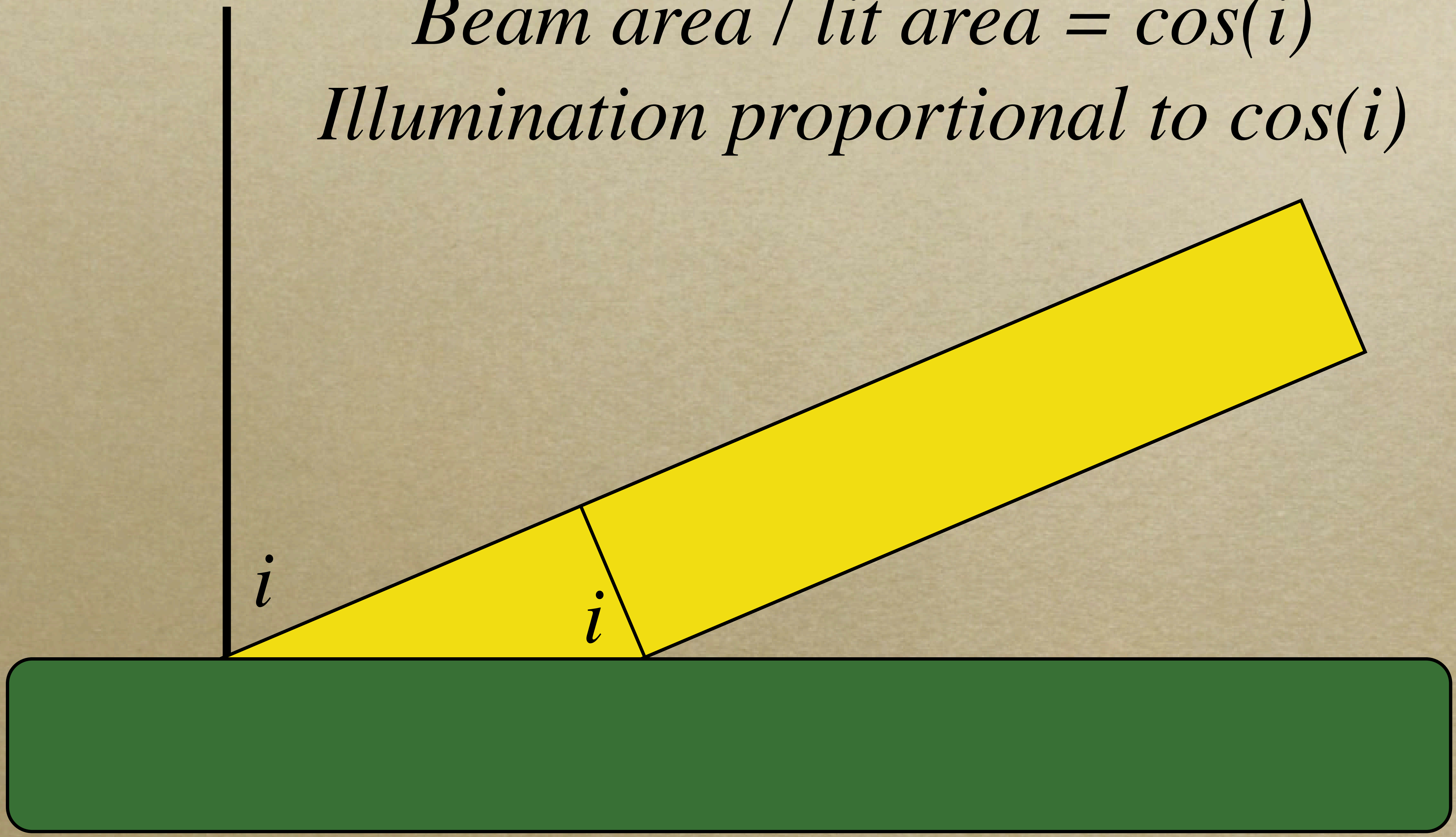
Most Basic Illumination



Lambert's Law

Beam area / lit area = cos(i)

Illumination proportional to cos(i)



But where does the energy go?

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Lambertian Illumination

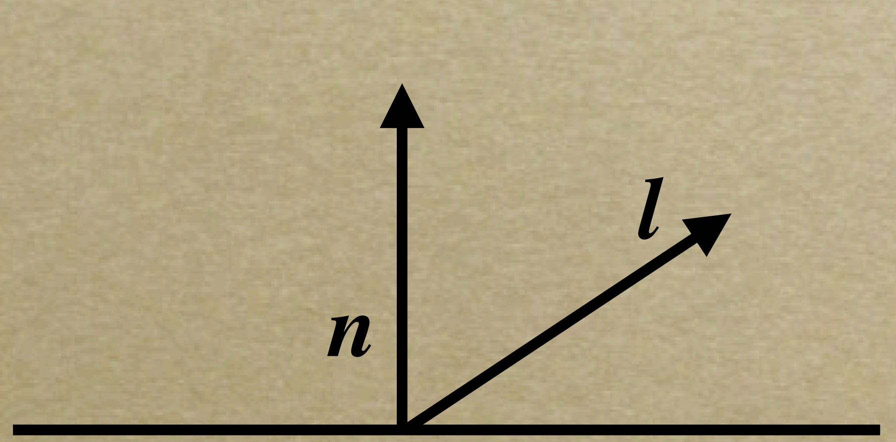
For a perfectly diffuse surface the luminous intensity is independent of the angle of view.

E.g: Chalk or tissue paper are nearly perfectly diffuse.

We can form a vector equation

If the incident light has intensity, I, and is from a source in direction, $\mathbf{l}$, and the surface reflects a proportion, k, the observed brightness is:

$$kI\ \mathbf{n}\cdot\mathbf{l}$$



Is that all?

Alas no.

Most surfaces are more complicated in their reflection properties. We will look at this next time.