Assignment 1 for COSC410 Logic for AI

Due 21 March 2016, based on lectures 1, 2, 3

1. Assume that we are dealing with the traffic system and have the language $L_{\{p,q\}}$ generated by $A = \{p, q\}$ with the set of states $S = W_A = \{11, 10, 01, 00\}$.

   Consider the following 16 sentences of $L_{p,q}$:

   - $p \vee \neg p$
   - $p \vee q$
   - $p \vee \neg q$
   - $\neg p \vee q$
   - $\neg p \vee \neg q$
   - p
   - q
   - $p \leftrightarrow q$
   - $\neg(p \leftrightarrow q)$
   - $\neg q$
   - $\neg p$
   - $p \wedge q$
   - $p \wedge \neg q$
   - $\neg p \wedge q$
   - $\neg p \wedge \neg q$
   - $p \wedge \neg p$

   Arrange these sentences to give the Lindenbaum-Tarski algebra, i.e. arrange the sentences in rows so that all the sentences in a given row have the same number of models and, as you descend from the top row, the number of models becomes one less at each step. Leave a nice big gap between rows. Now connect the sentences with lines according to the following rule. Each line segment goes upward from one row to the next row, and the line connects a lower sentence x to a higher sentence y if and only if $x \models y$. (5 marks)

2. Let L_A be arbitrary and let $\varphi, \psi \in L_A$. Show that $\varphi \equiv \psi$ if and only if it is the case that every state satisfies $\varphi \leftrightarrow \psi$. Note that an example is insufficient. (2 marks)

3. Let L_A be arbitrary and let $\alpha, \beta \in L_A$. Prove that $\models$ is contrapositive: if $\alpha \models \beta$ then $\neg\beta \models \neg\alpha$. Note that an example is insufficient. (2 marks)

4. Suppose $A = \{p_0, p_1, p_2\}$. Consider the sentence $p_1 \leftrightarrow p_2$. Give two equivalent sentences, one in strong disjunctive normal form (SDNF) and the other in conjunctive normal form. To avoid a common error, note that the 3 atoms start with p_0, not p_1, and there is no p_3. (2 marks)