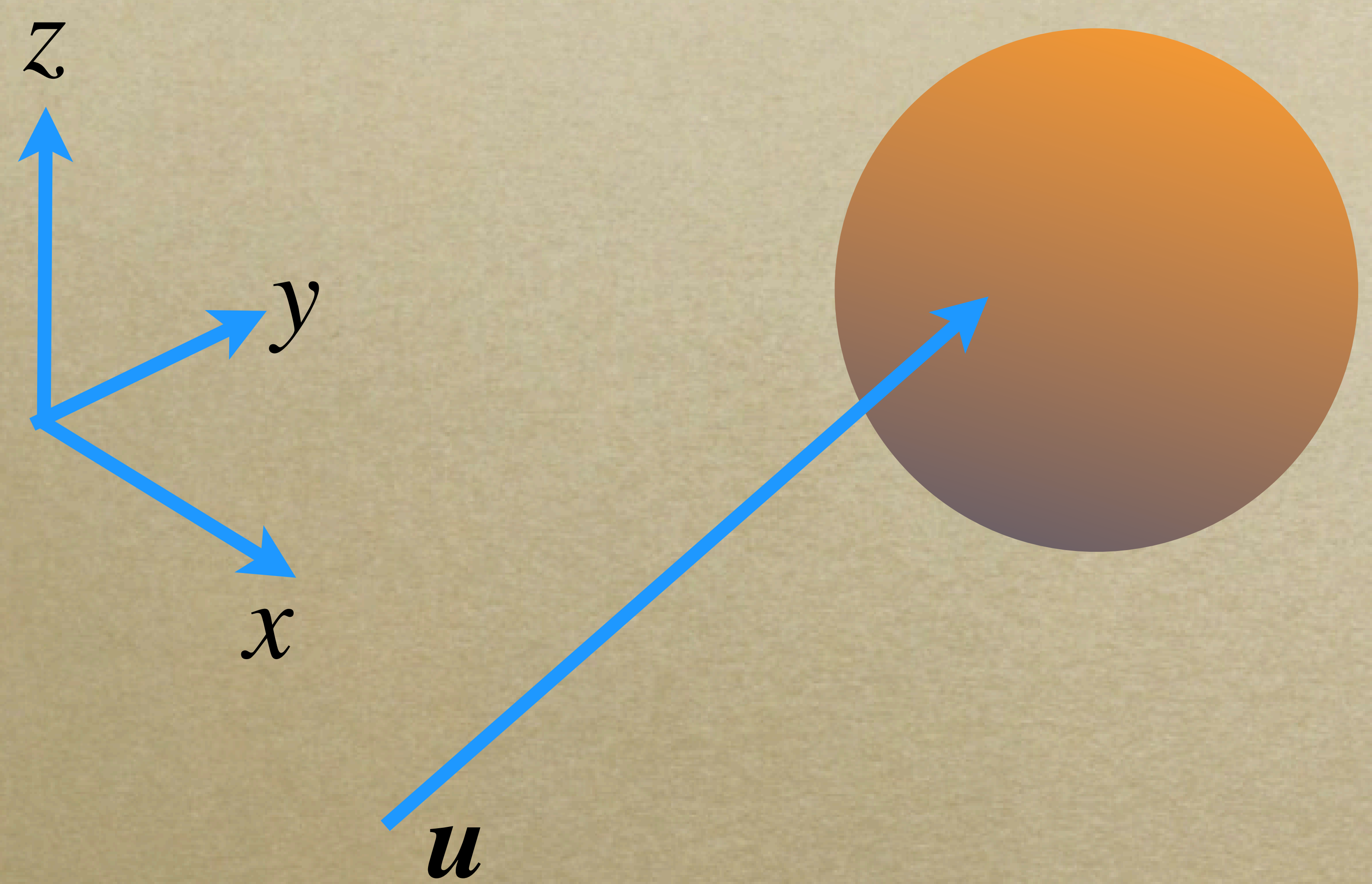
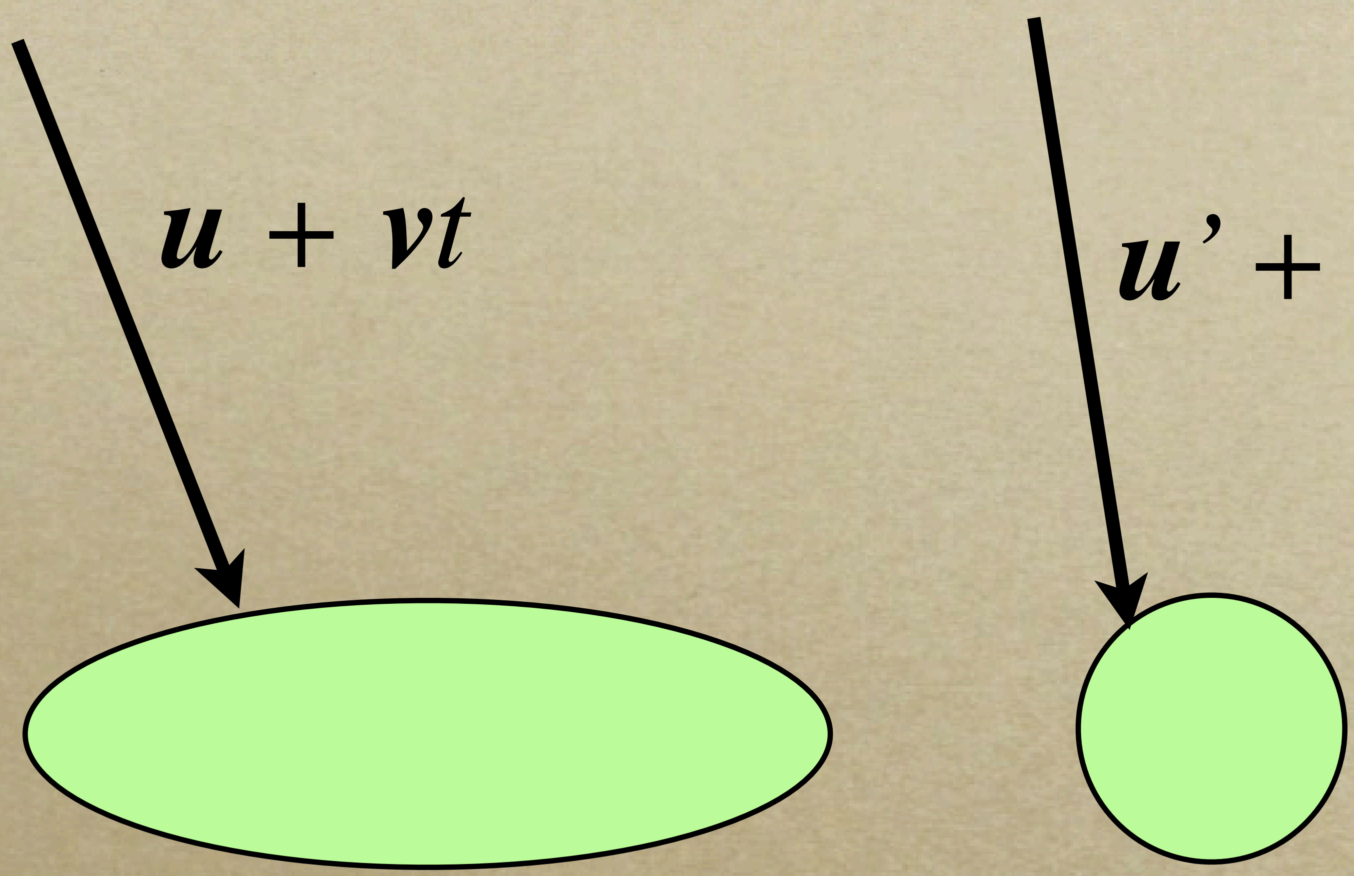
# *Efficient illumination of transformed objects and some global illumination*

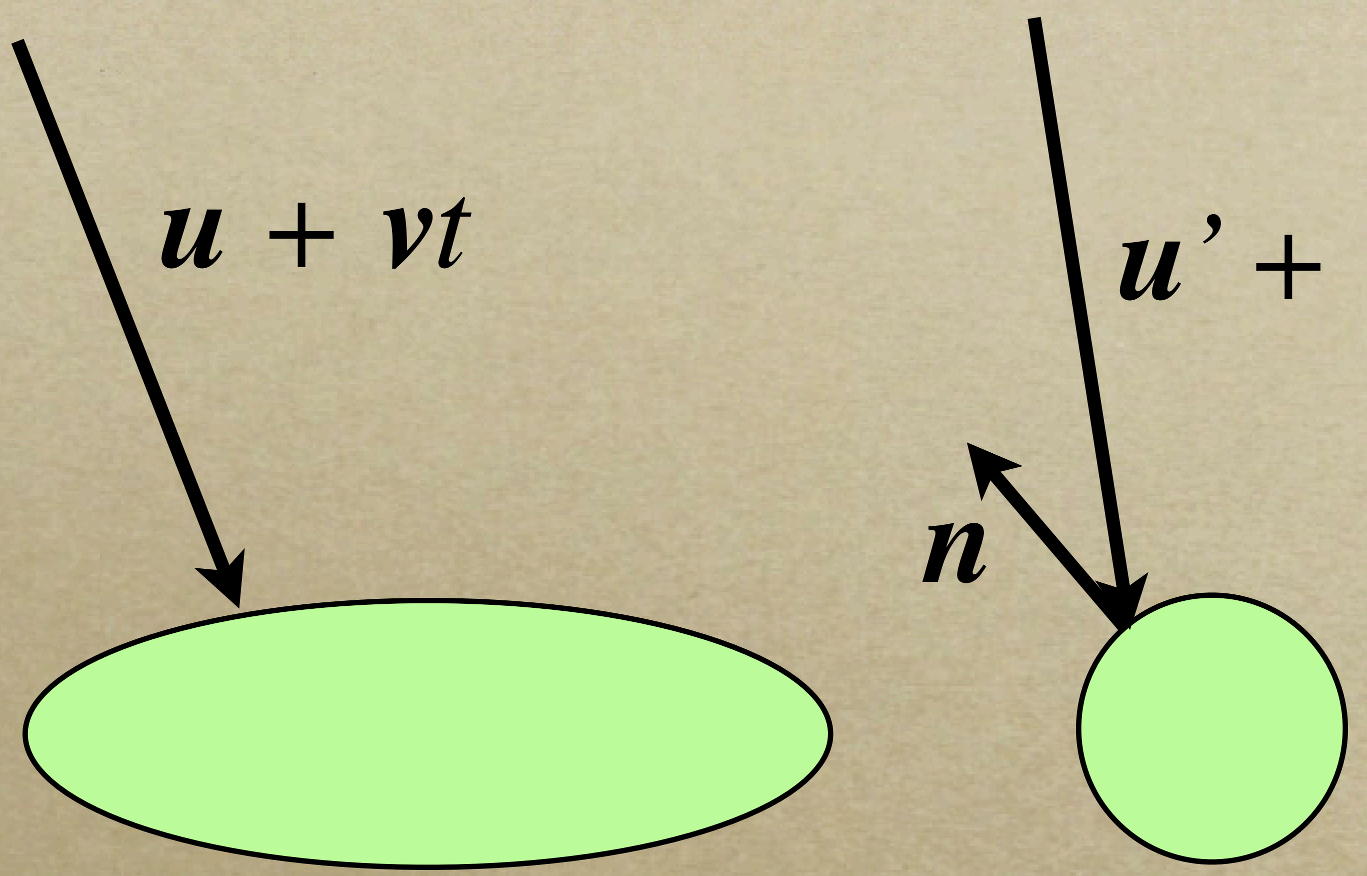
# Transformed objects



# Transform the ray but ...

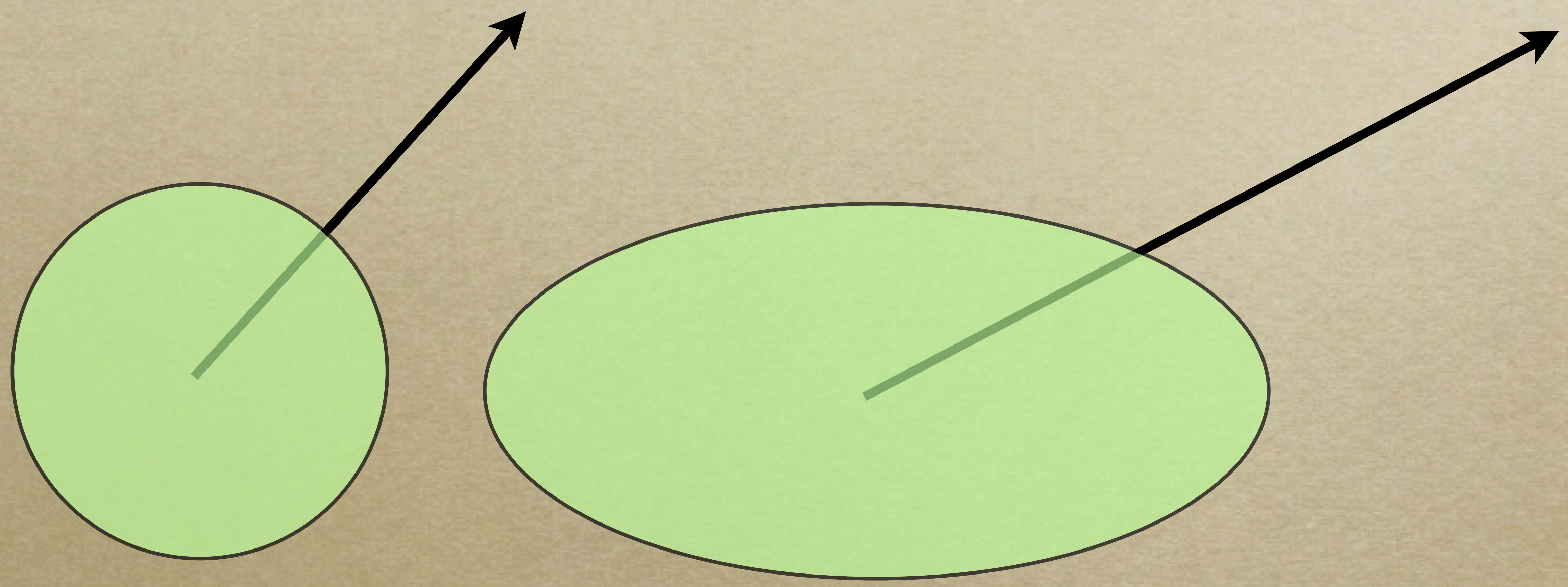


# Transform the ray but ...




*How do you get the surface normal back into world space?*

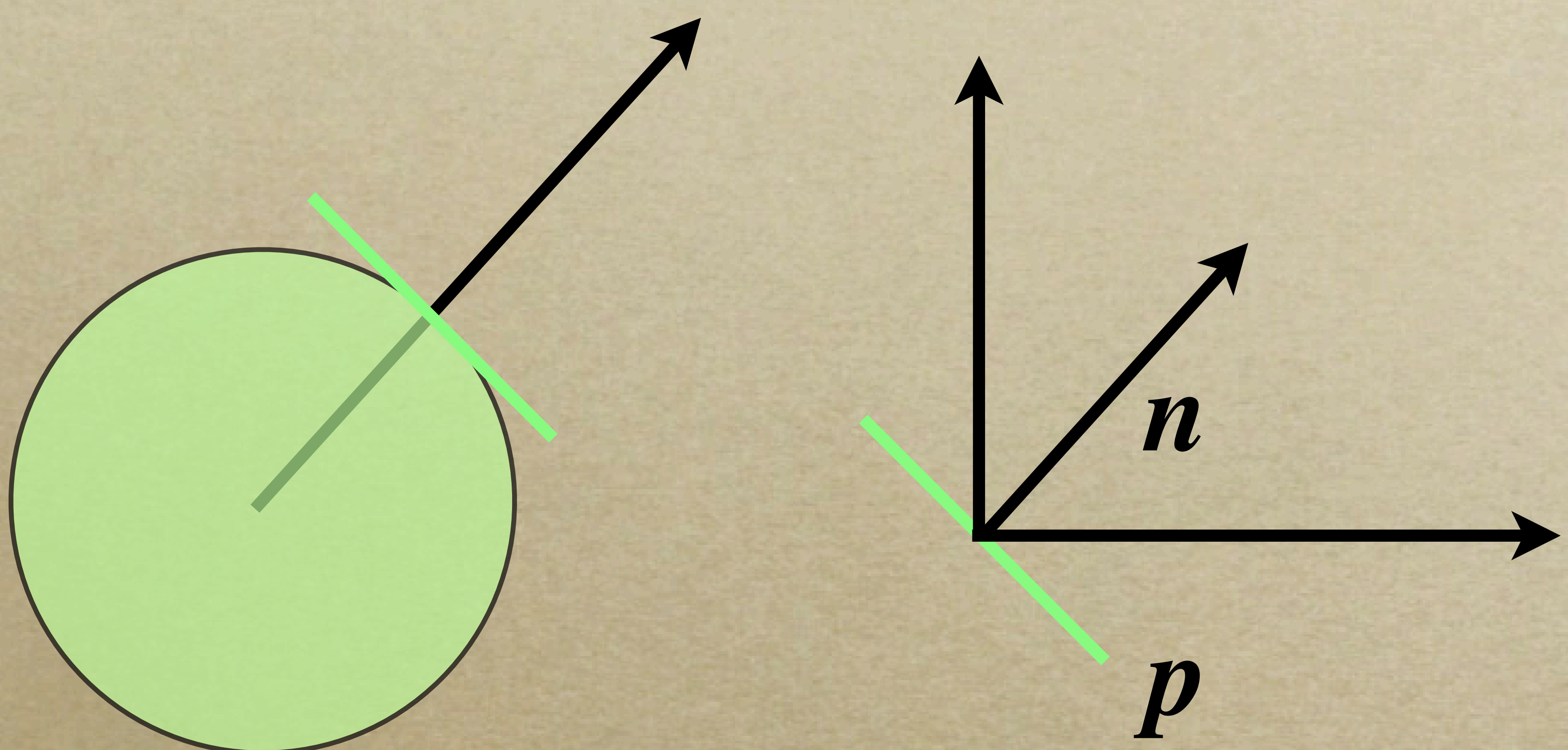
# Transformed *n* is not normal



# So what do we do?

- *When we apply a transformation matrix lines and planes are preserved but not angles.*
- *The normal defines a plane and the plane transforms to a plane.*

# Which Plane?




$\boldsymbol{p}.\boldsymbol{n} = 0$ *is a plane through the origin.*

*Suppose* $\boldsymbol{p}' = T\boldsymbol{p}$

# Dot product as matrix multiplication

$$\boldsymbol{p} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad \boldsymbol{v} = \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix}$$

$$\boldsymbol{p}^T\boldsymbol{v} = [\, x \;\; y \;\; z \;\; 1 \,] \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} = \boldsymbol{p}.\boldsymbol{v}$$

# Now we can do the maths

$\boldsymbol{p}.\boldsymbol{n} = 0$ *is a plane through the origin.*

*or in matrix terms* $\boldsymbol{p}^{\mathrm{T}}\boldsymbol{n} = 0$

*Suppose* $\boldsymbol{p}' = T\boldsymbol{p}$, *(T is any transformation)*

$\boldsymbol{p} = T^{-1}\boldsymbol{p}'$ *and* $T^{-1}\boldsymbol{p}'.\boldsymbol{n} = 0$

*So…* $T^{-1}\boldsymbol{p}'.\boldsymbol{n} = 0$ *or* $(T^{-1}\boldsymbol{p}')^{\mathrm{T}}\boldsymbol{n} = 0$

*Now* $(AB)^{\mathrm{T}} = B^{\mathrm{T}}A^{\mathrm{T}}$ *(prove that yourself)*

$(\boldsymbol{p}'^{\mathrm{T}}T^{-1\mathrm{T}})\,\boldsymbol{n} = 0$ $\quad \boldsymbol{p}'^{\mathrm{T}}\,(T^{-1\mathrm{T}}\,\boldsymbol{n}) = 0$

*i.e.:* $\boldsymbol{p}'.T^{-1\mathrm{T}}\boldsymbol{n} = 0$

# Beyond maths: see what it means

$$\mathbf{p}.\mathbf{n} = 0 \quad and \quad \mathbf{p}' = T\mathbf{p}$$

$\mathbf{p}$ *is a point on a plane with normal* $\mathbf{n}$.

$\mathbf{p}'$ *is a point on a transformed plane.*

*And we have shown that* $\mathbf{p}'.(T^{-1})^{\mathrm{T}}\mathbf{n} = 0$

*So* $\mathbf{p}'$ *is a point on a plane with normal*

$$T^{-1\mathrm{T}}\mathbf{n}$$

# And the application...

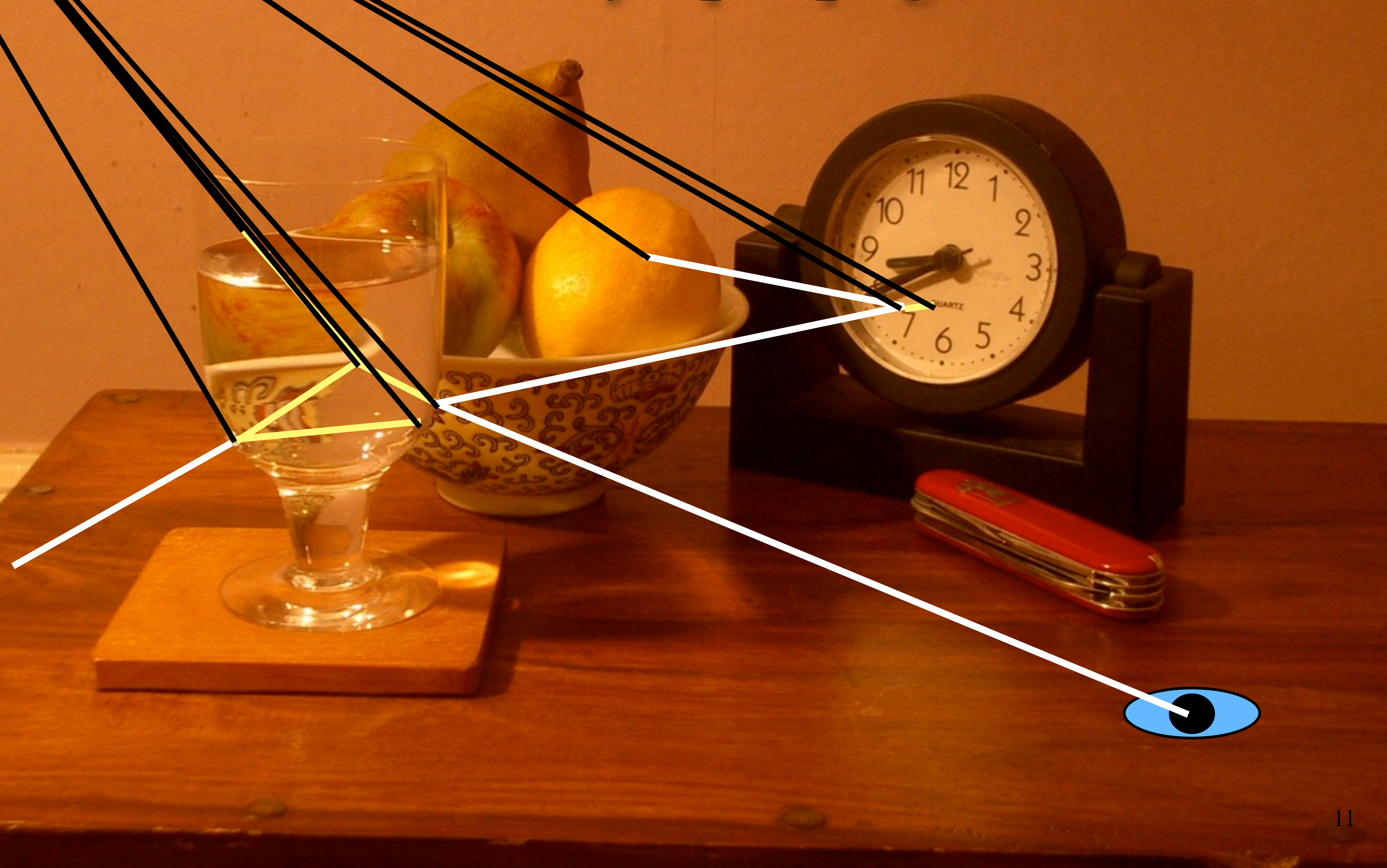
*Suppose we have an object that has been transformed by a matrix T.*

*We transform* $\mathbf{u} + \mathbf{v}t$ *by* $T^{-1}$ *and find t at the hit point, and a normal,* $\mathbf{n}$.

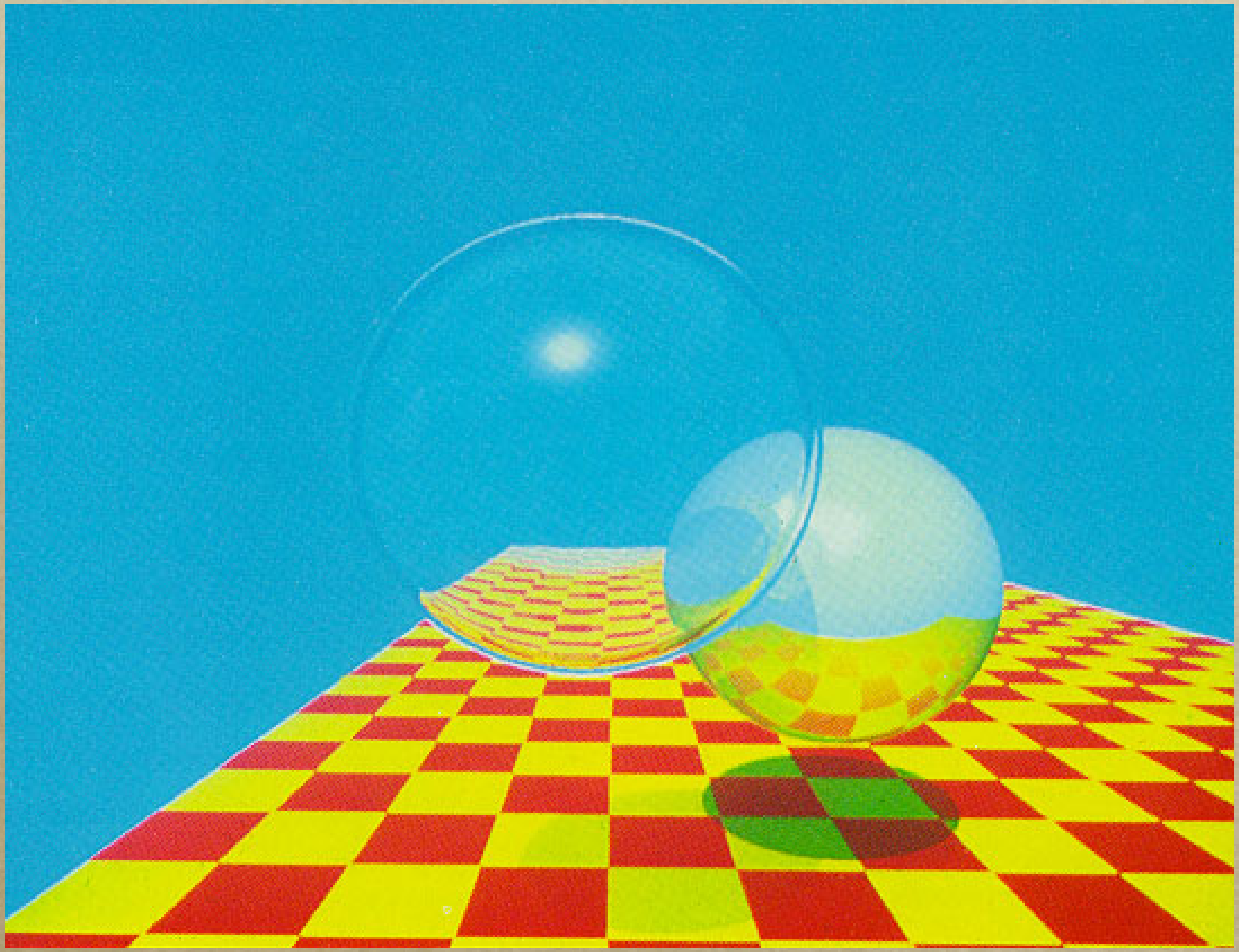
*The hit point in world space is* $\mathbf{u} + \mathbf{v}t$

*and the normal in world space is* $T^{-1\mathrm{T}}\mathbf{n}$.

# How rays propagate



# Whitted 1980



*Ambient, Lambert, Phong, reflection, refraction, point light sources.*

# Just the beginning...

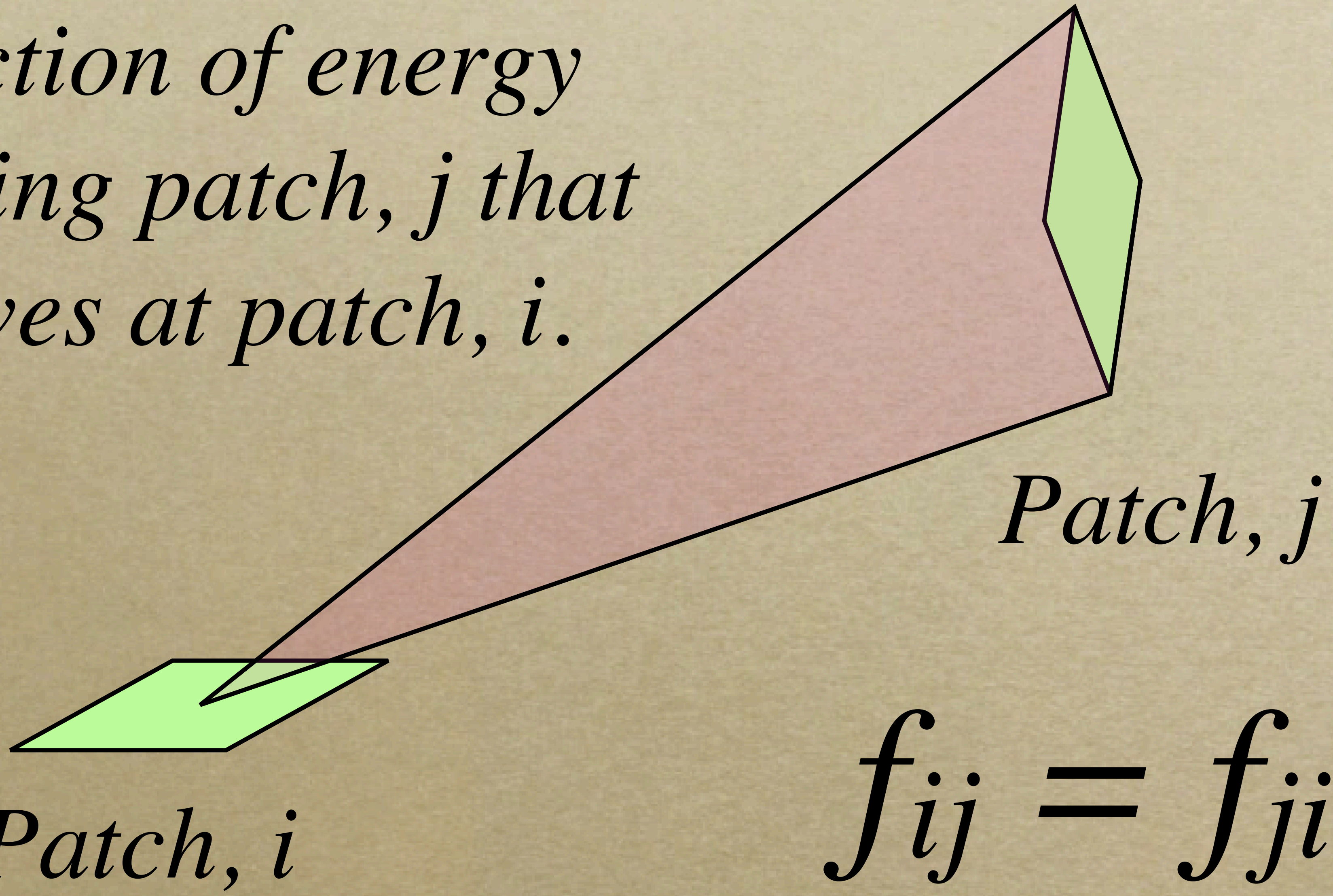
- *Aliasing artefacts*
- ***No surface/surface illumination***
- *No caustics*
- *Real shadows are soft*
- *Colour problems*
- *Very slow*

# Radiosity

- *Divide the scene into small surface patches.*
- *For every patch pair find form factor.*
- *Find radiosities*
- *Render picture*

# Form Factor, $f_{i,j}$

*Fraction of energy leaving patch, j that arrives at patch, i.*



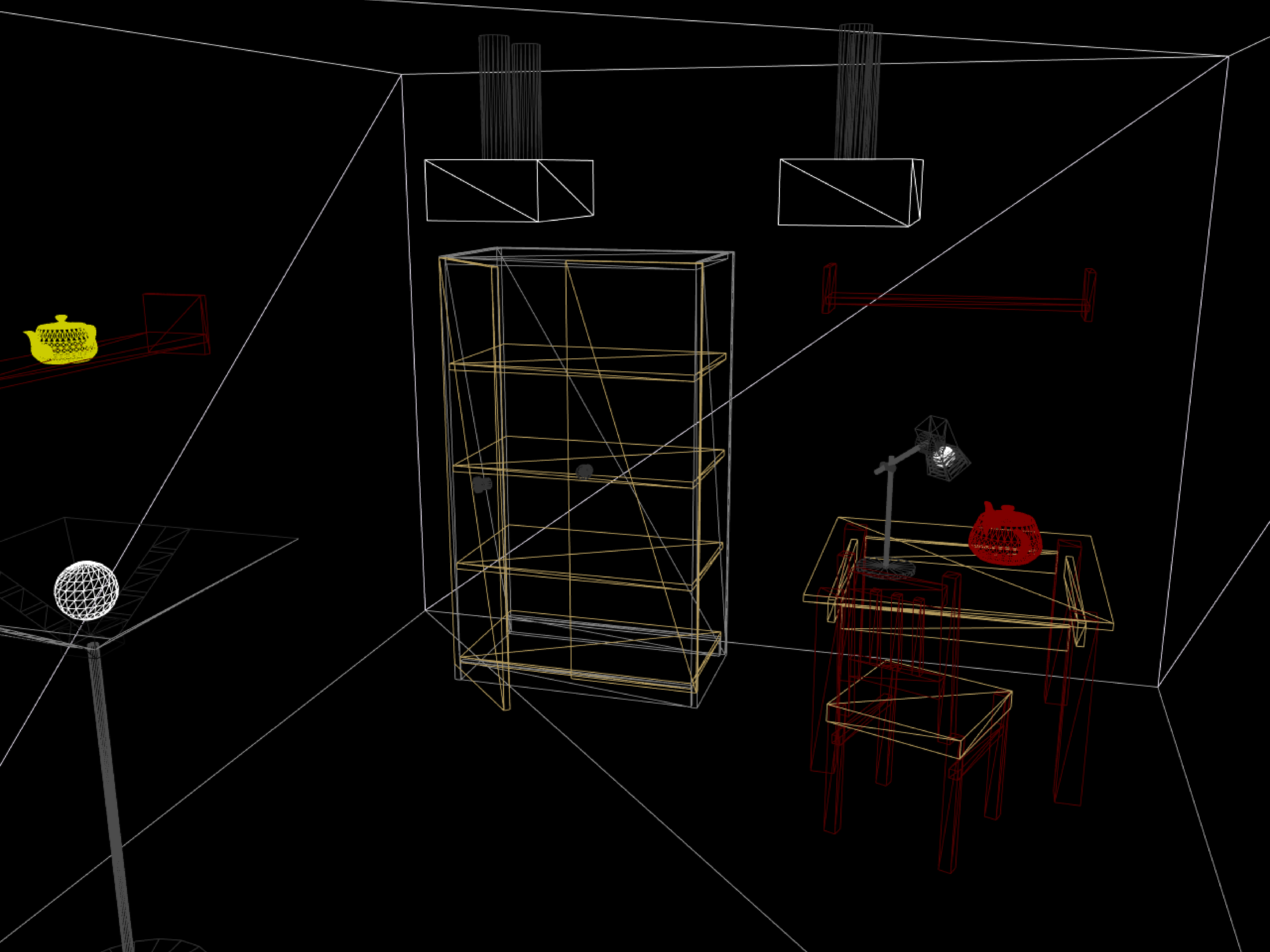

$$f_{ij} = f_{ji}$$

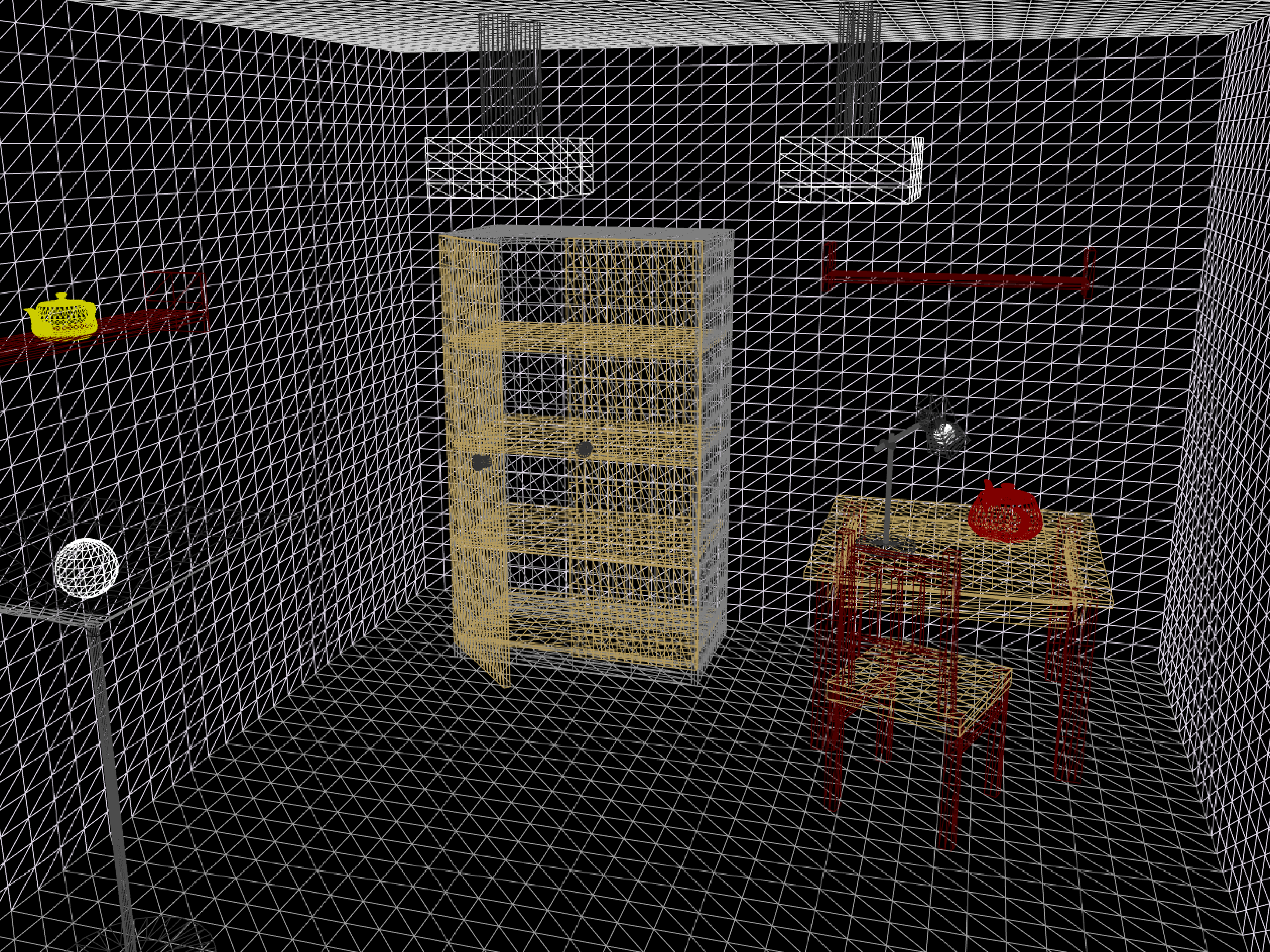
# Radiosity Equation

$$R_i = E_i + k_{d,i}(\sum f_{i,j}R_j)$$

*Set up this system of equations and solve.*

*Alternatively, don't solve whole equation in one go, instead use successive approximation (i.e. multiple passes that compute the most significant effects first).*





# Just the beginning...

- *Aliasing artefacts*
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- ***No caustics***
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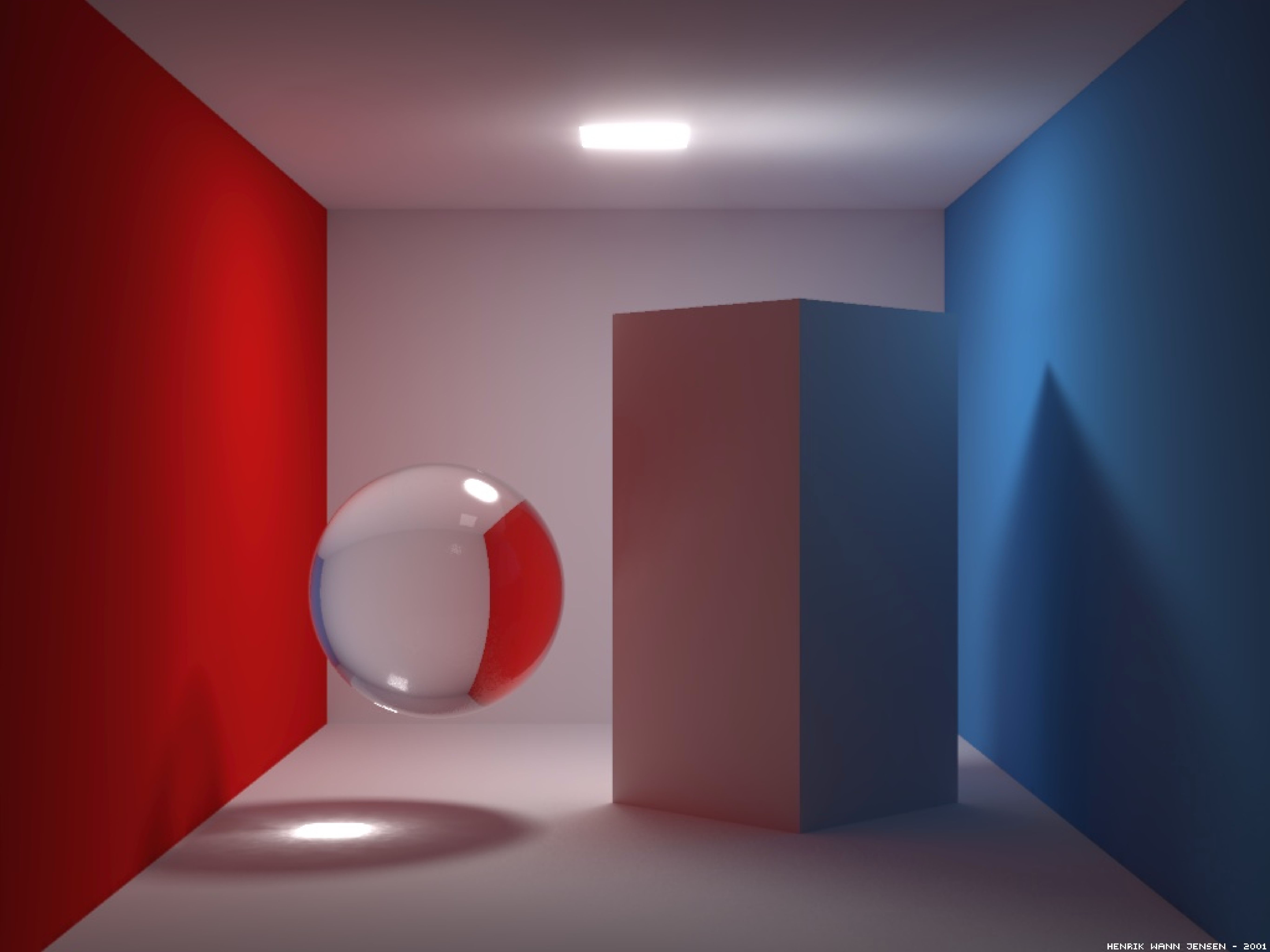
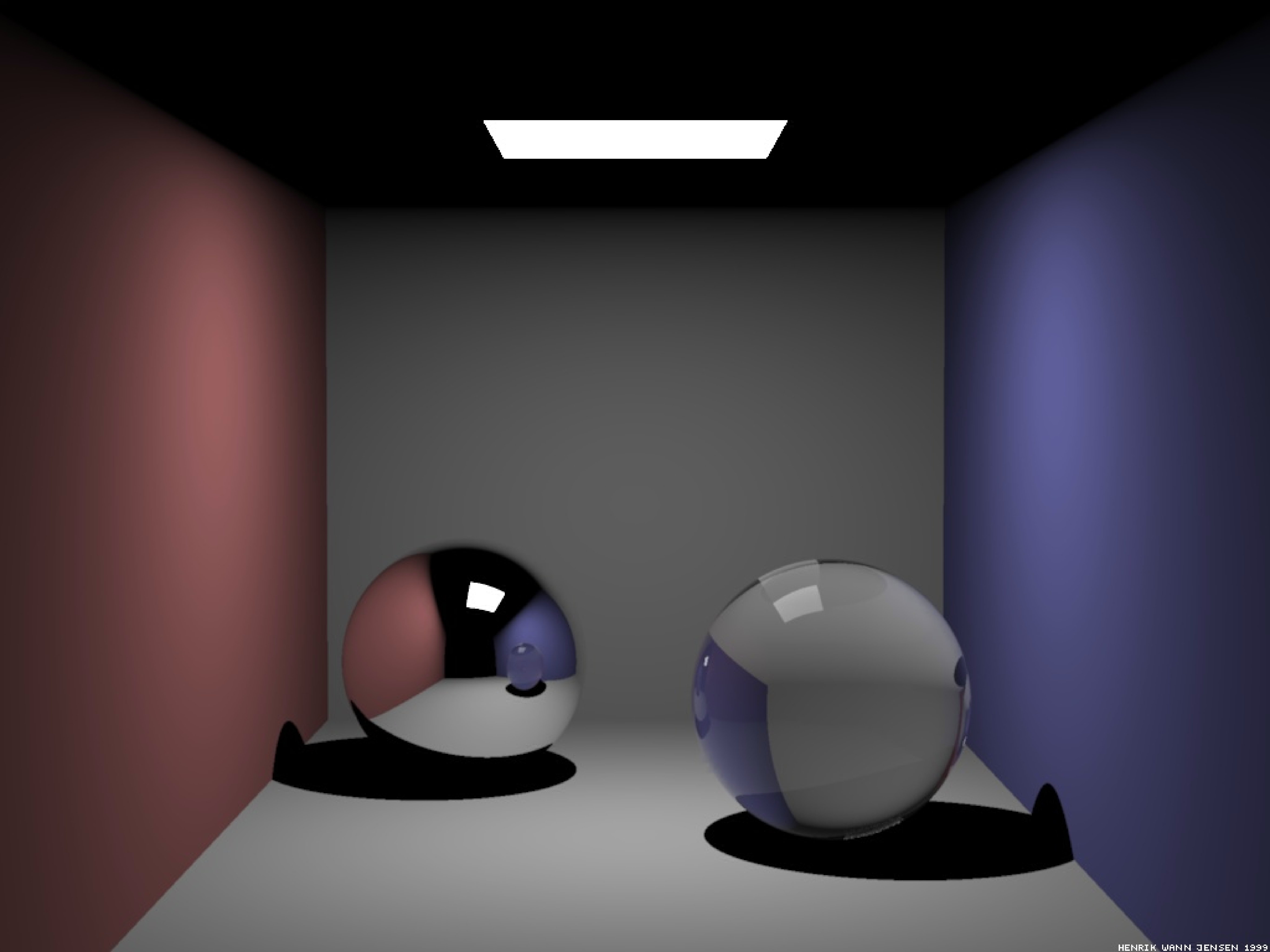
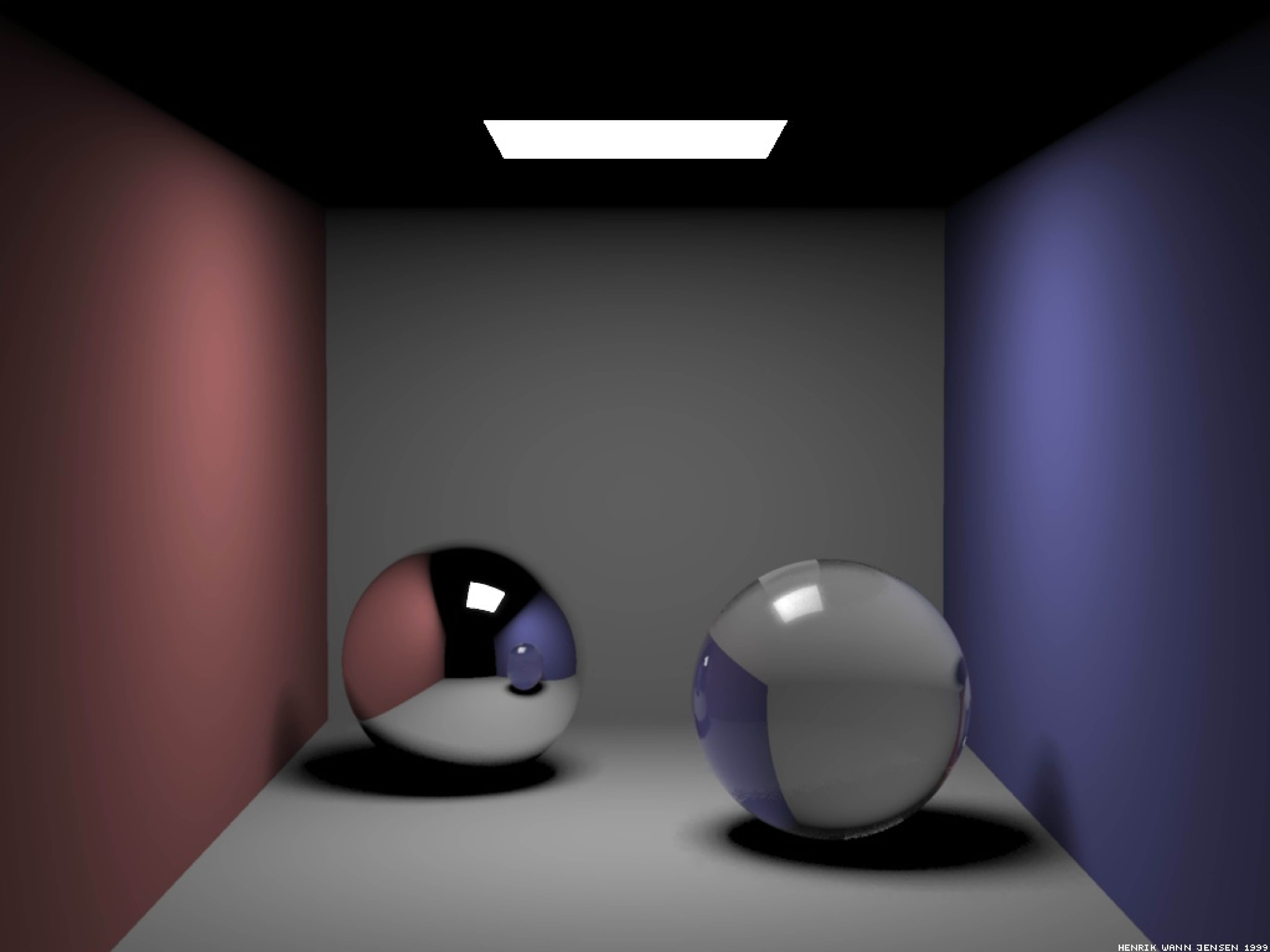
# Caustics

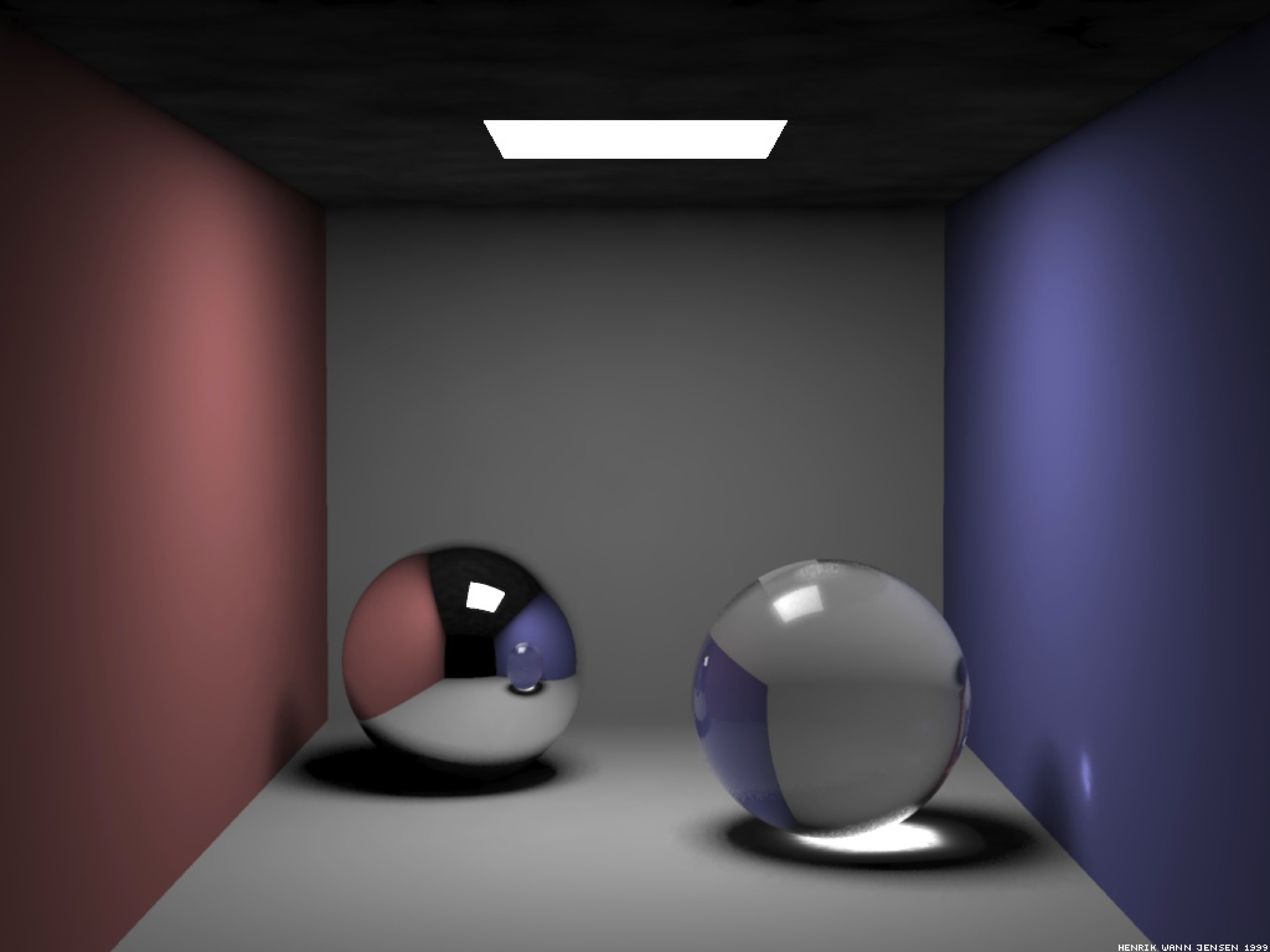
- *Can be done by photon mapping.*
- *Shoot light particles (photons) from light sources. They behave like rays.*
- *Store information where they hit surfaces.*
- *Render lighting from the map.*

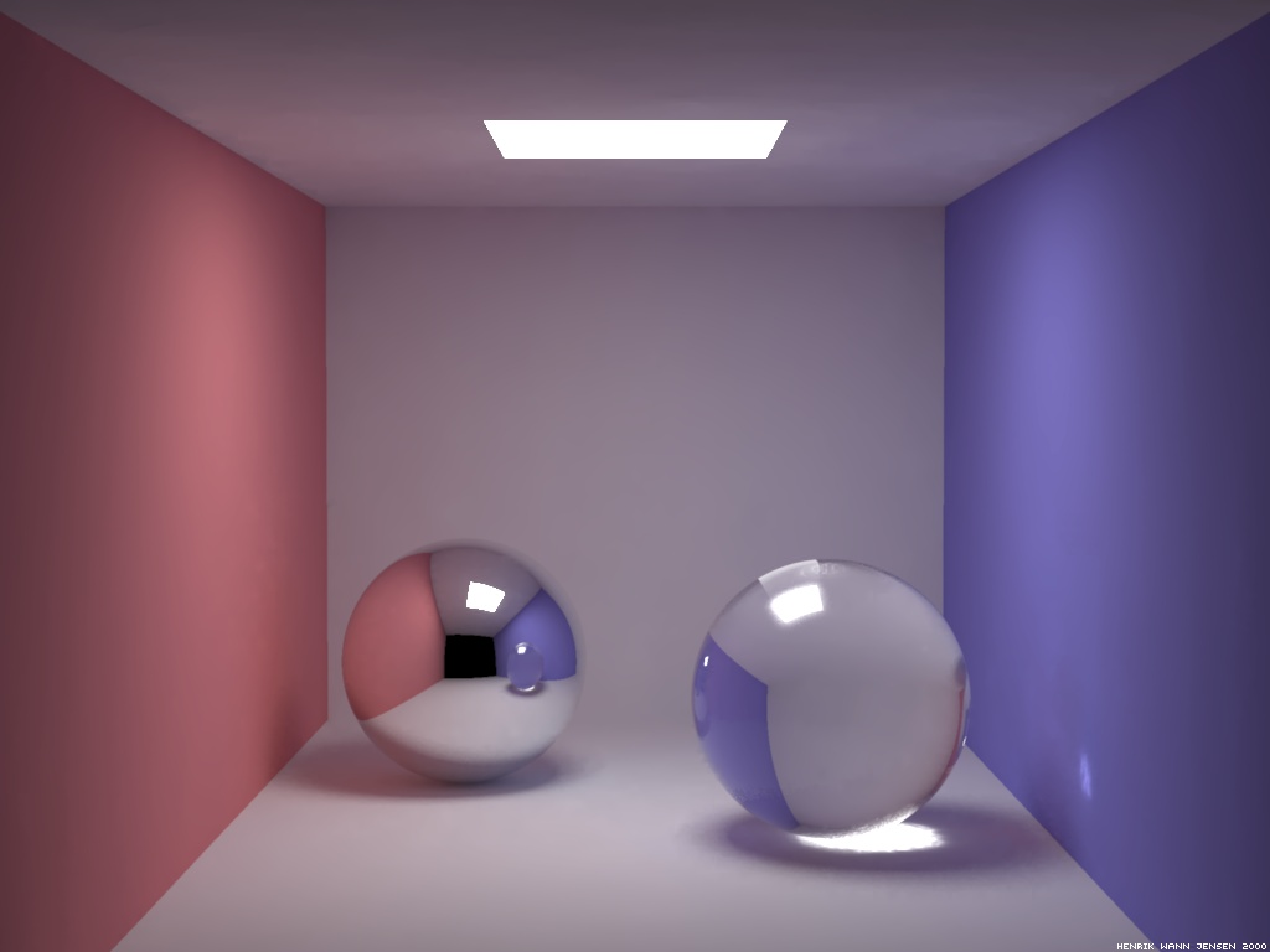

HENRIK WANN JENSEN - 2002


HENRIK WANN JENSEN - 2001


HENRIK WANN JENSEN 1999


HENRIK WANN JENSEN 1999


HENRIK WANN JENSEN 1999


HENRIK WANN JENSEN 2000

# Links

- *More on global illumination* http://escience.anu.edu.au/lecture/cg/GlobalIllumination/printNotes.en.html
- *Our radiosity example used:* http://dudka.cz/rrv/gallery?lang=cz
- *Cornell Box model:* http://graphics.ucsd.edu/~henrik/images/cbox.html