Willem's question for COSC410 assignment 02 (worth 2 marks out of 10):

Let L be a first-order language with Pred = {(P, 1)} and let (D, den) be an interpretation of L with D = {1, 2, 3} and den(P,1) = {2}.

Suppose v: Var $\rightarrow$ D is the variable assignment given by $v(x_j) = 1$ for all j.

Show that v satisfies $\exists x_1 P(x_1)$ in (D, den). Give details of your argument, and make use of the definition of satisfaction.

Solution

To show that v satisfies $\exists x_1 P(x_1)$ in (D, den), it is enough to show that we can find some $d \in D$ such that the variable assignment $v[x_1 \rightarrow d]$ satisfies $P(x_1)$.

For a variable assignment $v[x_1 \rightarrow d]$ to satisfy $P(x_1)$, the denotation of x_1 has to be in den(P,1) = {2}. But what is the denotation of x_1 when we have a variable assignment of the form $v[x_1 \rightarrow d]$?

Well, $v[x_1 \rightarrow d]$ is just the name of a function, namely the function that behaves exactly like v except for sending input x_1 to output d. To get the denotation of x_1 we apply the function to input x_1 and see what the output is. The output is

$$v[x_1 \rightarrow d](x_1) = d$$

because the name of the function tells you it's going to spit out d when fed x_1.

So, for $v[x_1 \rightarrow d]$ to satisfy $P(x_1)$ we want d to be in den(P,1) = {2}.

Choose d = 2. Then $v[x_1 \rightarrow 2]$ satisfies $P(x_1)$ since

$$v[x_1 \rightarrow 2](x_1) = 2 \in \{2\} = den(P,1).$$

Since there does exist some d in D, namely d=2, such that $v[x_1 \rightarrow d]$ satisfies $P(x_1)$, we may conclude that v itself satisfies $\exists x_1 P(x_1)$ in (D, den).