Dendrites

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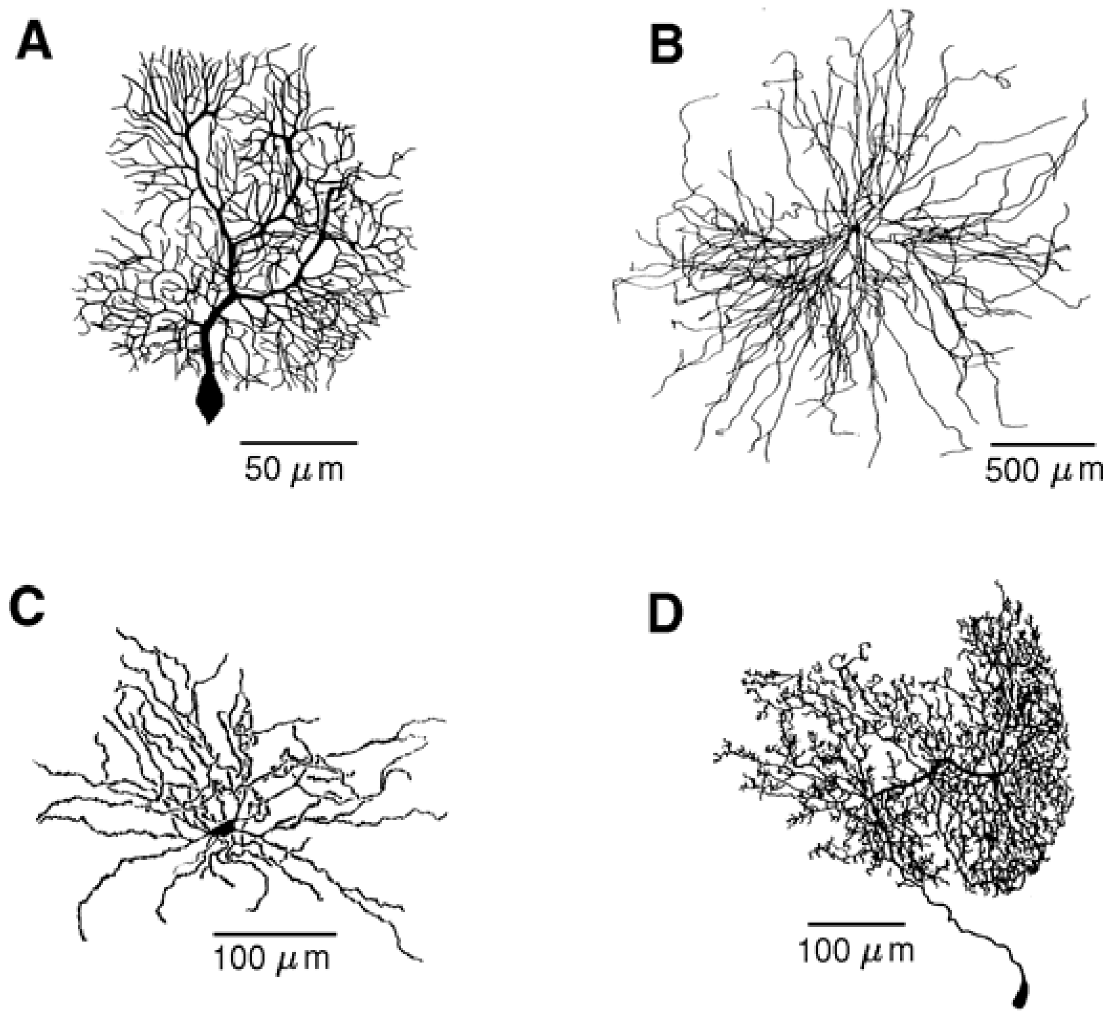
COSC422 – lecture 4

Dendritic trees

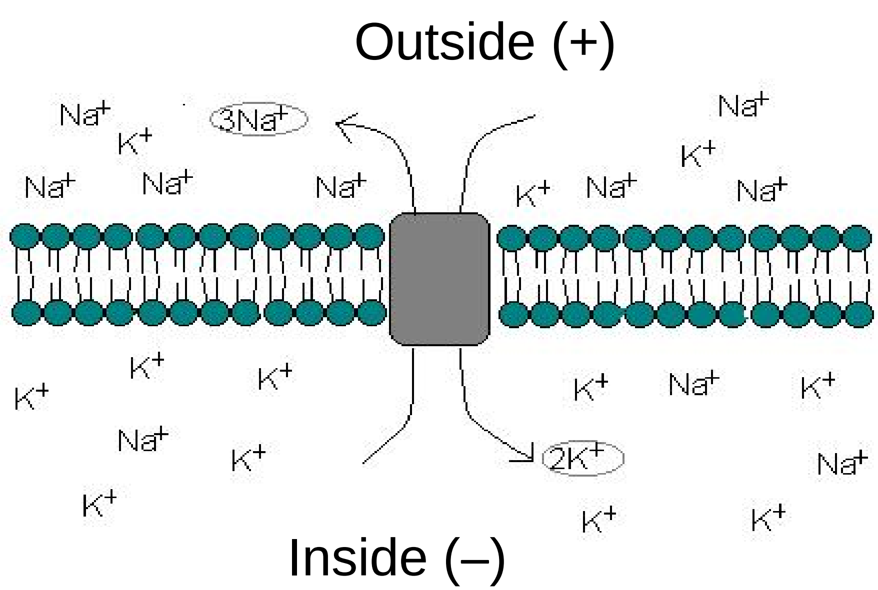
- Extensively branched dendritic trees are neurons' major input area for incoming signals.

- The distribution of membrane potential along each and every branch and the final total potential at the soma are complex functions of spatio-temporal summation of individual synaptic potentials and the geometry of the tree.



Ions and the membrane

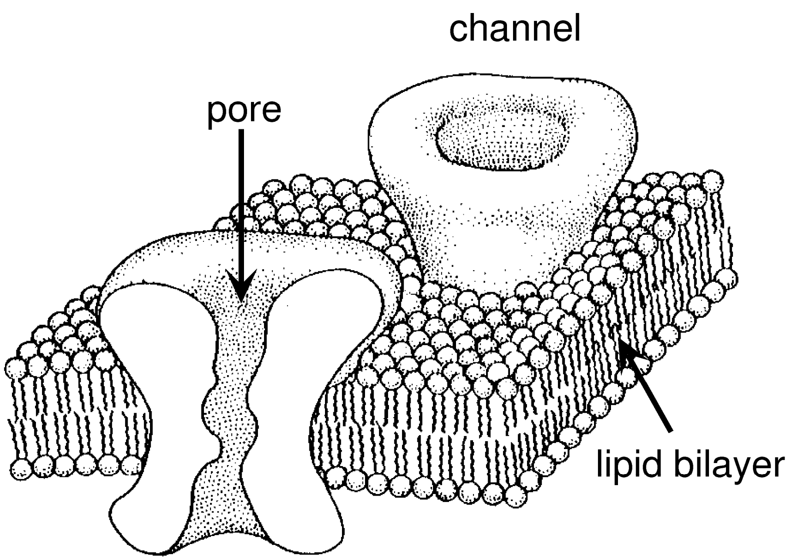
- Neurons are enclosed by a membrane separating interior from extracellular space.
- The concentration of ions of Na^+, K^+, Cl^- inside is different to that in the surrounding liquid (thanks to the "3Na+out & 2K^+ in" pumps).
- The difference in ionic concentrations (in vs. out) generates an electrical potential / voltage, called the membrane potential.



$$V = V_{in} - V_{out} \approx -65 \text{ mV}$$

Types of ion channels in the neuron membrane

- **Voltage-gated ion channels**: are located in the soma & axon, and play role in generation and propagation of action potentials. They open & close due to the changes in the membrane potential V.
- **Receptor-gated ion channels**: are located on the dendrites and soma. They open only when a particular chemical binds to them.
- **Permanently open or "passive" ion channels**: are everywhere. They allow "passive" flow of ions along the electro-chemical gradients. Passive ion channels underlie the theory of dendrites.



Equilibrium potential – Nernst equation

- Difference in concentration between inside and outside mean ions move through passive channels along their electro-chemical gradient.

- Equilibrium potential E_i for an ion i is the membrane potential at which current flow due to electric forces cancels diffusive flow.

- $$E_i = \frac{RT}{z_i F} \ln \frac{[\, i \,]_{\mathrm{out}}}{[\, i \,]_{in}}$$

- where z_{i} is the electrical charge of ion i, R is is the universal gas constant (= $8.314\ \mathrm{J\ K^{-1}\ mol^{-1}}$), T is the absolute temperature (K) and F is the Faraday constant (= $9.648\times10^{4}\ \mathrm{C\ mol^{-1}}$). [i] means concentration of ion i. Natural logarithm is denoted by "ln".

Equilibrium potential: Nernst equation

$$E_i = \frac{RT}{z_i F} \ln \frac{[\, i \,]_{\mathrm{out}}}{[\, i \,]_{in}}$$

- From the Nernst equation we get these equilibrium potentials:
  - E_K is typically between –70 and –90 mV,
  - E_{Na} is +50mV or higher,
  - E_{Ca} is around +150mV while
  - E_{Cl} is about – 65mV (near the resting potential of many neurons).

- If $V > E_K$ then K^+ ions flow out from neuron and **hyper-polarise** it, while Na^+ and Ca^{++} have +E's, thus if $V < E$, then these ions flow in and **depolarise** neuron.

Resting potential: GHK equation

- When the membrane is in thermodynamic equilibrium (i.e., no ion flux), the membrane potential is equal to the Nernst potential *E*.
- However, in physiology, due to active ion pumps, the inside and outside of a cell are **not** in equilibrium and there are many ions with different *E's*,
- thus the *resting potential* V_0 is determined from the **Goldman-Hodgkin-Katz (GHK) equation** (P_i is the permeability for ion *i*):

$$V_0 = \frac{RT}{F} \ln \frac{\left(\sum_{i^+=1}^{N^+} P_{i^+} [i^+]_{out} + \sum_{i^-=1}^{N^-} P_{i^-} [i^-]_{in} \right)}{\left(\sum_{i^+=1}^{N^+} P_{i^+} [i^+]_{in} + \sum_{i^-=1}^{N^-} P_{i^-} [i^-]_{out} \right)}$$

Resistance and capacitance of the membrane

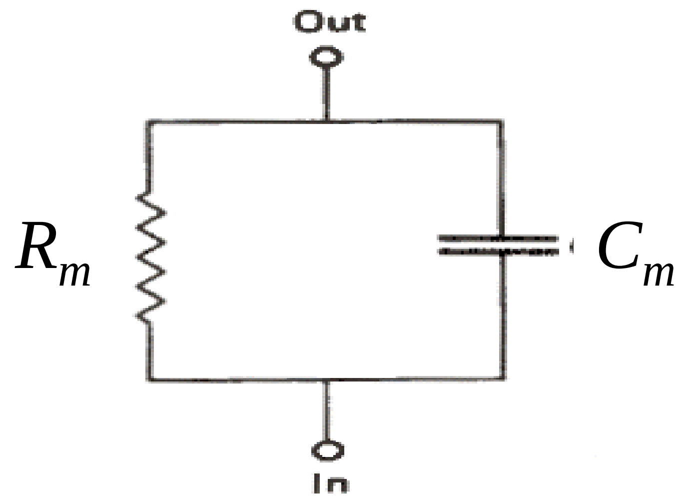
- Cell membrane: 2-3 nm thick and is impermeable to most charged molecules, thus acts as a **resistor** (however, the passive ion channels in the membrane lower the effective membrane resistance by a factor of 10,000).

- Membrane also acts as a **capacitor** by separating the charges lying on either side of the membrane.

- Capacitors store charge across an insulating medium. They don't allow current to flow across, but charge can be redistributed on each side leading to the current flow.

Electric circuit of the membrane patch

- The electric circuit representing a patch of the neuronal membrane looks like this:



- This is called an RC circuit in electronics since it consist of a resistor and capacitor.

- What is a patch of membrane? Where R_m and C_m stay the same.

(Trans)membrane electric current I_m

- Let us denote the membrane capacitance as C_m and the electric charge on the membrane as Q. From physics we have: $Q = C_m V$

- The electric current I_m through the membrane is equivalent to the change of electrical charge on the capacitor, i.e.:

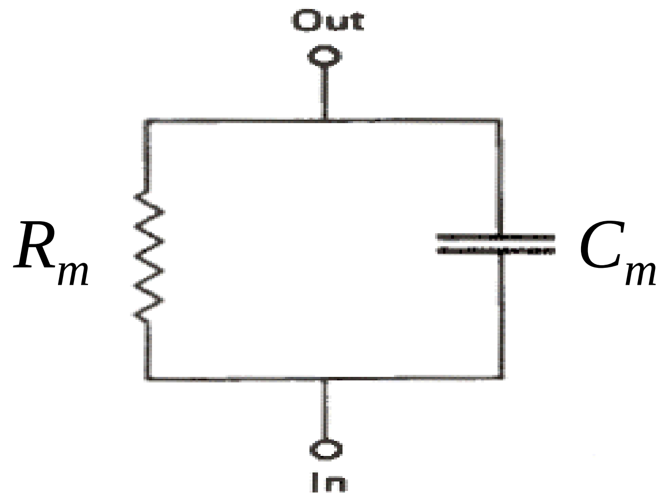
$$I_m = \frac{dQ}{dt} = C_m \frac{dV}{dt}$$

- Membrane also has a resistance R_m. Thus the membrane current I_m also equals to:

$$I_m = \frac{V}{R_m}$$

Electric circuit of the membrane patch

- In the RC circuit



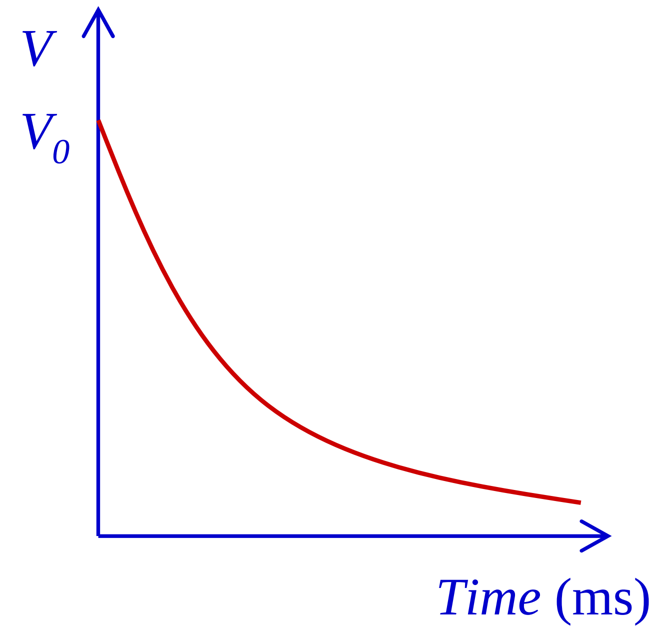
- By convention capacitance current is ‘plus’ while membrane current is ‘minus’:

$$I_m = C_m \frac{dV}{dt} = -\frac{V}{R_m}$$

- From which we derive an ODE for the voltage: $$\frac{dV}{dt} = -\frac{V}{R_m C_m}$$

Elementary solution for the membrane voltage

- Solution of ODE for voltage: $$\frac{dV}{dt} = -\frac{1}{\tau_m} V$$
- Is the exponential function: $$V = V_0 \, e^{-\frac{t}{\tau_m}}$$
- Where $V_0 = V(t = 0)$.
- Thus if the initial voltage $V_0 > 0$, then V decays in time exponentially with the so-called membrane time constant $\tau_m = R_m C_m$



Specific capacitance and resistance

- Both R_m and C_m are dependent on surface area of membrane A.

- Therefore we define the size-independent versions, specific membrane capacitance c_m and specific membrane resistance r_m per unit area, where $c_m = C_m / A$ and $r_m = R_m \ A$, respectively .

- Membrane time constant $\tau_m = R_m C_m = r_m c_m$ sets the basic time-scale for changes in the membrane potential (is typically between 10 and 100ms).

Membrane current

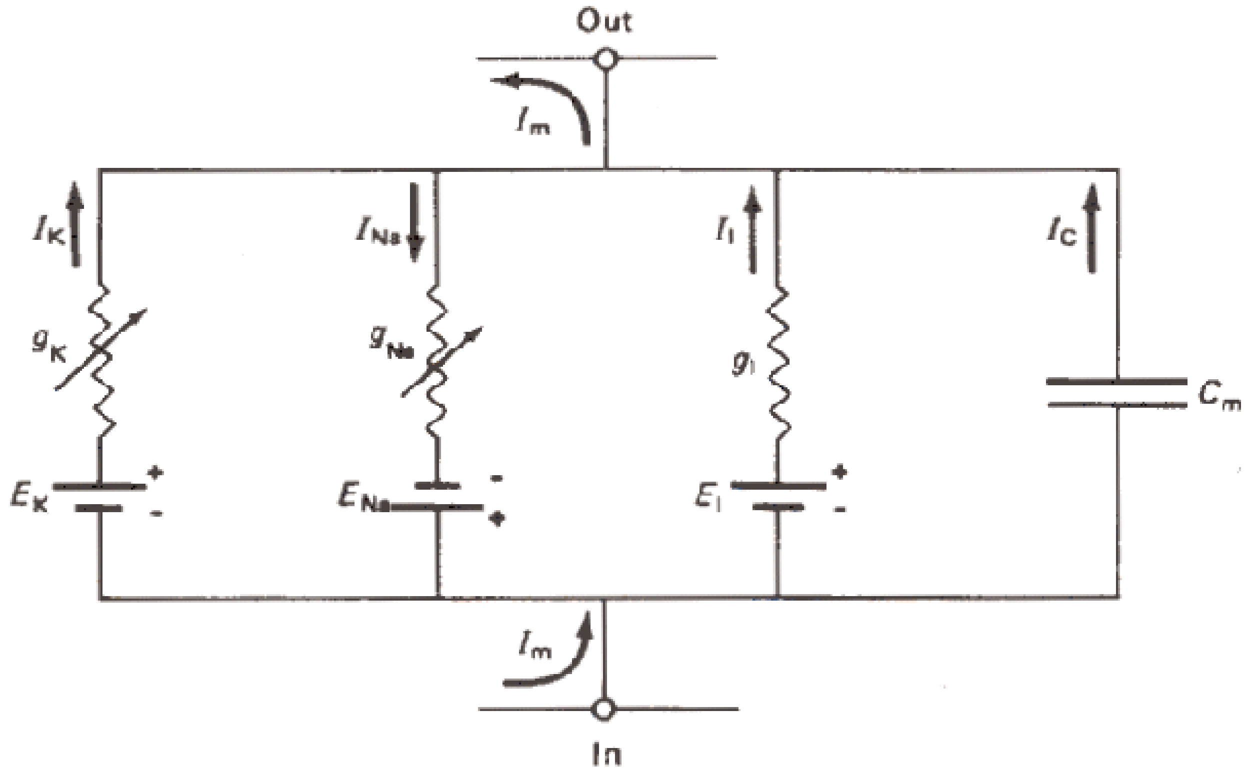
- (Specific) membrane current i_m is the total current flowing through all the ion channels per unit area of the membrane plus a leakage current (ion pumps, other ions, etc.)

$$i_m = i_L - \sum_k i_k \qquad i_L = \bar{g}_L (V - E_L)$$

- Current flowing through any type of ion channel (when it's open) for k = Na, K, and Cl is equal to driving force (difference between the membrane potential V and the equilibrium potential E_k) multiplied by channel *conductance* g_k:

$$i_k = g_k (V - E_k)$$

Single compartment model



- Rate of change of the membrane potential V is proportional to the total current entering into neuron $i_m = i_{Na} + i_K + i_L$
- Here it still holds that:

$$c_m \frac{dV}{dt} = -i_m$$

"Passive" membrane model

Dendrite model is a passive model, which assumes NO voltage-gated Na & K conductances, i.e.:

$$c_m \frac{dV}{dt} = -\bar{g}_L (V - E_L) + I_e$$

Where all synaptic currents per unit area are denoted by I_e.

Multiplying through by $r_m = 1/g_L$ we get the so-called **cable equation** for one compartment:

$$\tau_m \frac{dV}{dt} = E_L - V + r_m I_e$$

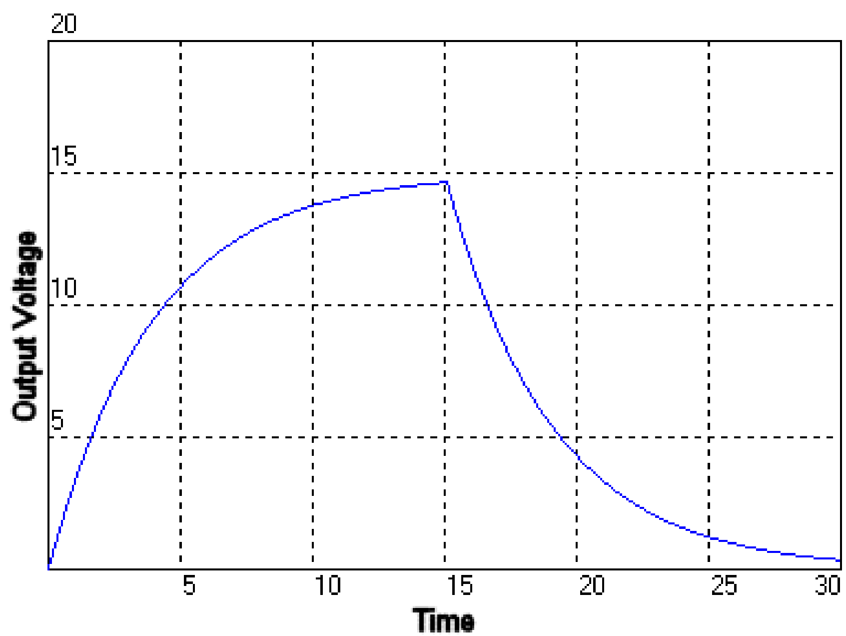
If $I_e = 0$, V decays exponentially to E_L with time constant $\tau_m = c_m r_m$

"Passive" membrane model

- Solution of the cable equation when we apply a rectangular current pulse

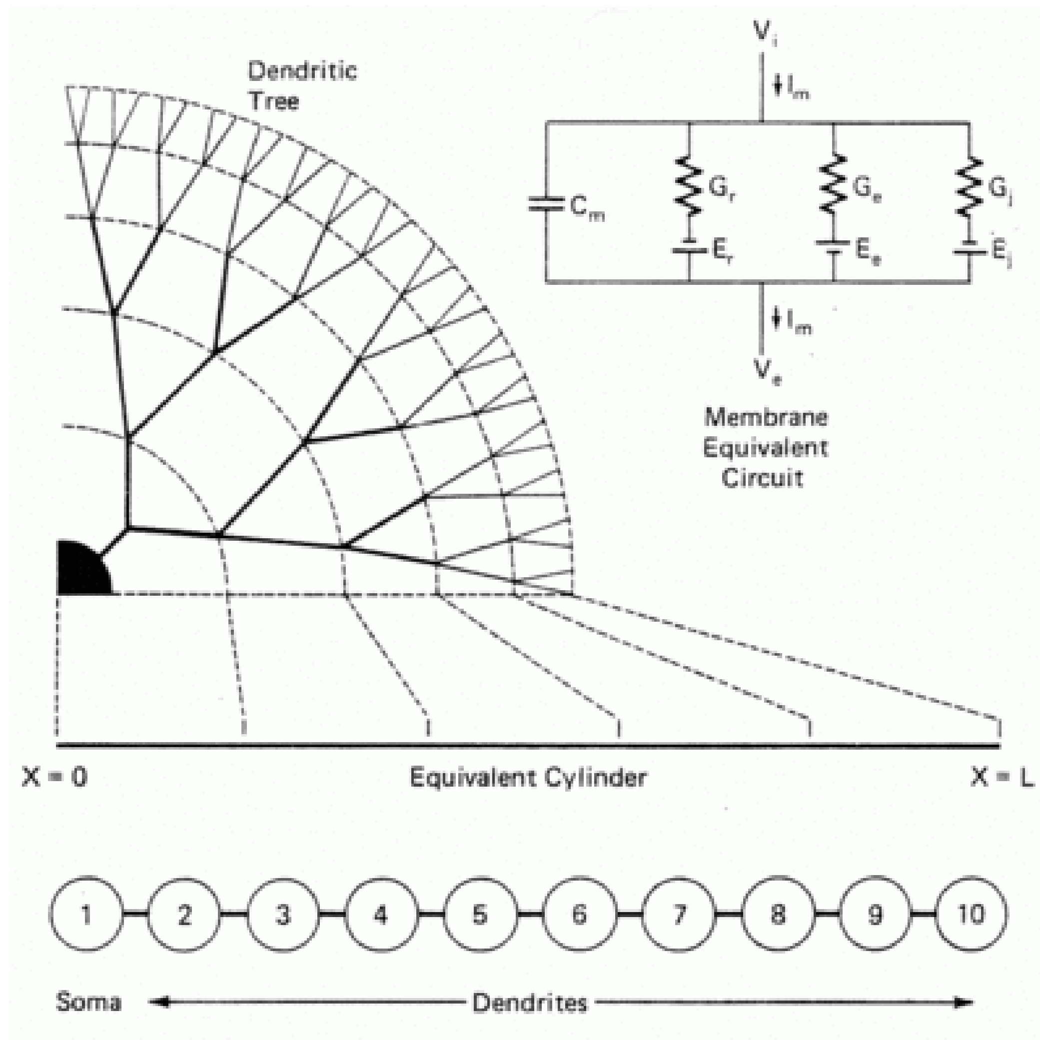
$$\tau_m \frac{dV}{dt} = E_L - V + r_m I_e \quad \Rightarrow \quad V = E_L + r_m I_e \left(1 - e^{-\frac{t}{\tau_m}}\right)$$

- & looks like this:



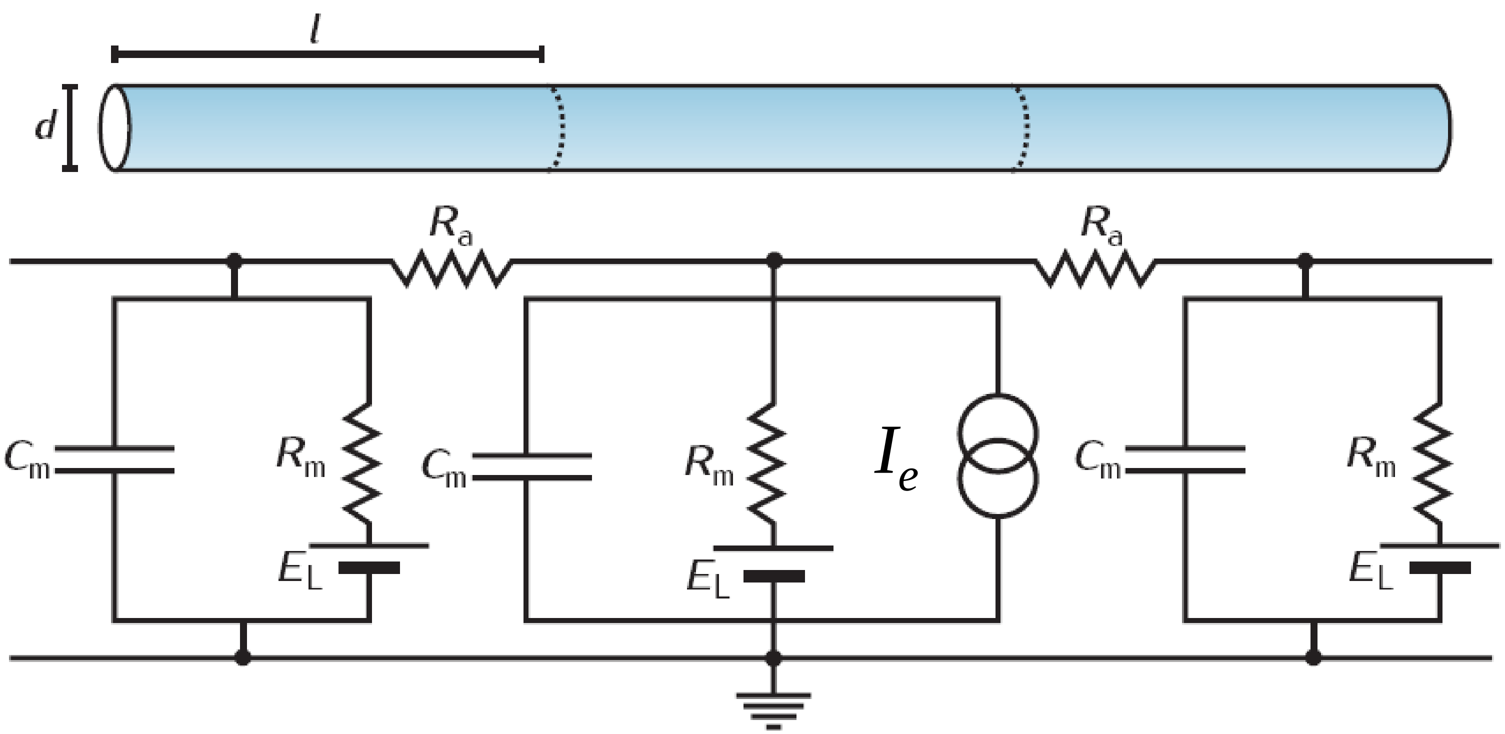
Rall's model

- Is a simplified biophysical model of a dendritic tree.
- Rall mathematically showed that the tree can be mapped to an equivalent single electric cable comprised of 10 compartments.
- It was possible to obtain analytical solutions to many of interesting problems.

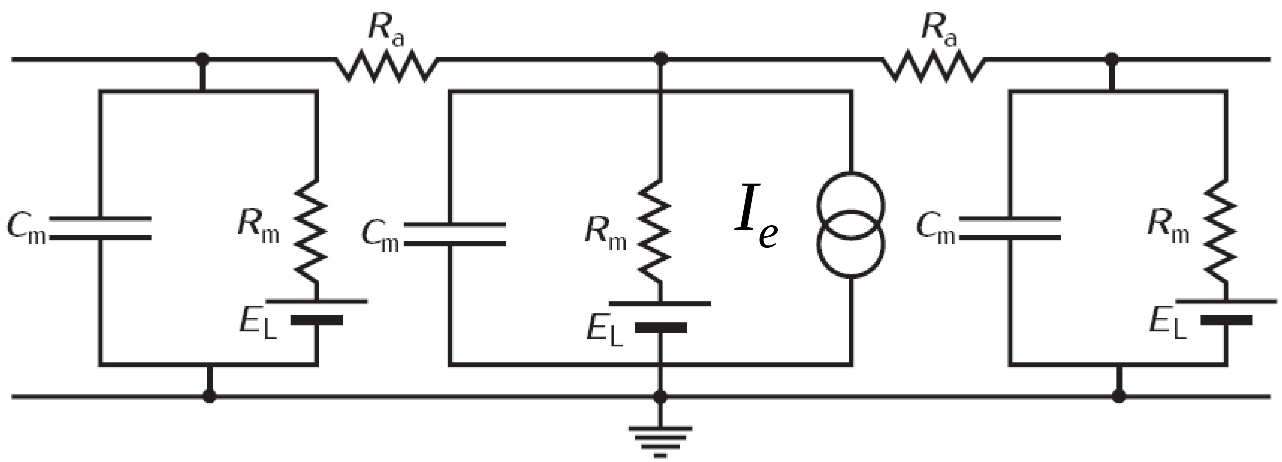


Electrical scheme of a cable

- An equivalent cable consist of n compartments.
- Each compartment has a length l and diameter d.
- Each compartment is represented by the RC circuit.
- Compartments are connected through axial resistance R_a.



Cable equation for a compartmental model

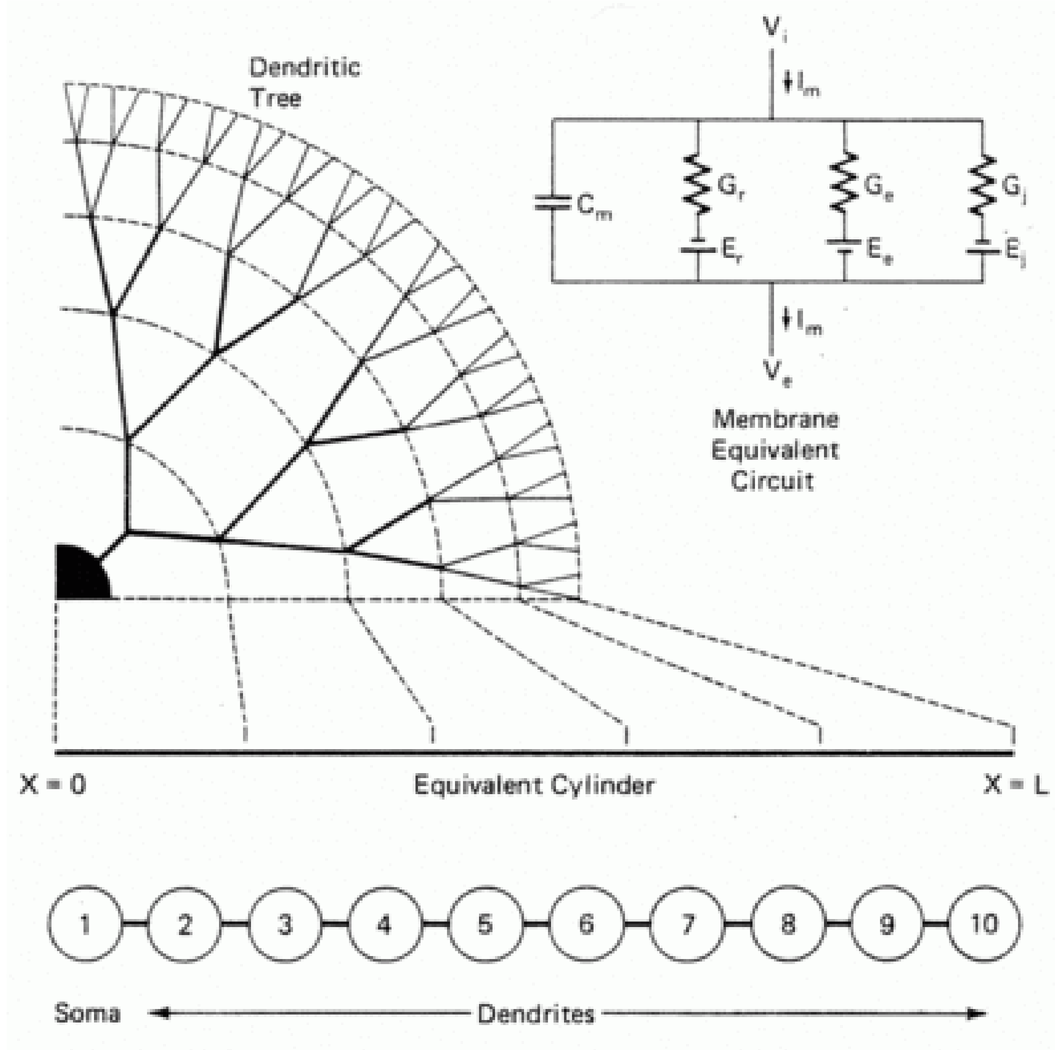


- For the compartment j, the total electric current is the sum of the capacitive current, leftward and rightward axial currents and in general also external current (from electrode or synapses). Thus:

$$C_m \frac{dV_j}{dt} = -\frac{V_j - E_L}{R_m} + \frac{d}{4R_a}\left(\frac{V_{j+1} - V_j}{l^2} + \frac{V_{j-1} - V_j}{l^2}\right) + I_e$$

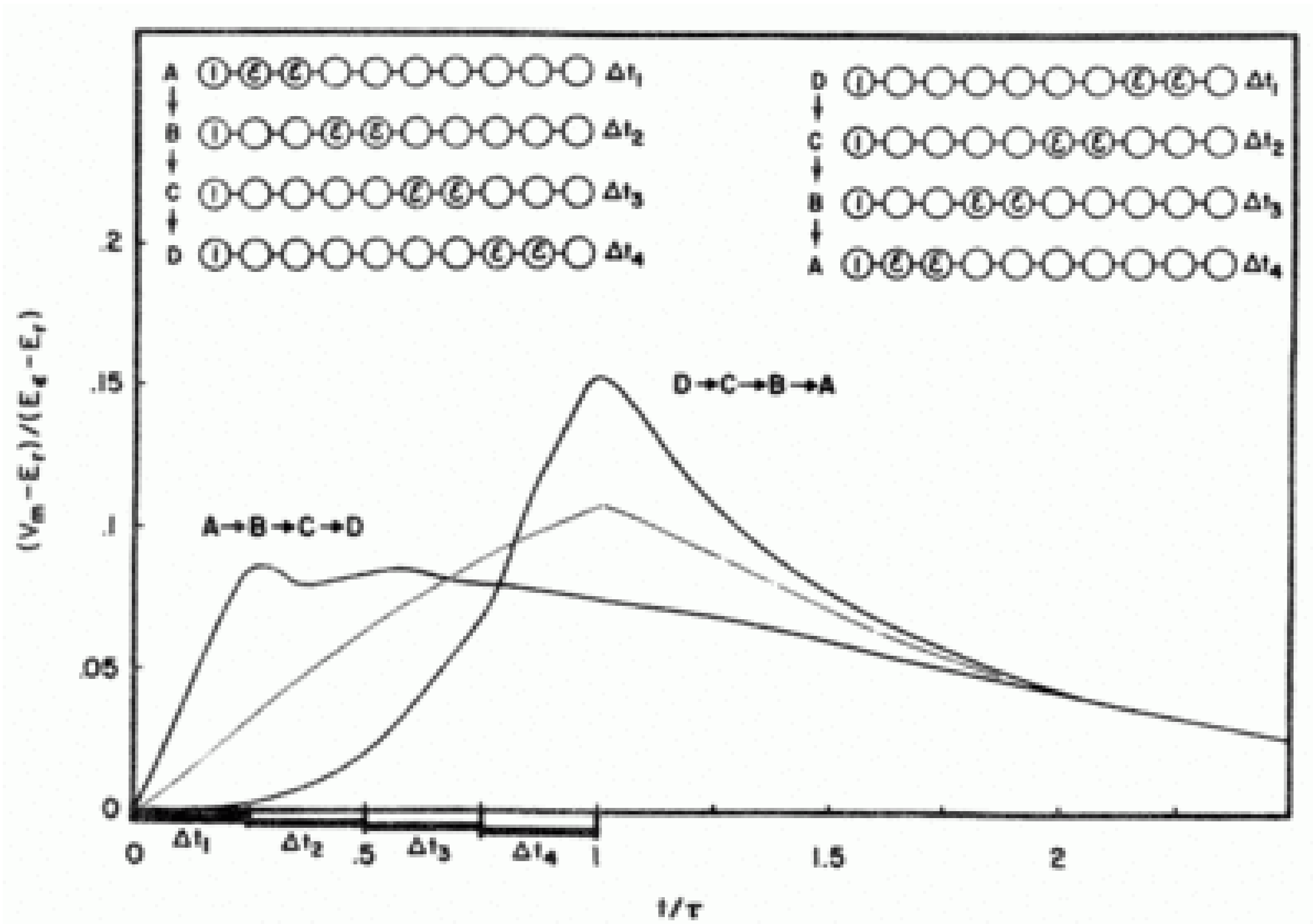
Rall's model contd.

- Synaptic excitation and synaptic inhibition can be specified to occur in particular compartments to explore the consequences of different input locations and different spatio-temporal patterns of synaptic activation upon the total potential at soma.
- With this 10 compartmental model, it was possible to obtain analytical solutions to a number of neurophysiologically interesting problems.



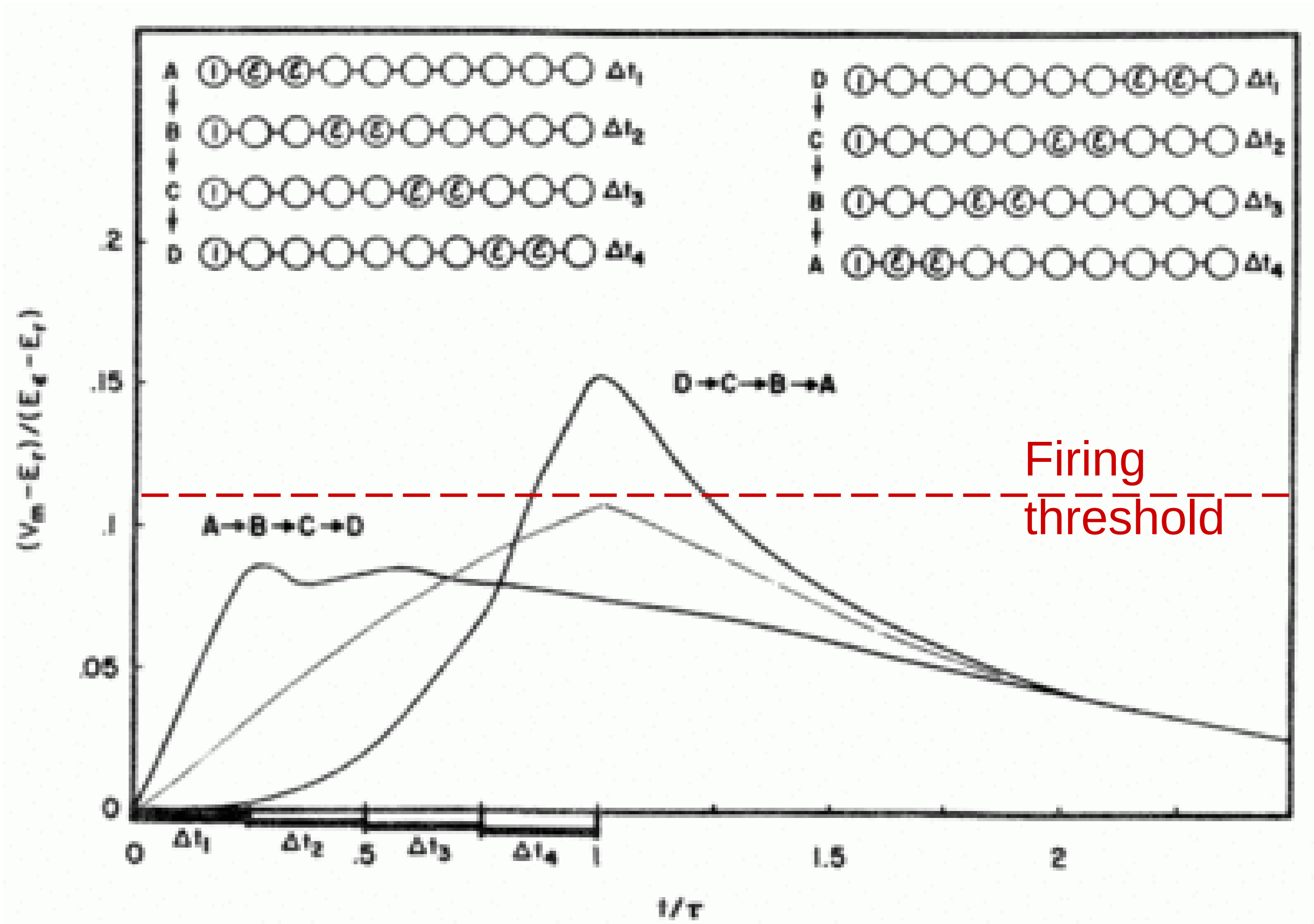
Order of synaptic activation matters

- An example of one of Rall's results: course of somatic potential as a function of different spatio-temporal orders of synaptic activation.



Order of synaptic activation matters

- If the potential at soma > firing threshold, then HH conductances are activated and neuron fires a series of output spikes.



Multi-compartmental models

- (a) actual cell morphology
- (b) represented by a set of connected cylinders;
- (c) equivalent electric circuit consisting of interconnected RC circuits is built from the geometrical properties of the cylinders and from their membrane properties.