Illumination

Scott King 2003 Revised 2004.

Minor editing by Geoff Wyvill 2005

Local vs Global

- **Local Illumination** considers light that goes directly from the light sources to an object and reflected toward the eye? What terms does this include?
  - ⋆ Shading of a surface is independent from shading of all other surfaces.
- **Global Illumination** considers light reflected from other surfaces.
- Which category is ray tracing in?
- What are the pros and cons of each?

The illumination model

For an object where does light come from?

The illumination model

For an object where does light come from?

- From some light source.
- Through the object.
- Reflected from another object.
- Incident illumination (ambient light).

Incident illumination

Where does this come from?

Incident illumination

Where does this come from?

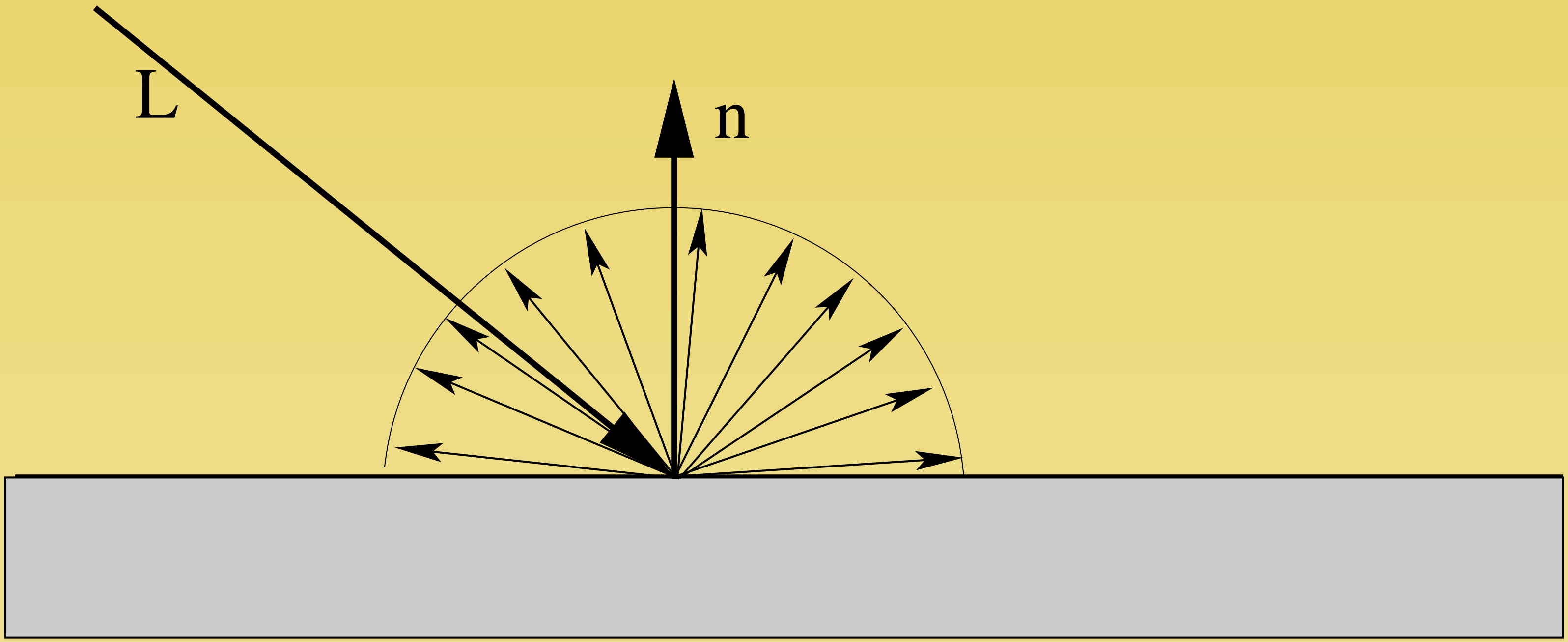
- How about light transmitted (refracted) through another object.
- How about light bouncing off of a non-reflective surface.

For now we won't worry about this incident illumination, it is the subject of other methods (*radiosity, global illumination*). We'll just call it ambient light.

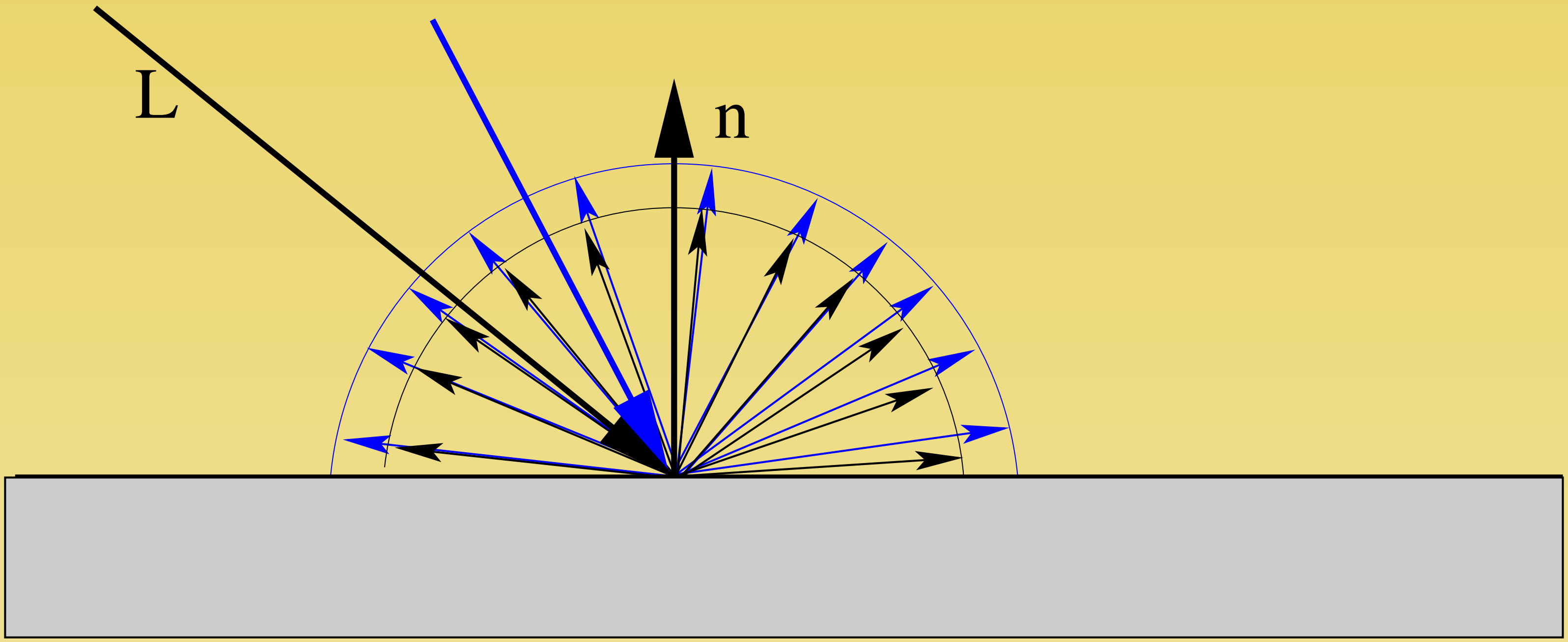
Diffuse Reflection

- An ideal diffuse reflector (a Lambertian Reflector, e.g. chalk) is the simplest to model.
- Incoming light is scattered equally in all directions, so brightness does not depend on the viewing direction.
- Reflected brightness depends on the direction and brightness of illumination (cos of light/normal angle)

Diffuse Reflection



Diffuse Reflection



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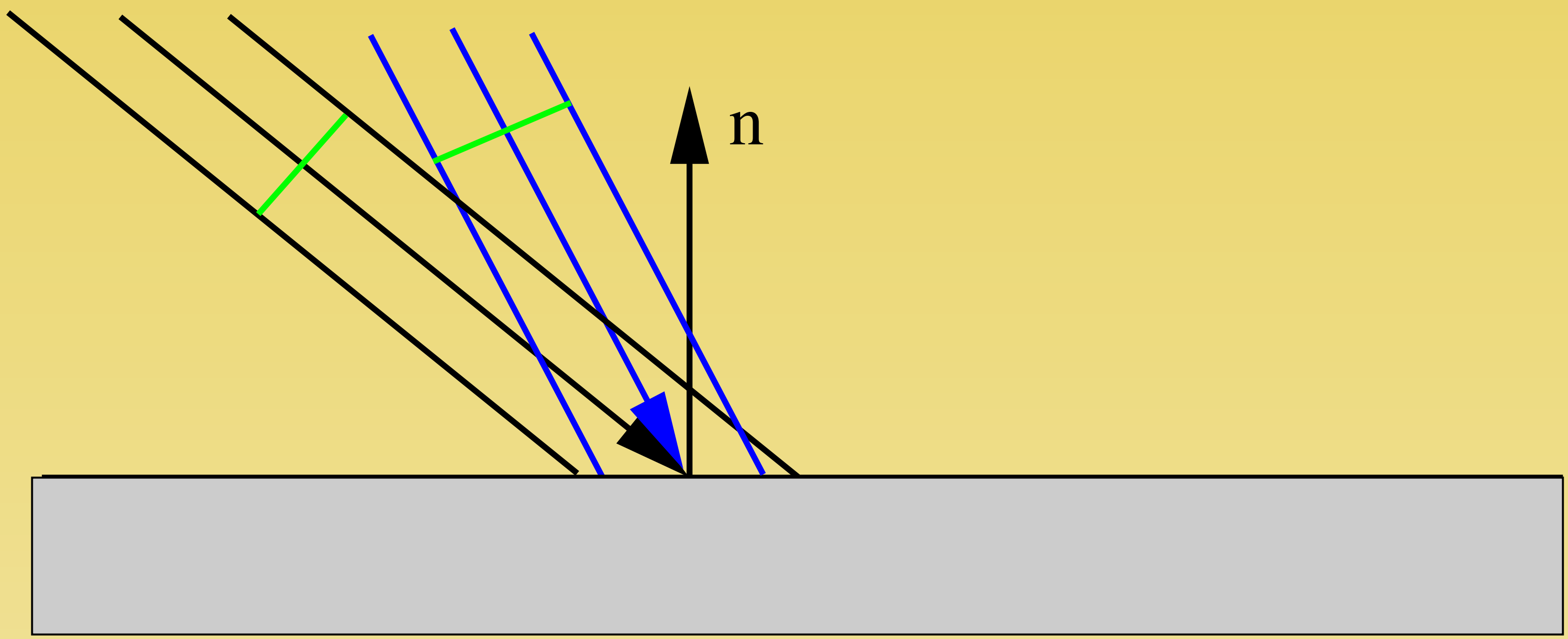

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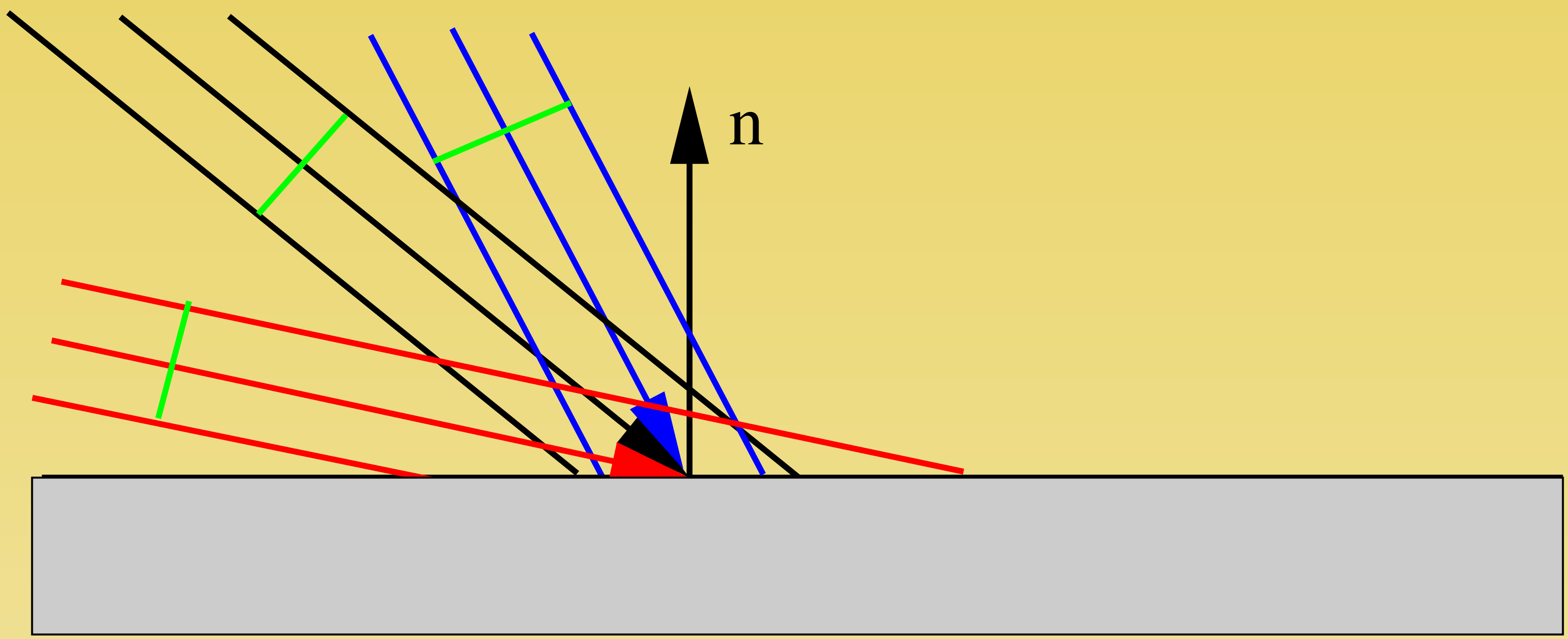

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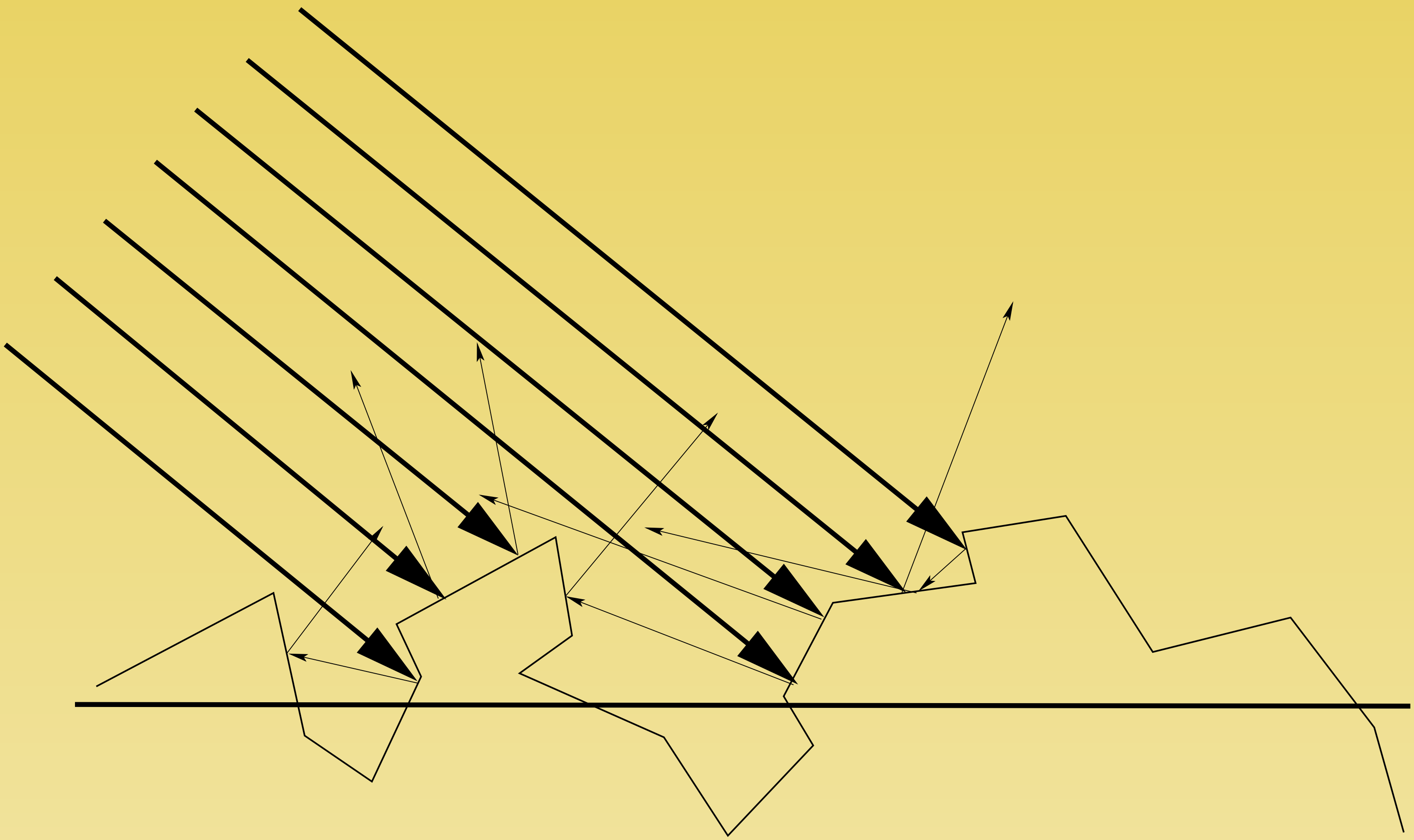


Lambertian Illumination

- Use Lambert's law, which says the Intensity of the reflected energy (light) depends upon the angle between the incoming light and the surface normal.
- The intensity is view independent!

$$I = I_i k_d N.L$$

where
I_i is the intensity of the incindent light, and
k_d is the diffuse constant of the surface (0-1).

Illumination Equation

- We have ambient light.
- We have Lambertian reflection.

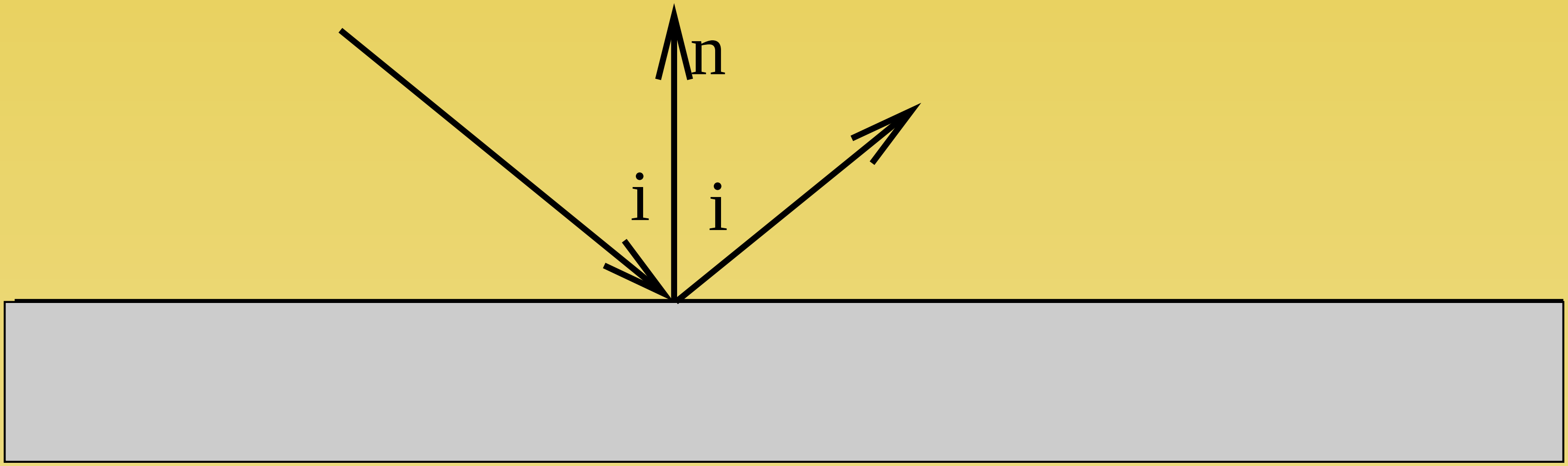
$$I = I_a k_a + I_i k_d N.L$$

where
I_i – intensity of the incident light,
k_d – diffuse constant of the surface (0-1),
I_a – ambient intensity for the object, and
k_a – ambient constant of the surface (0-1).
note: These can all be wavelength dependent.

Specular Reflection

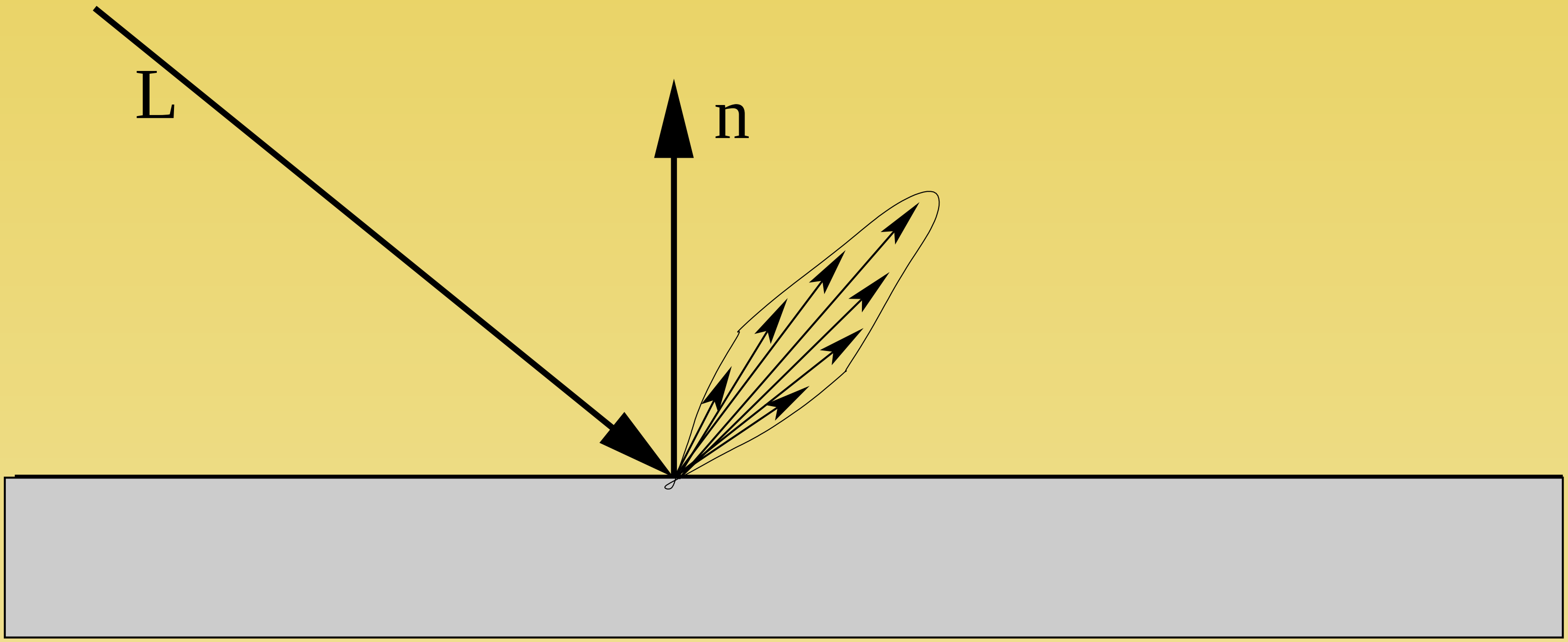
- Shiny surfaces reflect light coherently - the light is reflected in a narrow beam around a single direction, called the *specular* direction. The specular direction is the angle of incident light reflected about the surface normal.
- If your eye is in that cone, the surface looks brighter (a highlight).
- Specular reflection isn't perfect so the highlight is a blob with brightness reducing gradually away from the center.

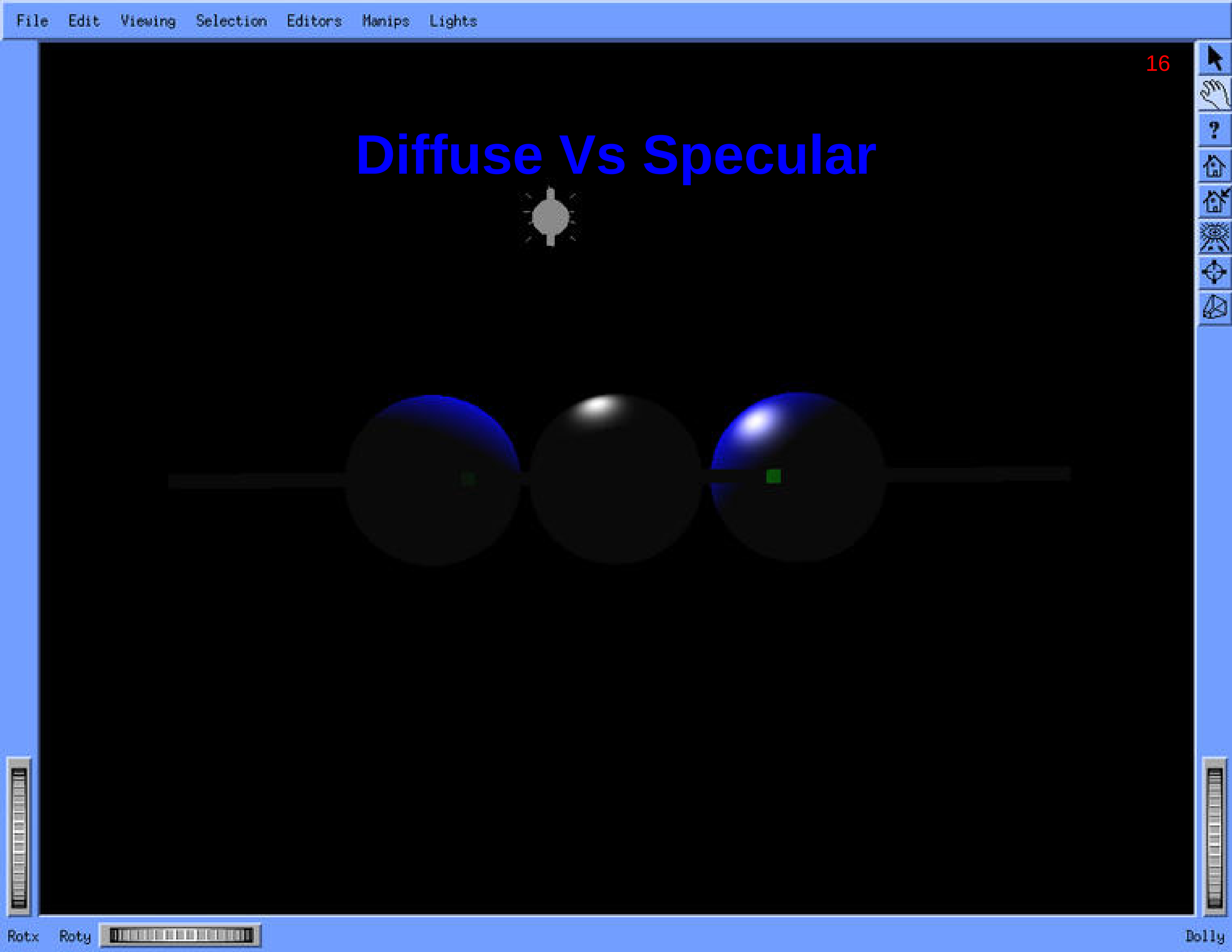
A Perfect Reflector



- The angle between the normal and incoming ray is maintained for the outgoing ray.

Specular Reflection

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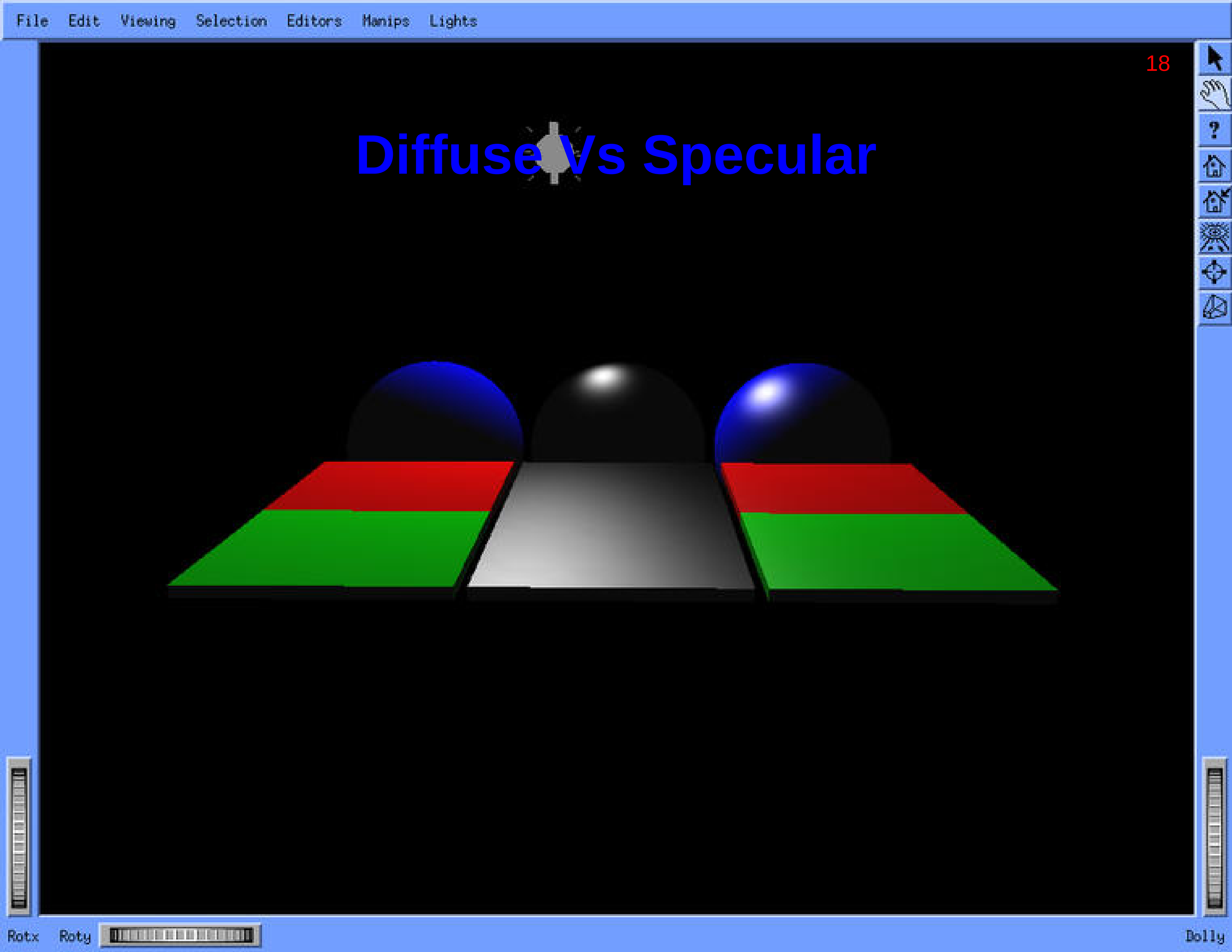
Diffuse Vs Specular
Rotx
Roty
Dolly

Diffuse Vs Specular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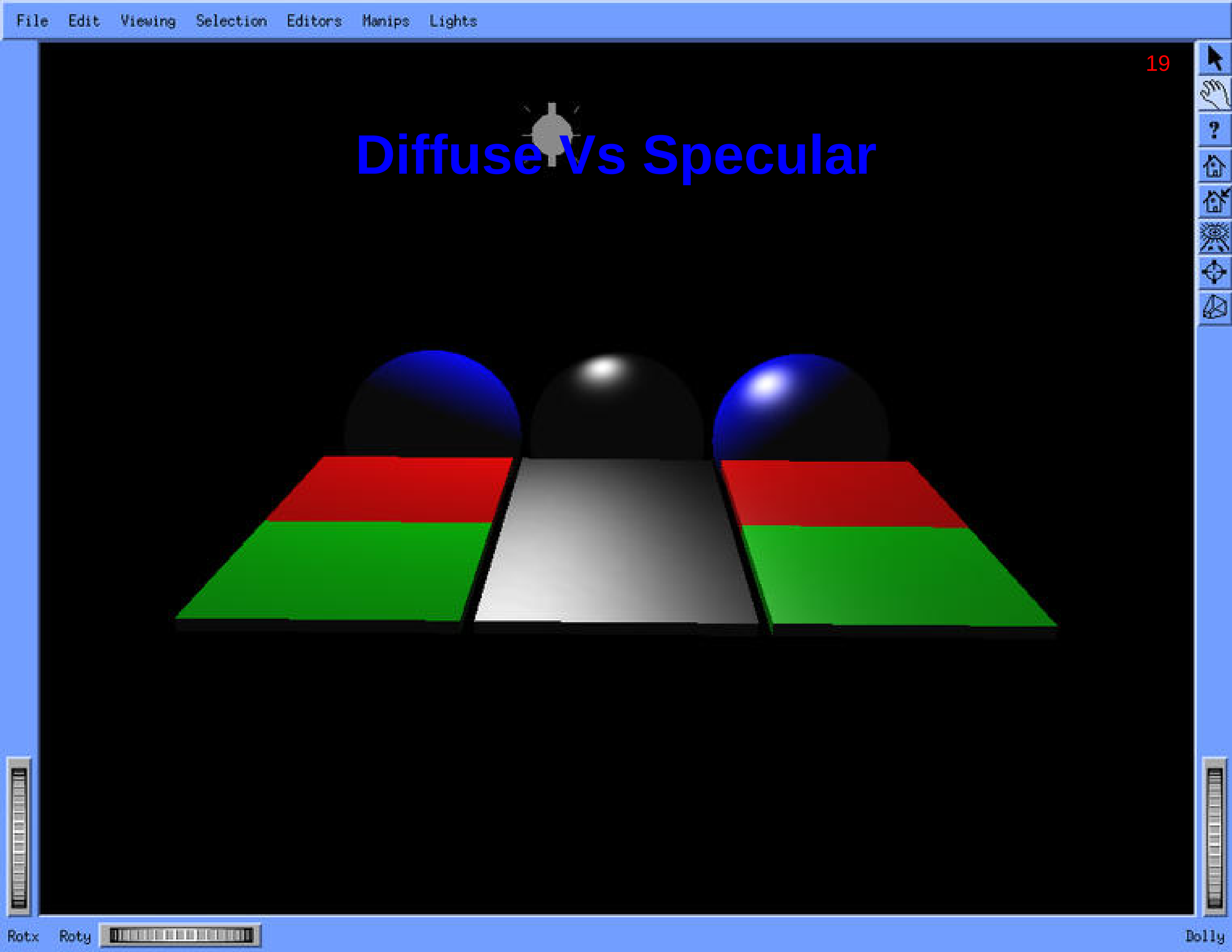

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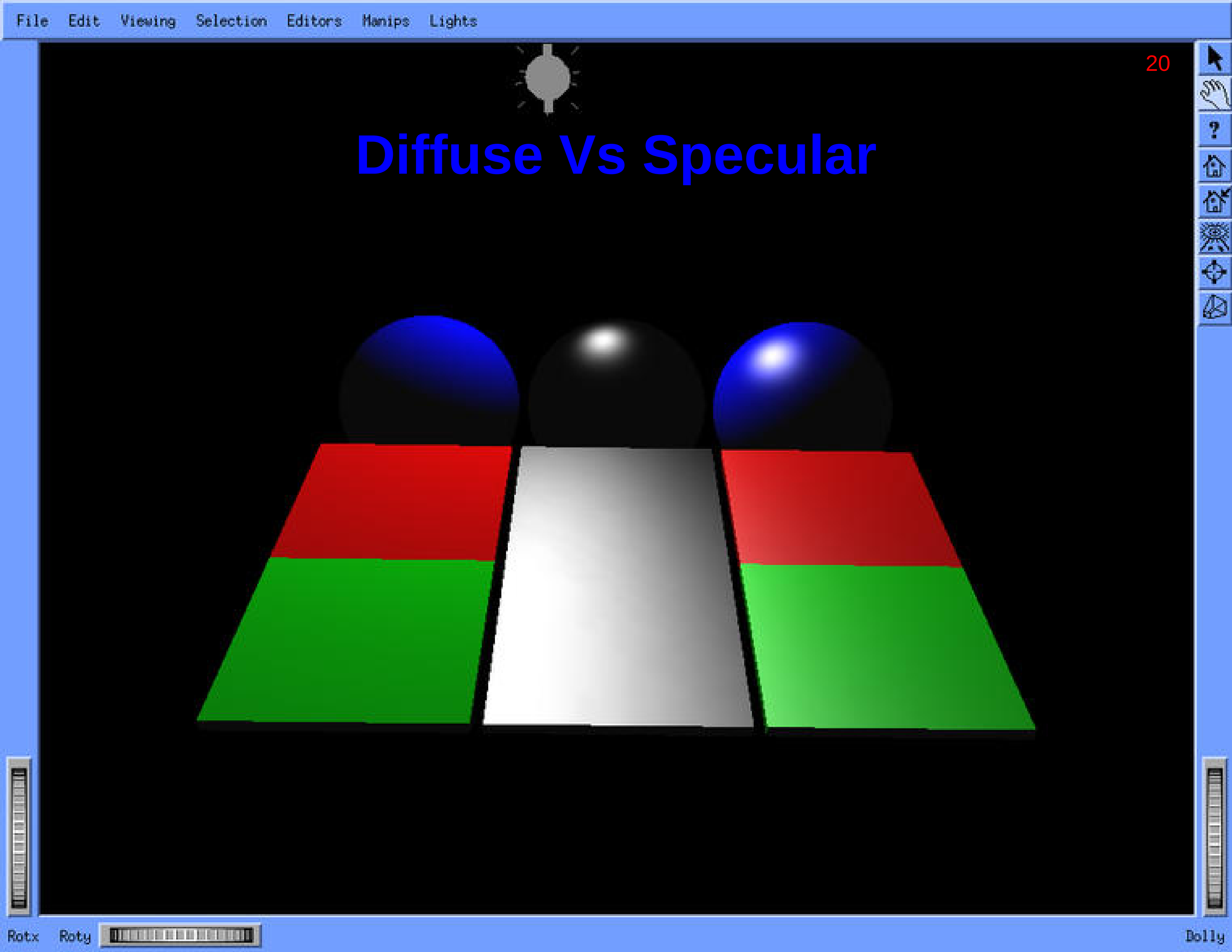

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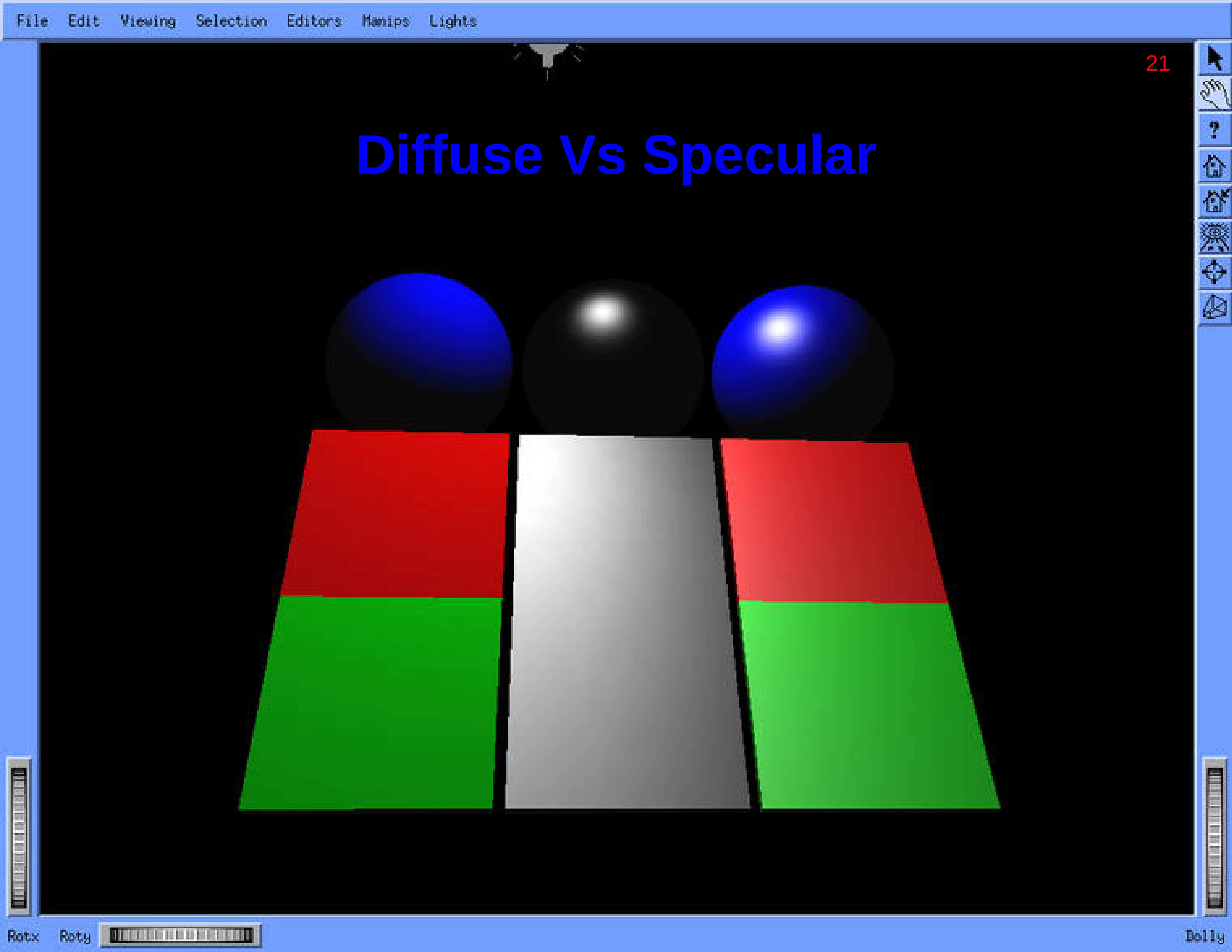

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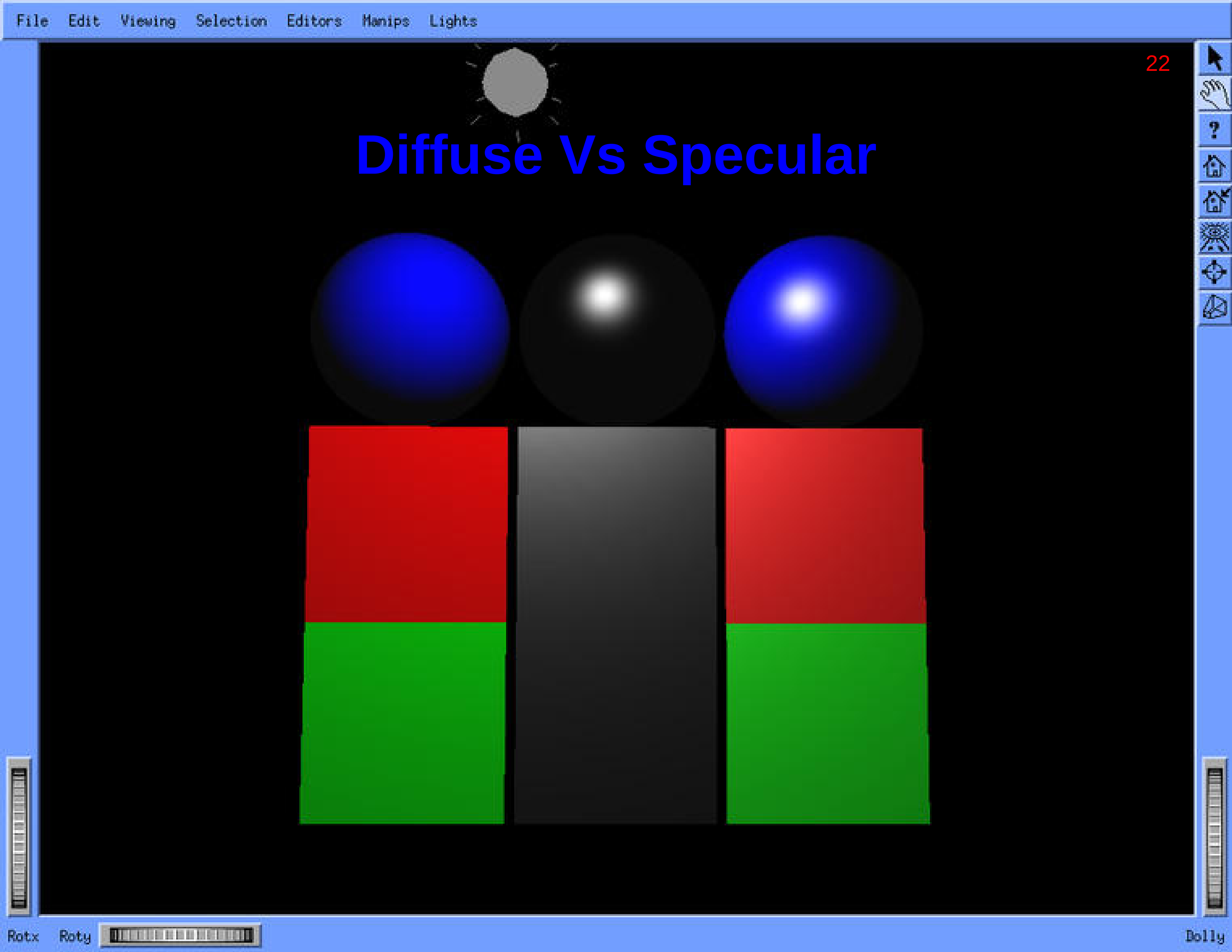

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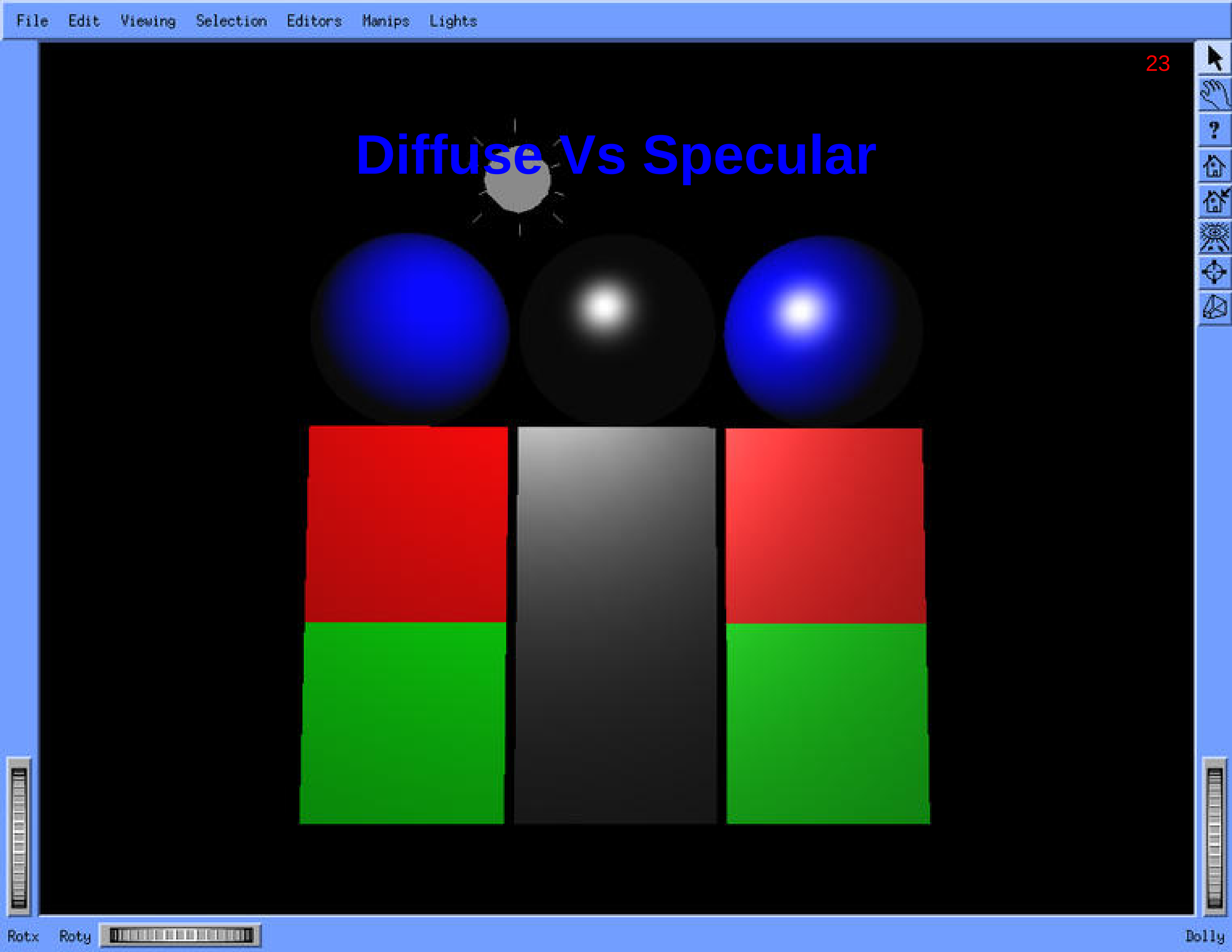

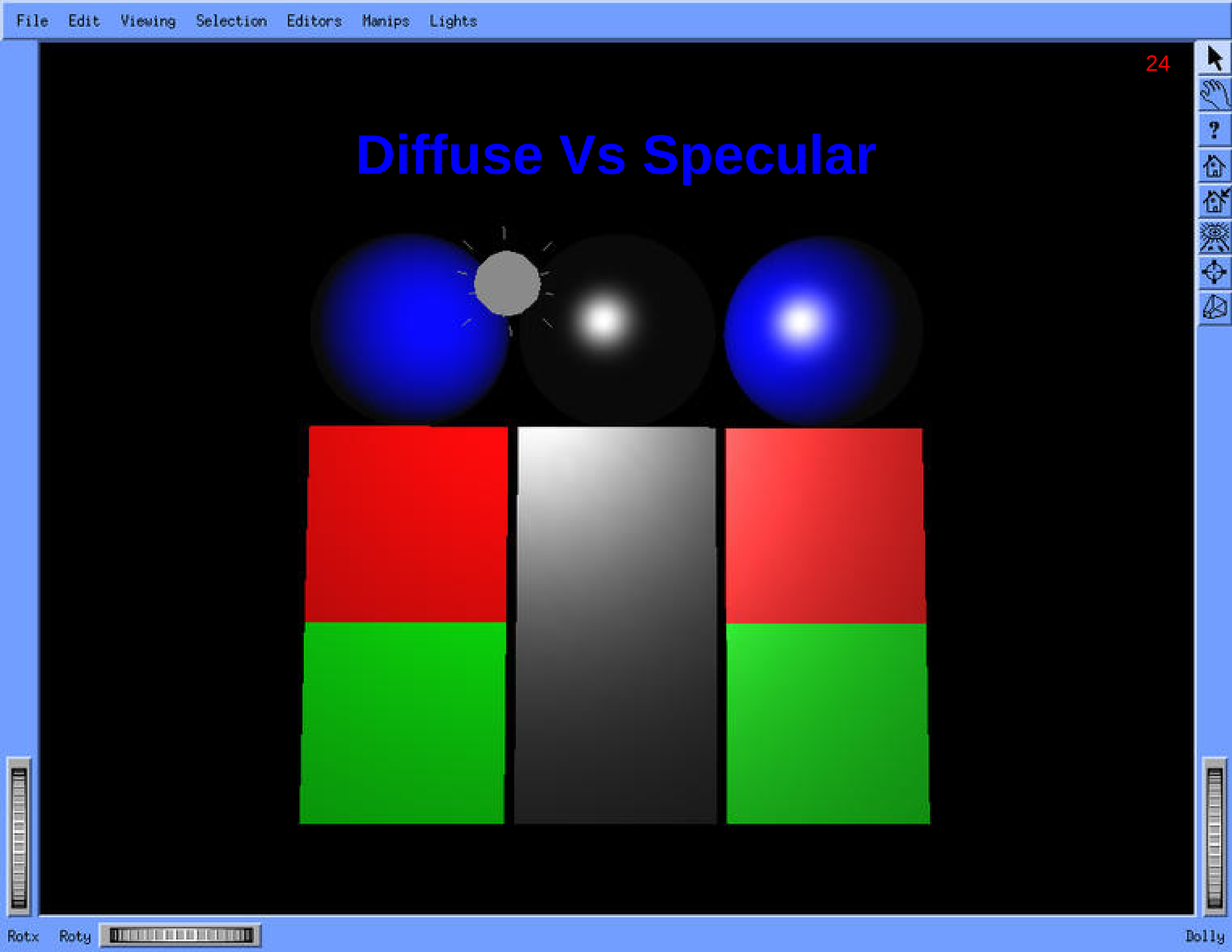

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Diffuse Vs Specular
Rotx Roty
Dolly

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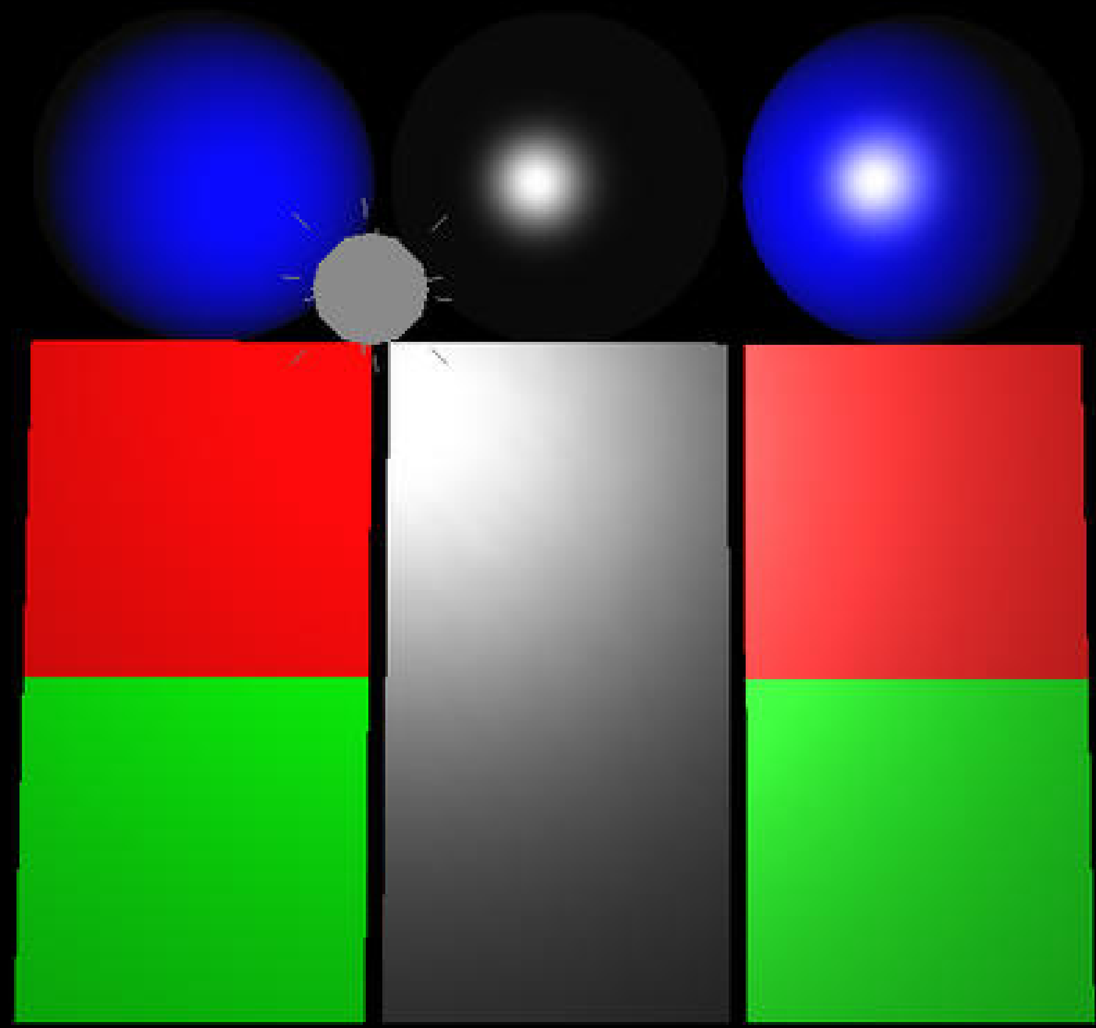
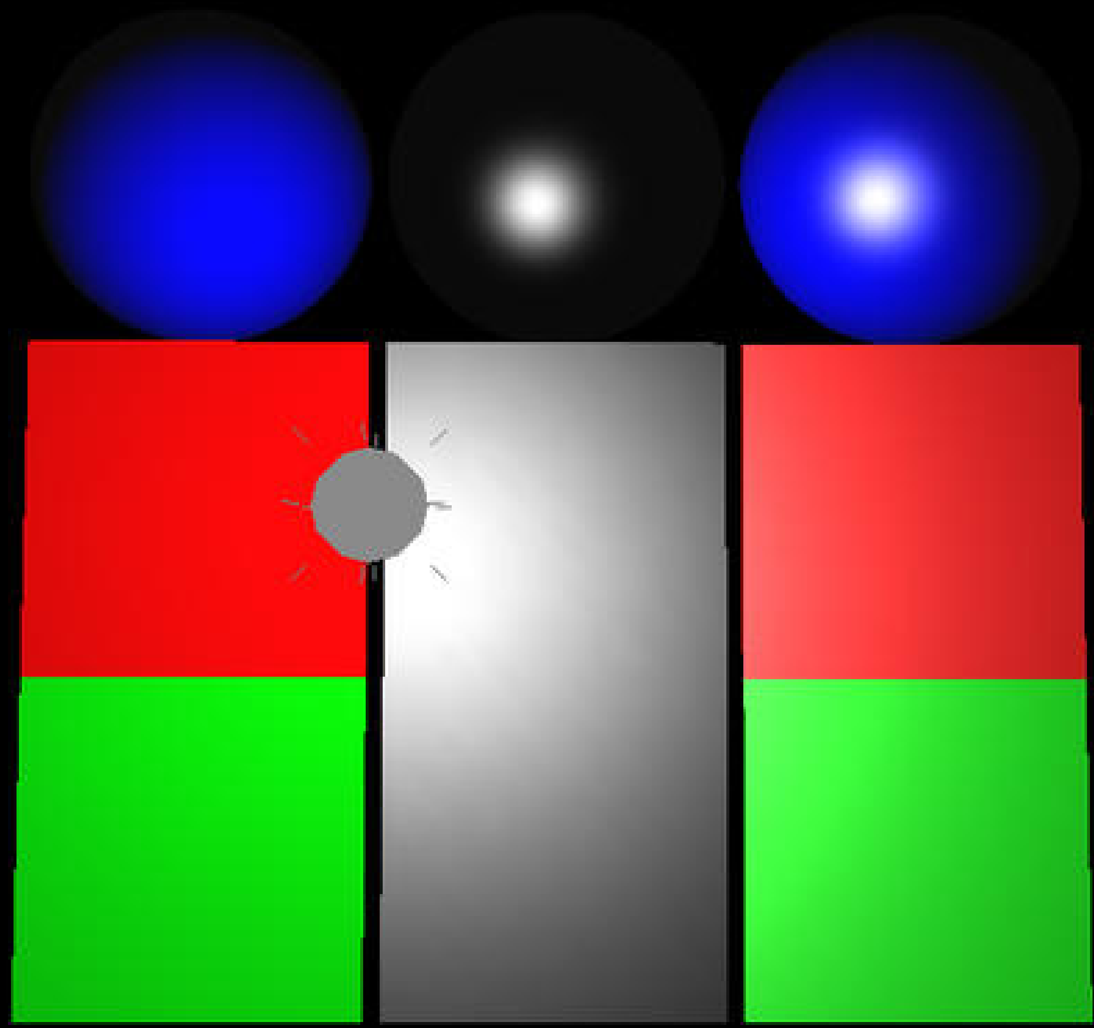
Diffuse Vs Specular



Rotx Roty Dolly

Diffuse Vs Specular



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Rotx Roty Dolly

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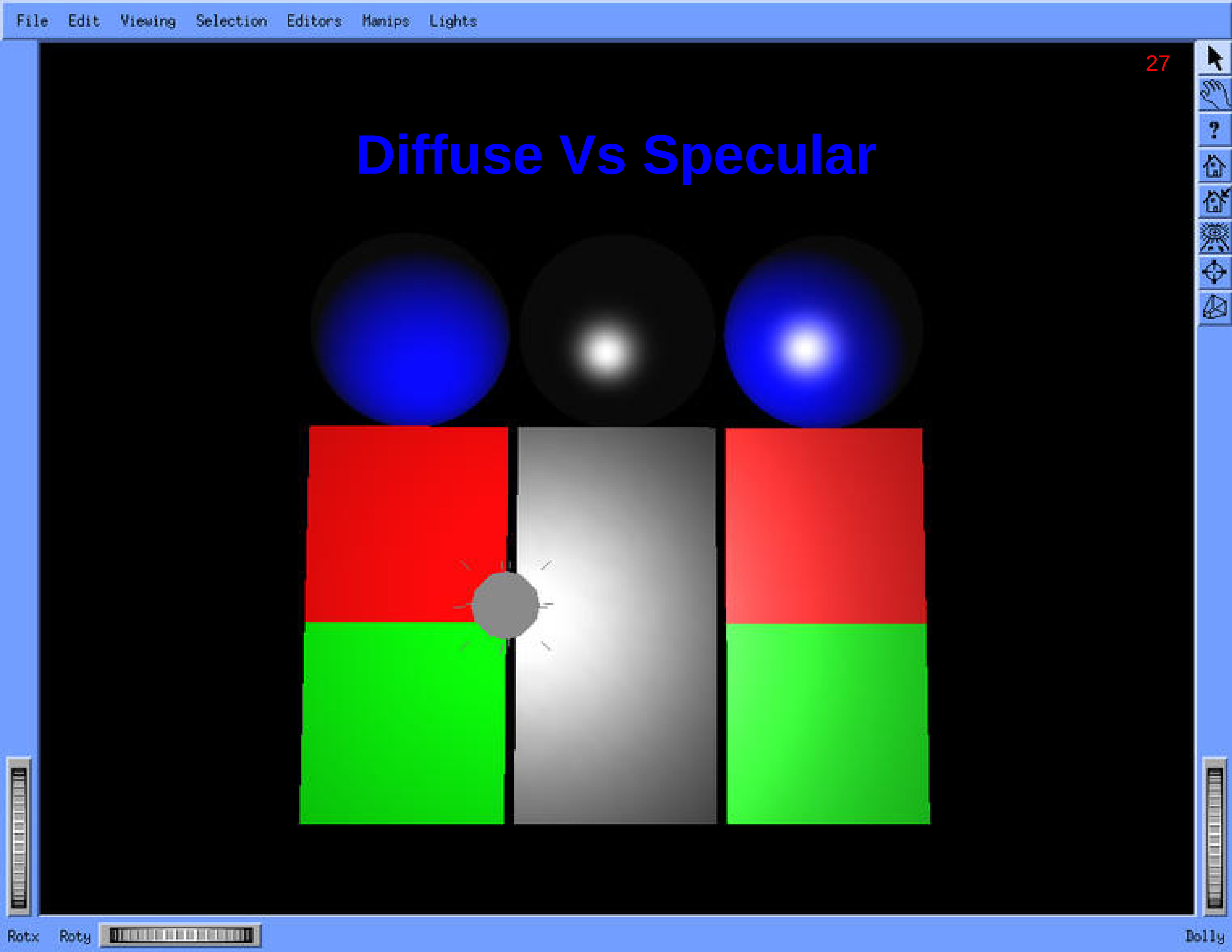

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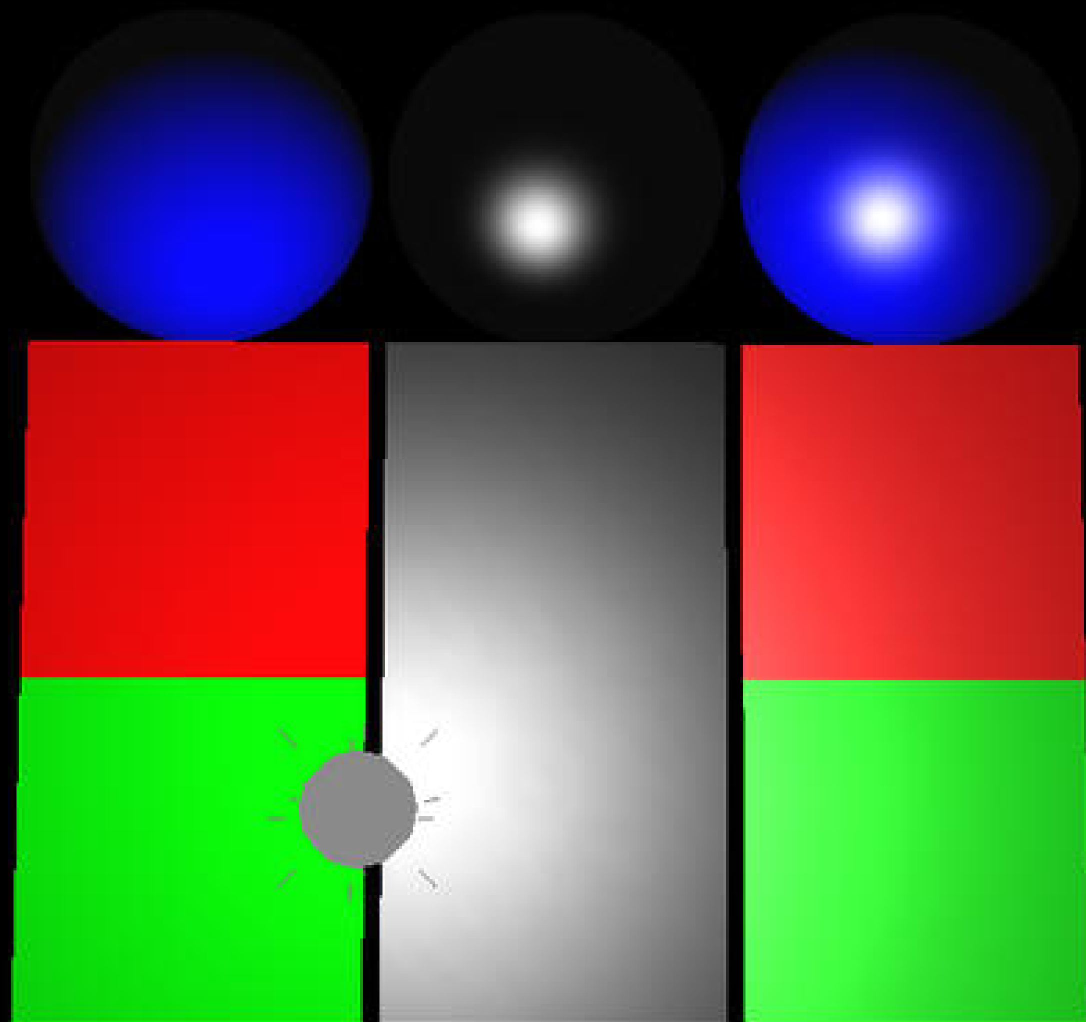

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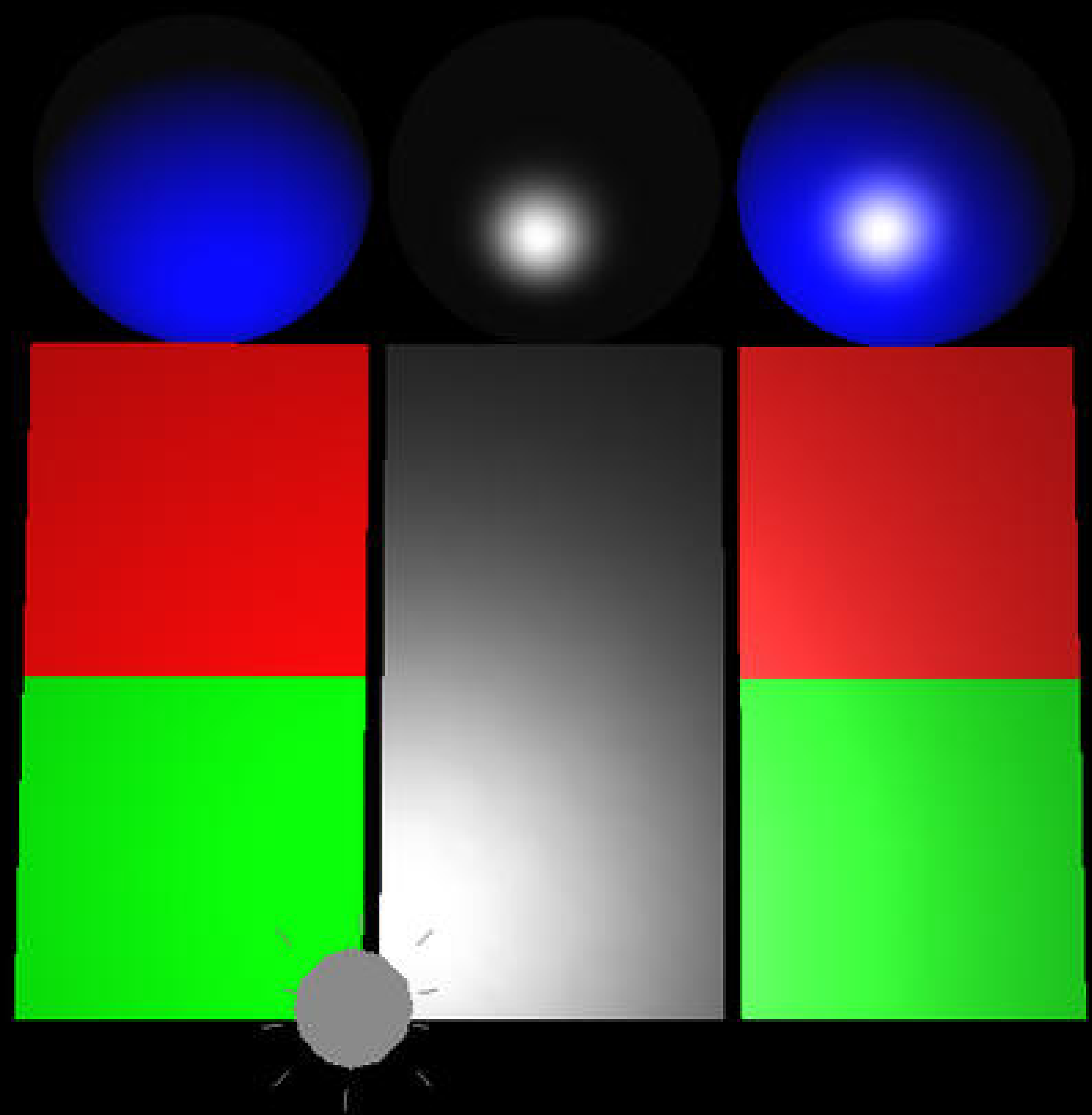

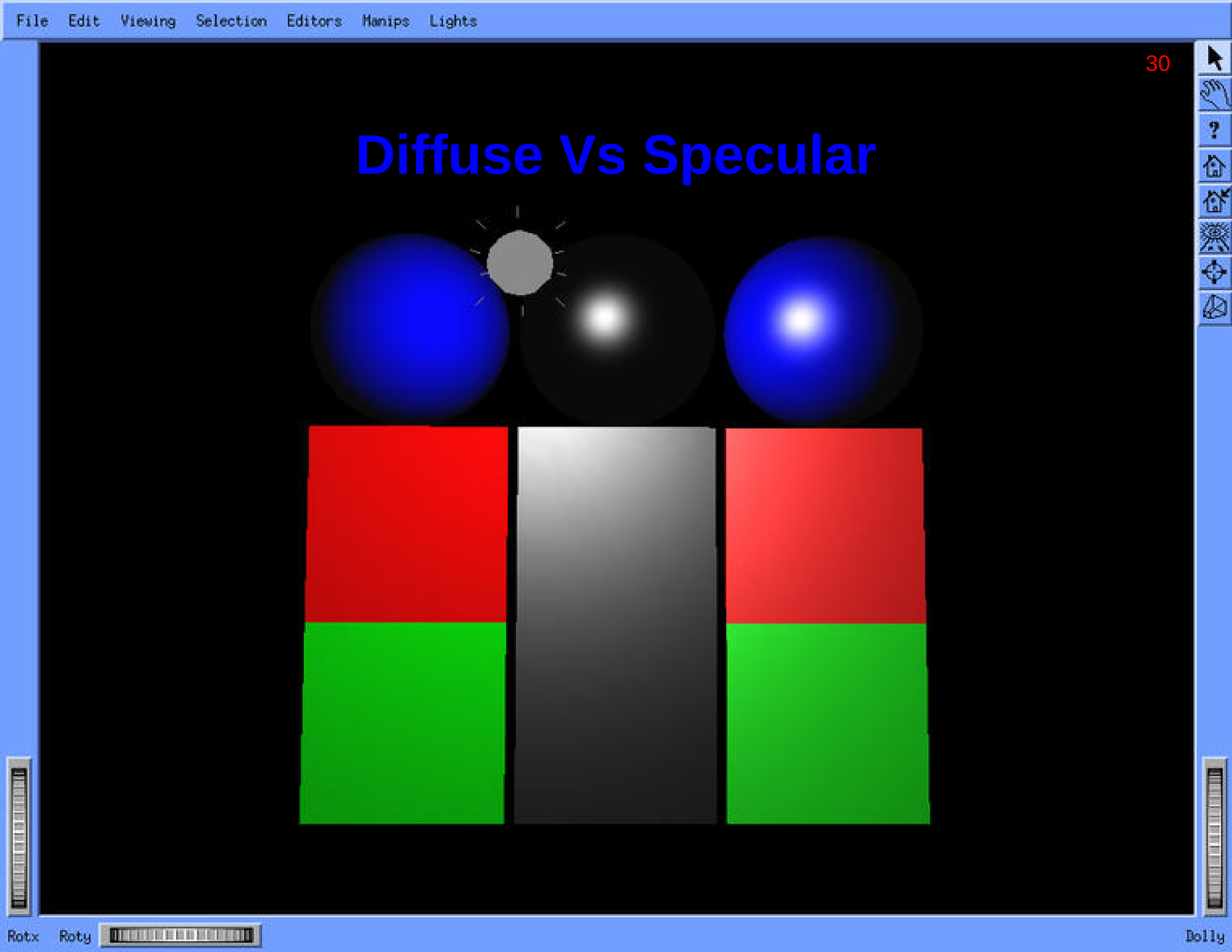
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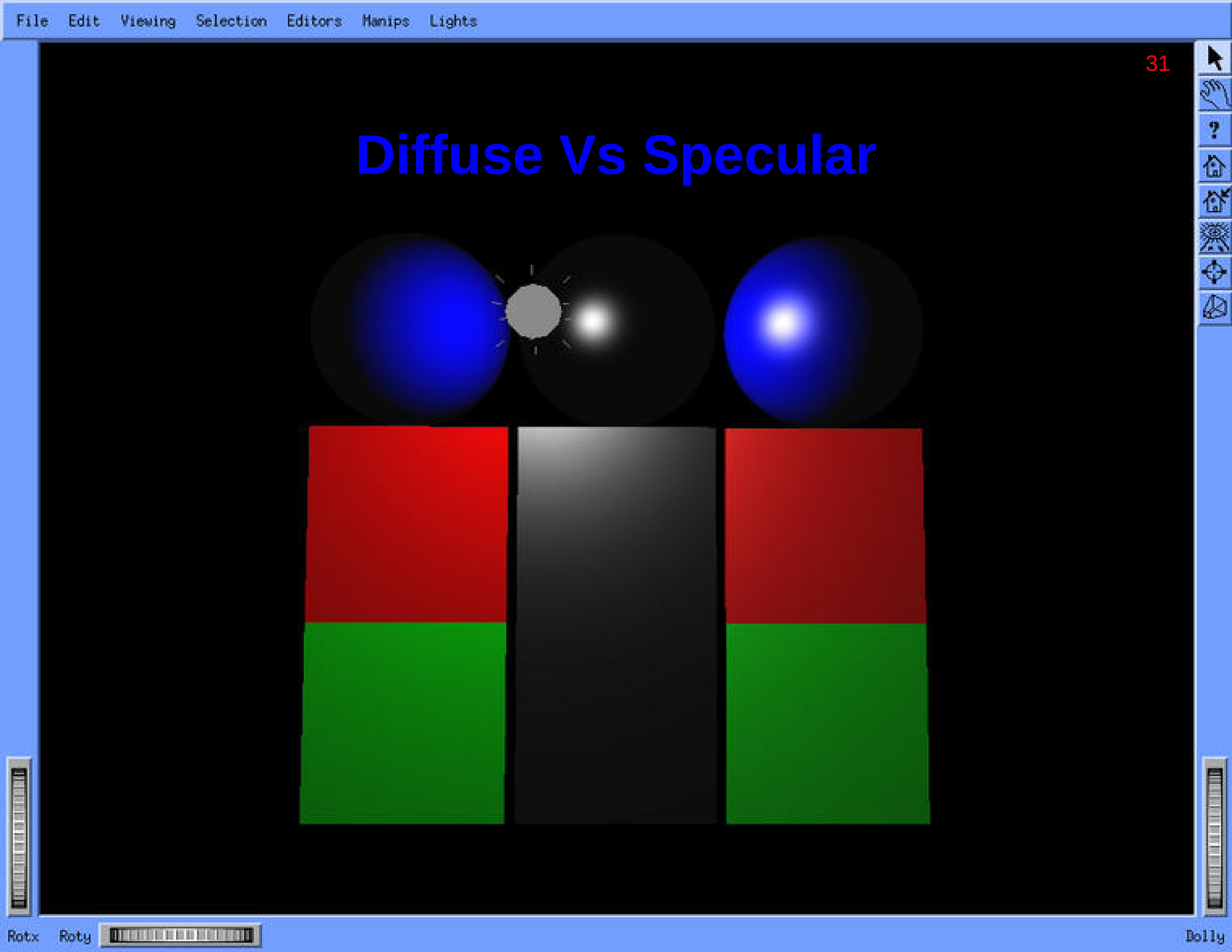

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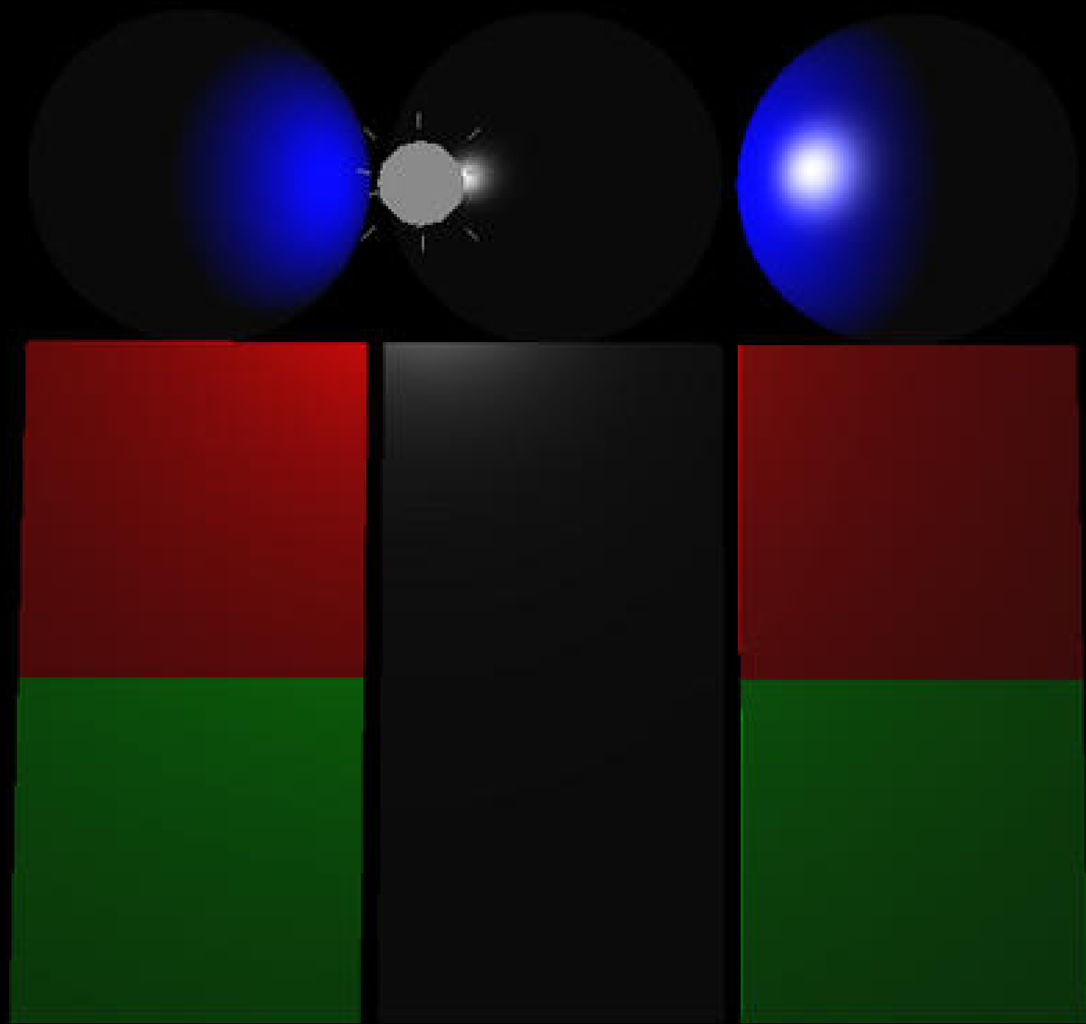

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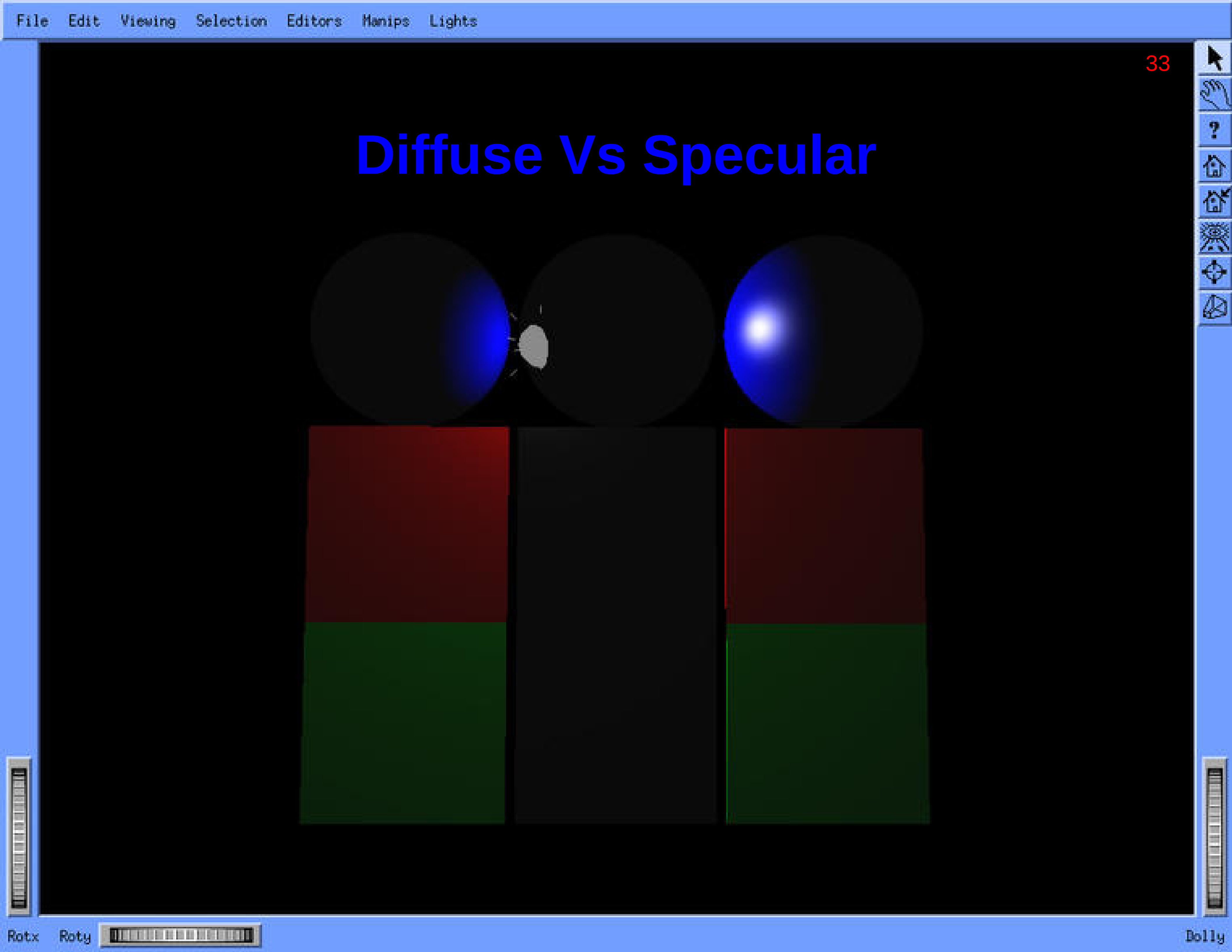


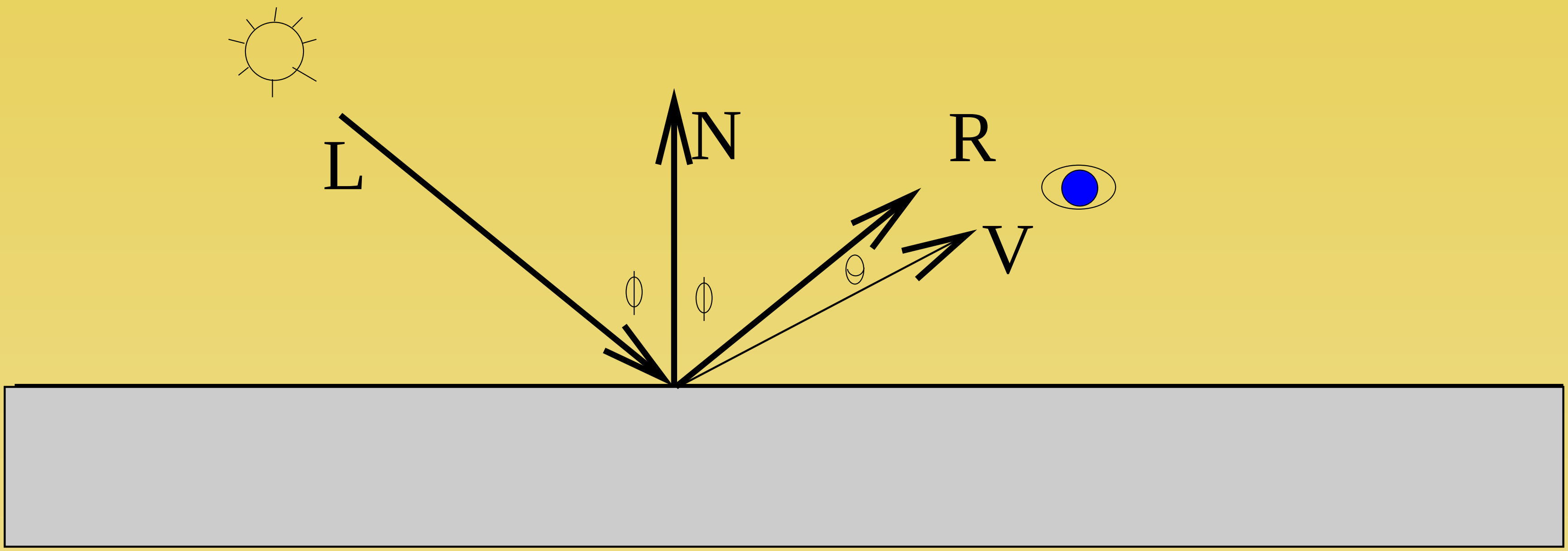
Specular Reflection

- The width of the reflective cone depends upon the smoothness of the surface.
- We can model this behavior using the Phong Illumination model.

$$I = k_p \cos^n \theta$$

where θ is the angle between the viewing direction and the natural angle of light reflection.

Phong Illumination: Specular

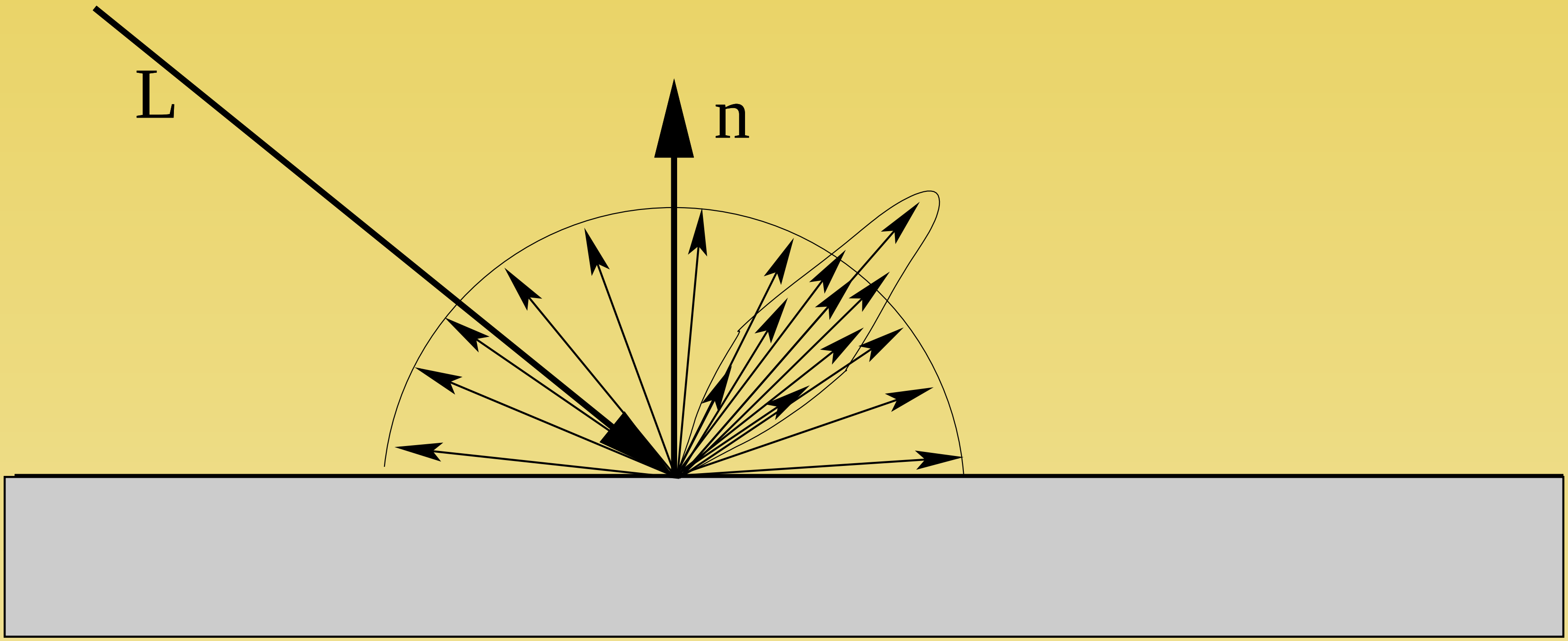


$$I = k_p \cos^n \theta$$

n denotes the shininess – $\uparrow n \downarrow$ highlight (cone) size

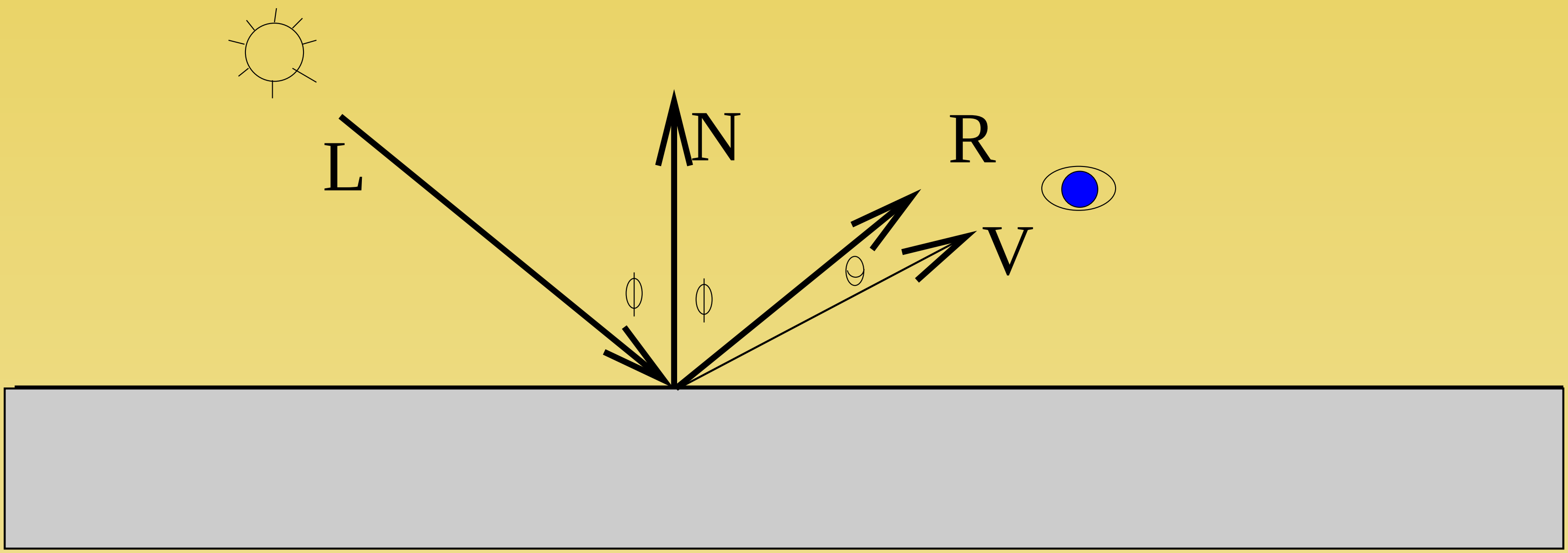
k_p is the Phong (specular) coeffecient (should be dependent on ϕ).

Diffuse + Specular Reflection



$$I = I_i k_d \cos\phi + k_s \cos^n \theta$$

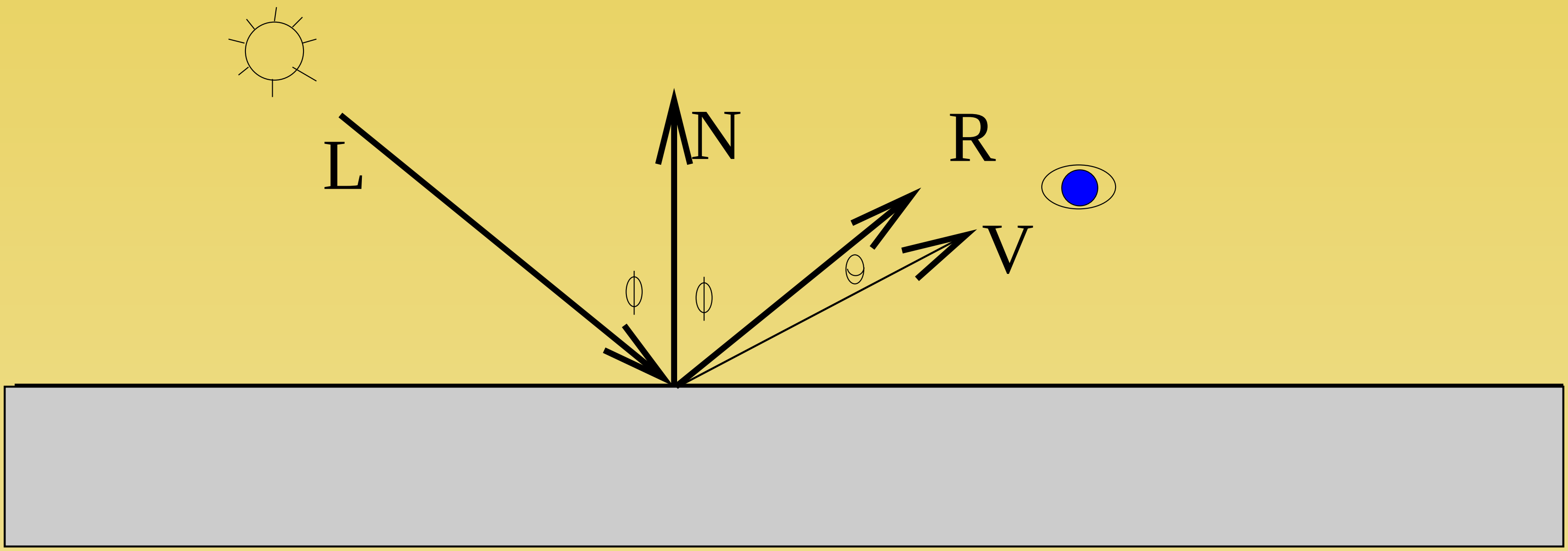
Phong Illumination



$$I = I_i k_d \cos\phi + k_s \cos^n \theta$$

$$I = I_i(k_d(L.N) + k_s(R.V)^n)$$

Phong Illumination Model



$$I = I_a k_a + \sum_j I_j (k_d (L.N) + k_s (R.V)^n)$$

Only lights that aren't in shadow are in summation.

Phong Illumination Model

- Does glass reflect the same for all viewing angles?

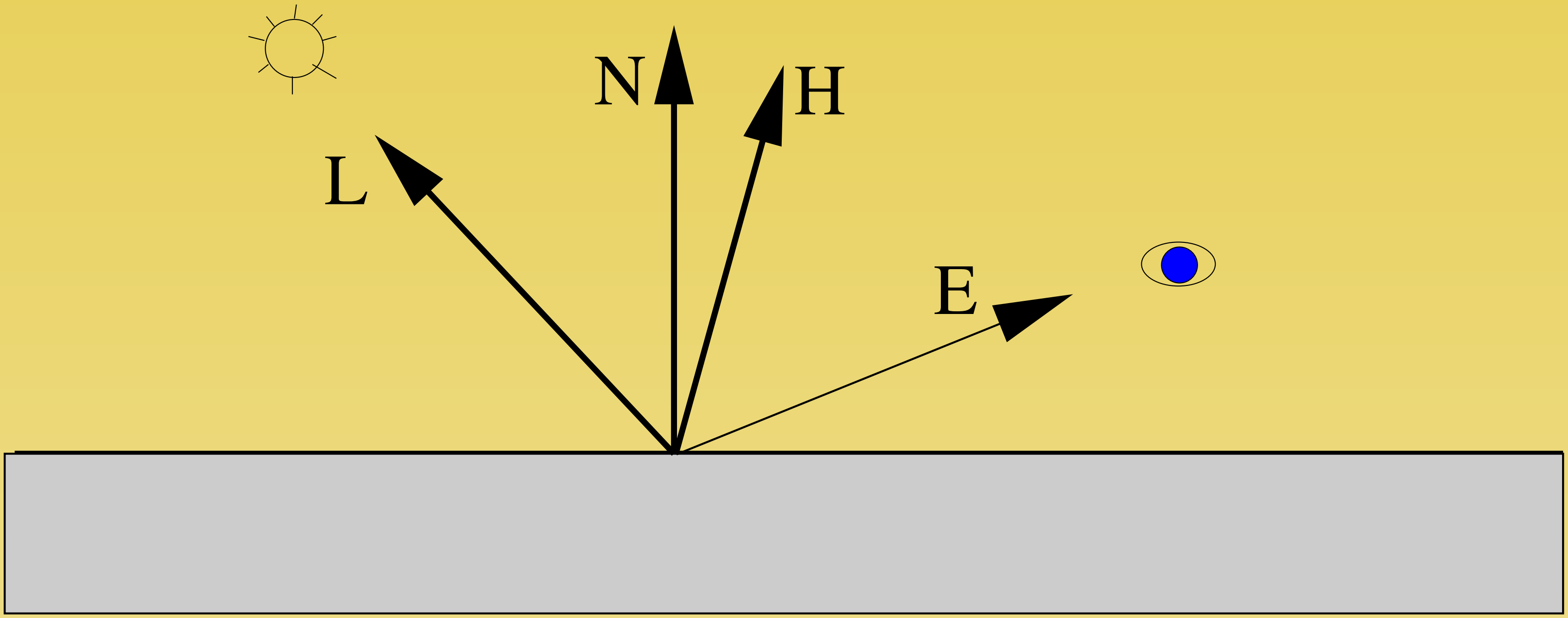
Phong Illumination Model

- Does glass reflect the same for all viewing angles?
- So why does Phong Illumination look like plastic?

Phong Illumination Model

- Does glass reflect the same for all viewing angles?
- So why does Phong Illumination look like plastic?
- Because its assumptions (k_s constant for all viewing angles, k_s dependent on incoming light and not the material.)

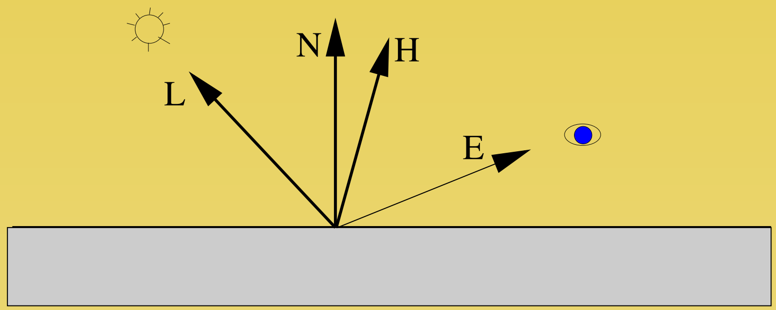
Blinn Illumination Model



$$H = \frac{L+E}{|L+E|} = \frac{L+E}{2}$$

H is the normal to a perfect specular reflector oriented such that the incident light is reflected to the eye.

Blinn Illumination Model



Comes from Torrance and Sparrow (1967) and matches experimental data.

Think of the surface as having a lot of micro facets in all directions. Those that have a normal pointing towards H, will reflect the light towards E.

Blinn Illumination Model

$$Specular = \frac{DGF}{N.E}$$

D - distribution function of micro facet directions.

G - amount the facets shadow and mask each other.

F - is the Fresnel reflection law.

Blinn Illumination Model

$$Specular = \frac{DGF}{N.E}$$

D - distribution function of micro facet directions.

G - amount the facets shadow and mask each other.

F - is the Fresnel reflection law.

$D(\alpha) =$ how many facets are pointing in the direction of the angle α. Where $cos\alpha = N.H$, $\beta = \alpha : D(\alpha) = \frac{1}{2}$

$D_1 = cos^{c_1}(\alpha)$	$c_1 = \frac{ln2}{lncos\beta}$	Phong
$D_2 = e^{-(\alpha c_2)^2}$	$c_2 = \frac{\sqrt{ln2}}{\beta}$	Torrance-Sparrow
$D_3 = (\frac{{c_3}^2}{cos^2\alpha({c_3}^2-1)+1})^2$	$c_3 = (\frac{cos^2\beta-1}{cos^2\beta-\sqrt{2}})^{\frac{1}{2}}$	Trowbridge-Reitz

Blinn Illumination Model

$$Specular = \frac{DGF}{N.E}$$

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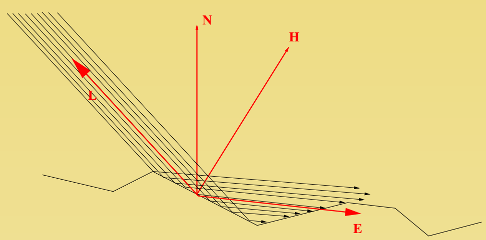
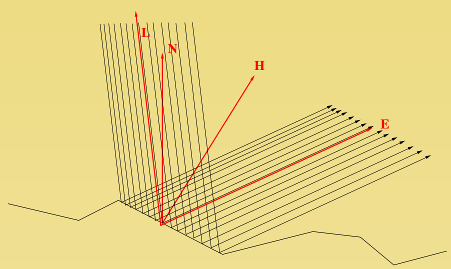
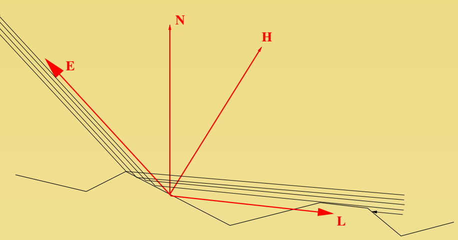
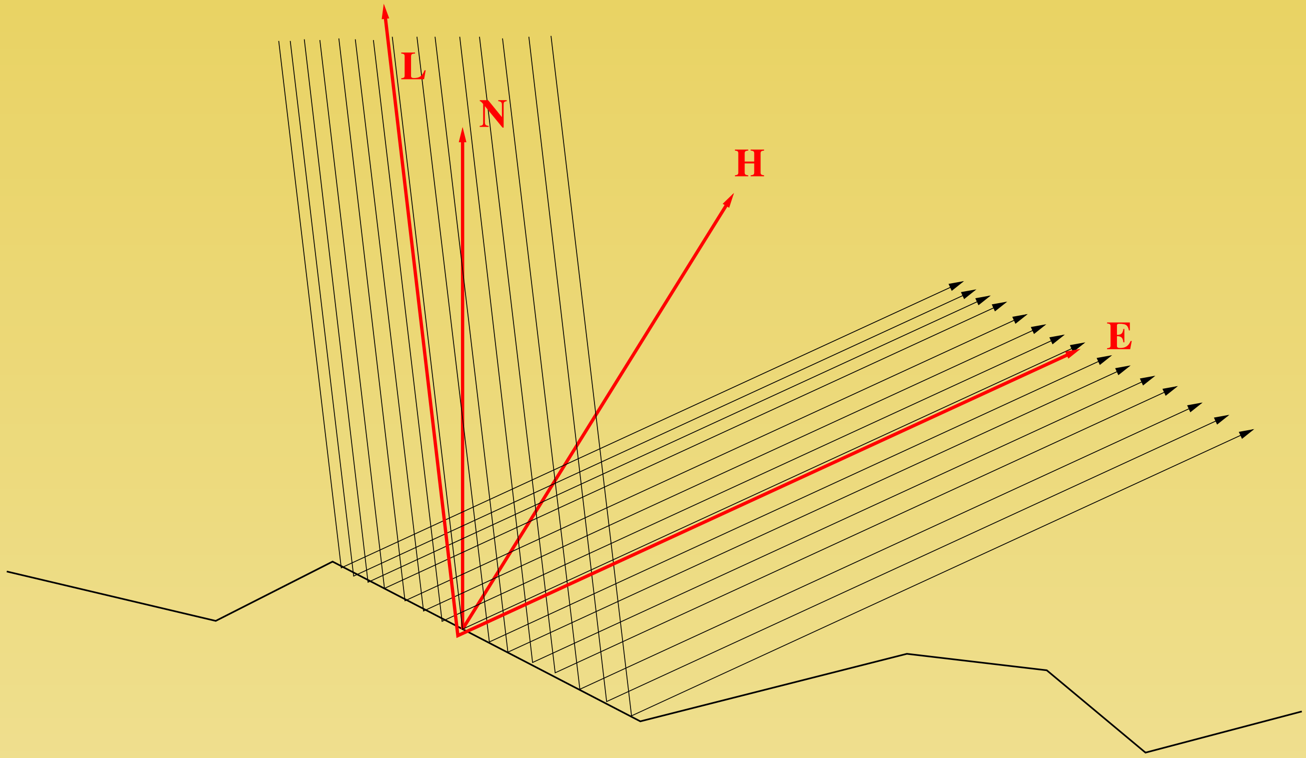






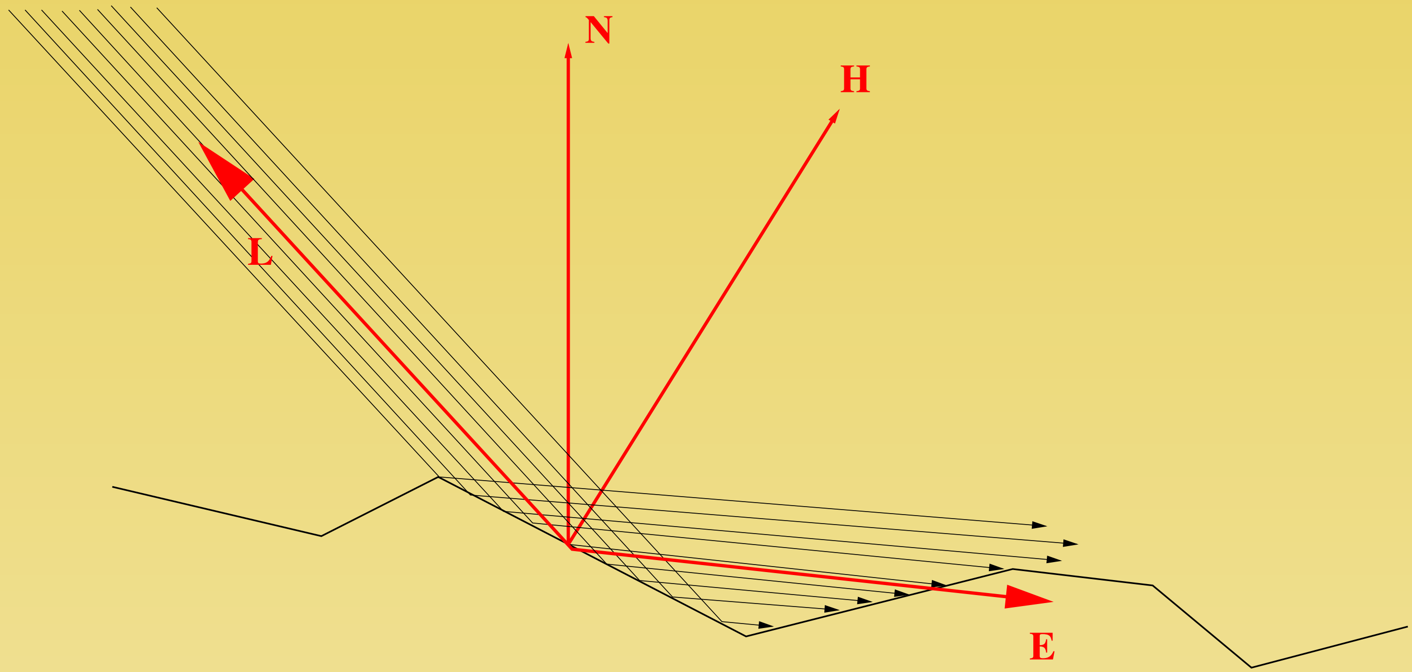
Geometric Attenuation

No attenuation - $Ga = 1.0$



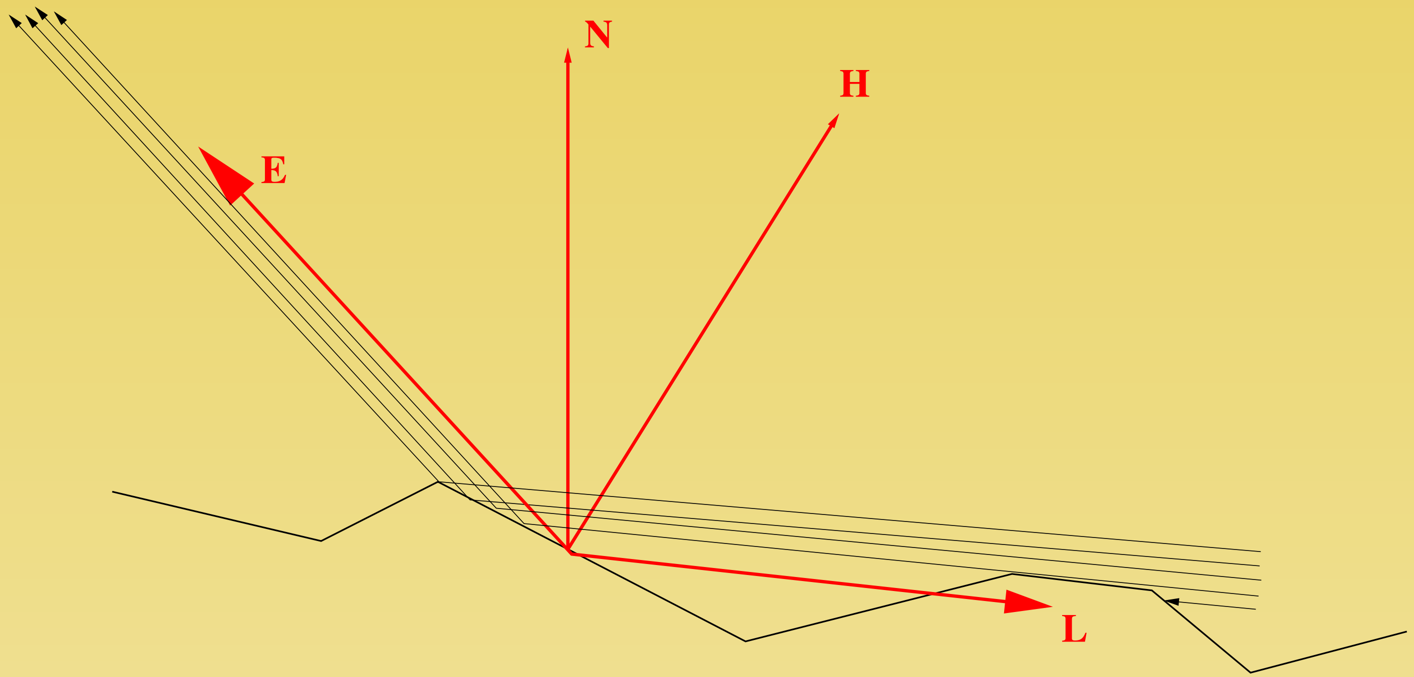
Geometric Attenuation

Blocked reflection - *Masking* $Gb = \frac{2(N.H)(N.E)}{E.H}$



Geometric Attenuation

Blocked incoming light - *Shadowing* $Gc = \frac{2(N.H)(N.L)}{E.H}$



Blinn Illumination Model

$$Specular = \frac{DGF}{N.E}$$

D - distribution function of micro facet directions.

G - amount the facets shadow and mask each other.

F - is the Fresnel reflection law.

$$G = min(Ga, Gb, Gc)$$

Blinn Illumination Model

$$Specular = \frac{DGF}{N.E}$$

D - distribution function of micro facet directions.

G - amount the facets shadow and mask each other.

F - is the Fresnel reflection law.

$$F = \frac{1}{2}\left(\frac{sin^2(\phi - \theta)}{sin^2(\phi + \theta)} + \frac{tan^2(\phi - \theta)}{tan^2(\phi + \theta)}\right)$$

Blinn Illumination Model

F determines how much light is reflected/absorbed.

$$F = \frac{1}{2}\left(\frac{sin^2(\phi - \theta)}{sin^2(\phi + \theta)} + \frac{tan^2(\phi - \theta)}{tan^2(\phi + \theta)}\right)$$

$sin(\theta) = sin\frac{\phi}{n}$

$\phi =$ angle of incidence

$n =$ index of refraction

Blinn Illumination Model

By some trig IDs:

$$F = \frac{(g-c)^2}{(g+c)^2}\left(1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right)$$

$c = E.H$

$g = \sqrt{n^2 + c^2 - 1}$

$n =$ index of refraction

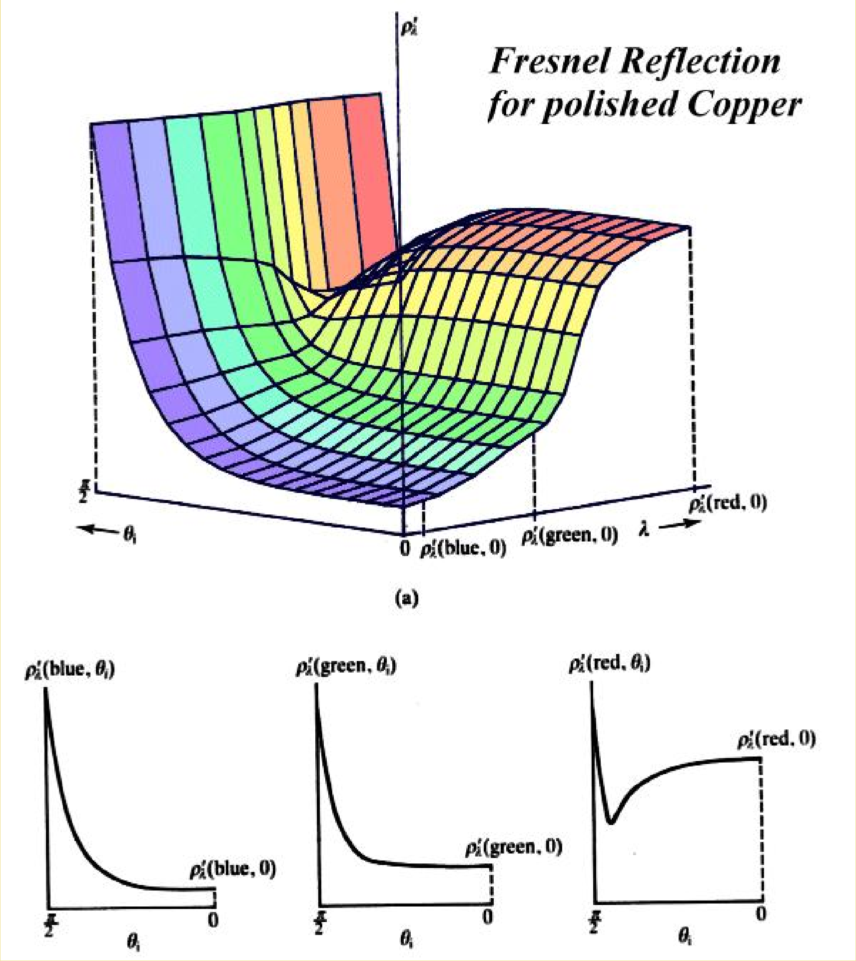
Cook Torrance Illumination Model

$$Specular = \frac{DGF}{\pi(N.L)(N.E)}$$

$$D = \frac{1}{m^2 cos^4\alpha} e^{-\frac{tan^2\alpha}{m^2}}$$

$$F = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right)$$

Cook Torrance Illumination Model



The reflectance curve for copper.

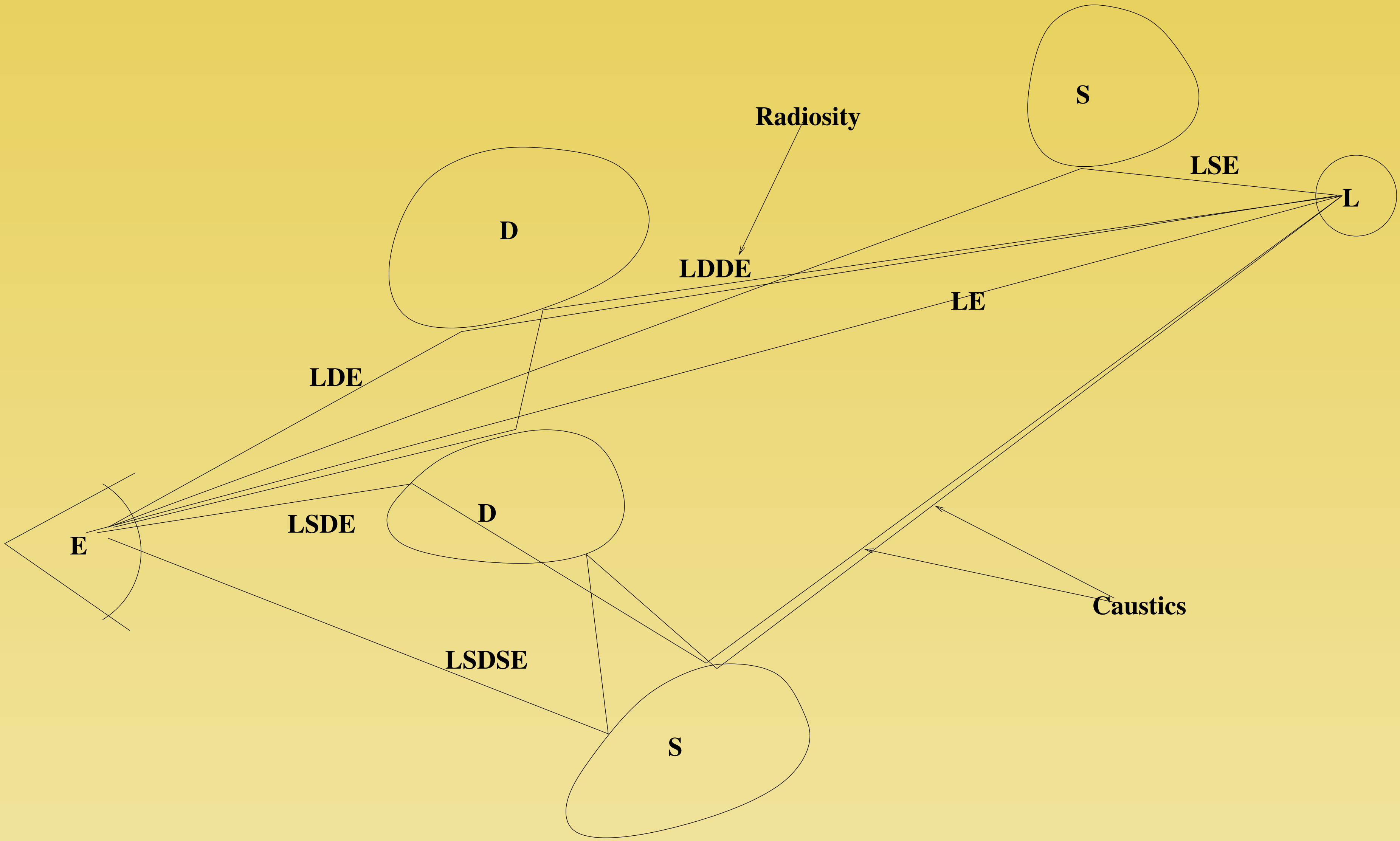
Types of light in a scene

- Light emittors (light bulb, sun)
- Light reflectors (any surface that reflects light)
- Don’t forget that light also scatters (this is hard to model.)

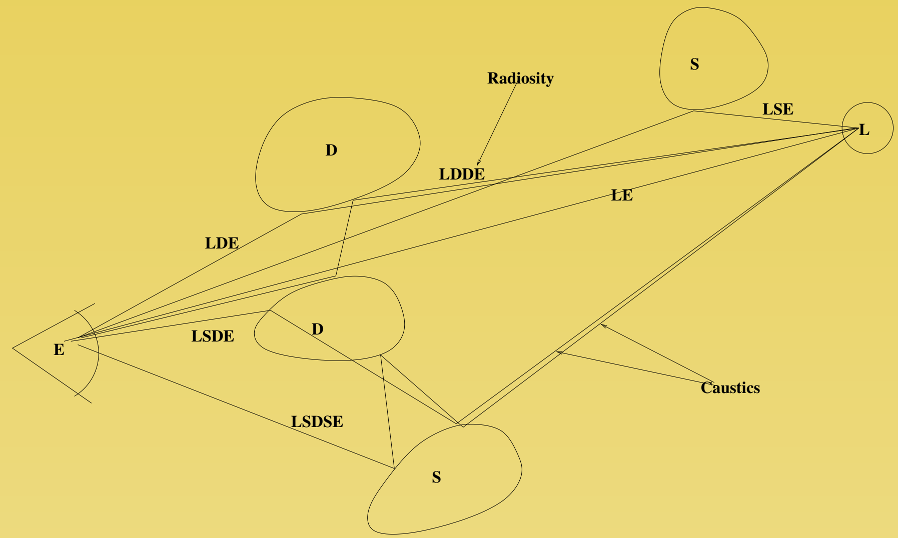
Light Transport Language

- If we denote:
  L - Light
  E - Eye
  D - Diffuse reflection/refraction(translucent)
  S - Specular reflection/refraction(transparent)
- We can specify the transport with a regular expressi-on, i.e. LDDE means light to diffuse to diffuse to eye.

Light Transport Language

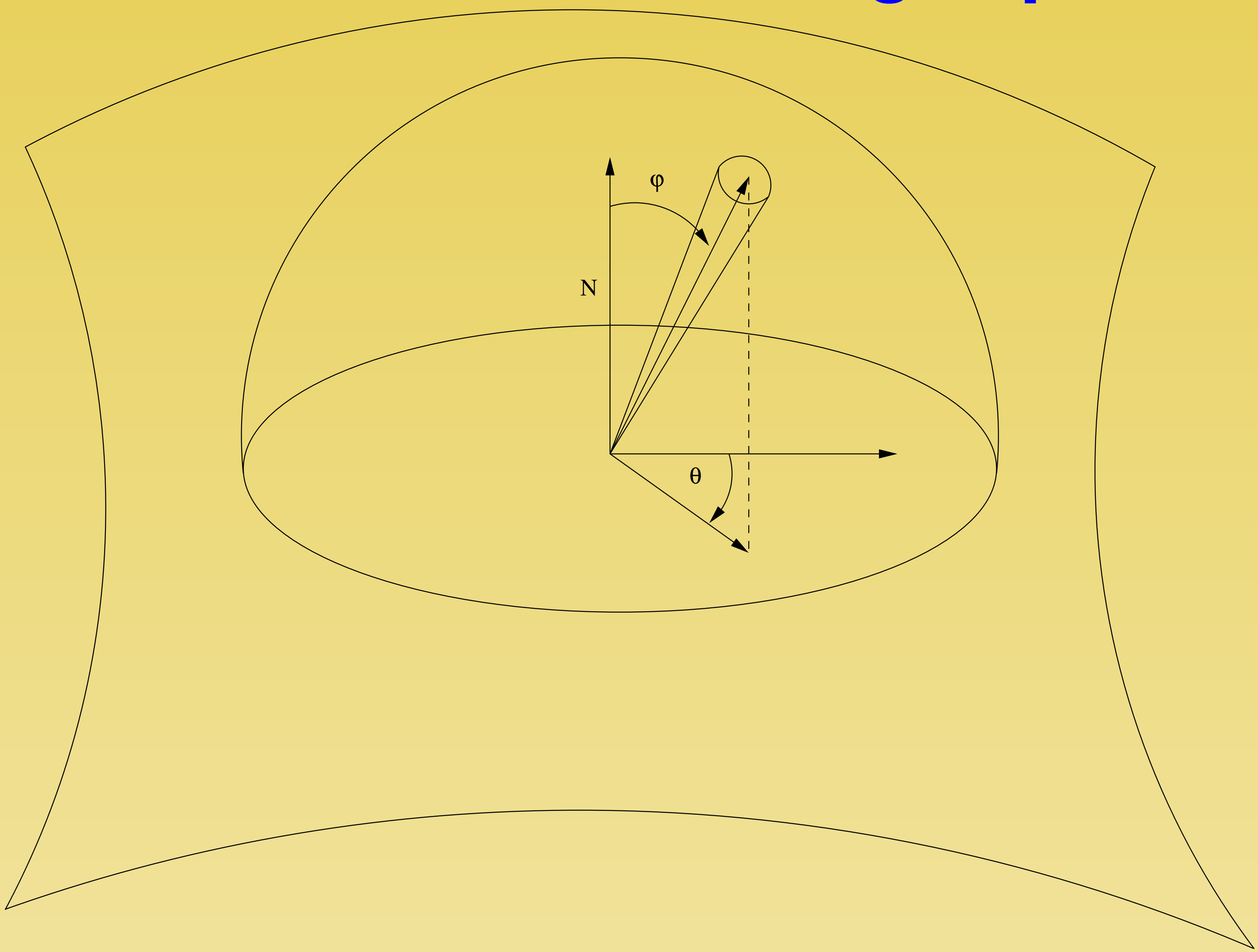


Light Transport Language



- What kind of interactions does raytracing do?
- What kind does radiosity do?
- What does photon tracing/light tracing do?

The Rendering Equation



The Rendering Equation

$$I_r(x,y,z) =$$

$$\int_{t=-\infty}^{\infty} \int_{\lambda=400}^{700} \int_{\varphi=0}^{\frac{\pi}{2}} \int_{\theta=0}^{\pi} L(t,x,y,z,\varphi,\theta,\lambda) R(t,\varphi,\theta,\lambda)\, d\theta\, d\varphi\, d\lambda\, dt$$

where:

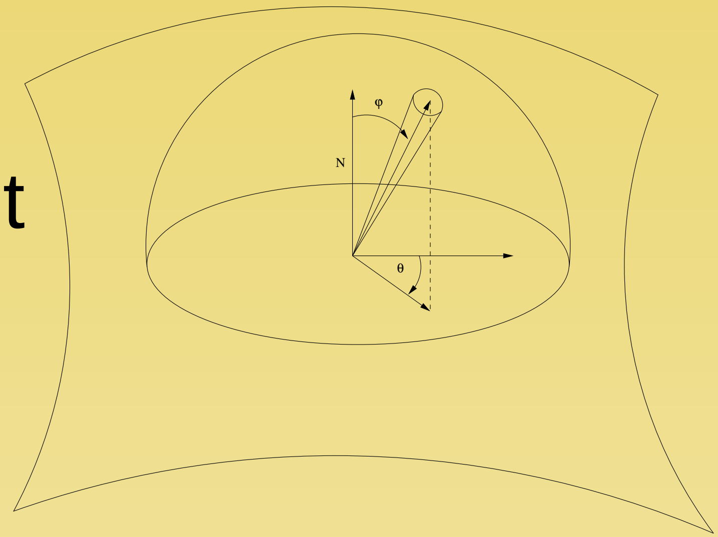
(x,y,z) - coords of surface point

t - time

λ - wavelength of light

φ - azimuthal angle (from N)

θ - angle about N



The Rendering Equation: Kajiya

$$I(x,x') = g(x,x')\left[e(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right]$$

The Rendering Equation: Kajiya

- Approximation to Maxwell’s Equations.
- A geometrical optics approximation.
- Time-averaged transport intensity.
- Balances the energy flow through scene.

The Rendering Equation: Kajiya

- Doesn’t deal with time. No phase.
- No diffraction.
- Assumes constant index of refraction between surfa-ces.

The Rendering Equation: Kajiya

$$I(x,x') = g(x,x')\left[\epsilon(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right]$$

$I(x,x')$ - Intensity of light passing from pt. x' to pt. x
$g(x,x')$ - Geometry term.
$\epsilon(x,x')$ - Emitted light from x' to x.
$\rho(x,x',x'')$ - Light transported from x'' to x via x'

Integral taken over $S = \bigcup S_i$, where S_i are the surfaces, with special background surface S_0.

The Rendering Equation: Kajiya

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$I(x,x')$ - Intensity of light passing from pt. x' to pt. x

- *Unoccluded two point transport intensity*.
- Energy of the radiation per unit time per unit area of source dx' per unit ared dx of the target.
- $\frac{Joule}{m^4 s} = \frac{watt}{m^4}$

The Rendering Equation: Kajiya

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$g(x,x')$ - Geometry term.

- Encodes inclusion of surface pts by other surface points.
- 0 if x and x' are not mutually visible.
- If they are visible then $g = \frac{1}{r^2}$ where $r = |x - x'|$

The Rendering Equation: Kajiya

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$I(x,x')$ - Intensity of light passing from pt. x' to pt. x

$g(x,x')$ - Geometry term.

$\epsilon(x,x')$ - Emitted light from x' to x.

$\rho(x,x',x'')$ - Light transported from x'' to x via x'

The Rendering Equation: Kajiya

$$I(x,x') = g(x,x')\left[\epsilon(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right]$$

$\epsilon(x,x')$ - Emitted light from x' to x.

- *Unoccluded two point transport emittance.*
- Energy emitted per unit time per unit area of source dx' per unit ared dx of the target.
- $\frac{Joule}{m^4 s} = \frac{watt}{m^4}$

The Rendering Equation: Kajiya

$$I(x,x') = g(x,x')\left[\epsilon(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right]$$

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The Rendering Equation: Kajiya

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$\rho(x,x',x'')$ - Light transported from x'' to x via x'

- The *scattering* term.
- Intensity of energy scattered by a surface element
- *unoccluded three point transport reflectance* from x'' to x through x'.

Maxwell's equations for Electromagnetics

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{D} = \rho$$

E - electric field ($\frac{volts}{meter}$)

D - electric flux density (displacement) ($\frac{coulombs}{meter^2}$)

$D = \epsilon_0 E + P$ ϵ_0 - permittivity of free space,

P the polarization.

μ_0 - permeability of free space.

Maxwell's equations for Electromagnetics

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{D} = \rho$$

H - magnetic field ($\frac{amps}{meter}$

$H = B/\mu_0 - M$

B - magnetic flux density (induction) ($\frac{webers}{meter^2}$)

J - electric conduction current density ($\frac{amps}{meter^2}$)

r - volume charge density ($\frac{coulombs}{meter^3}$).

Maxwell’s equations for Electromagnetics

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{D} = \rho$$

$\nabla \cdot \vec{D} = \sum_{i=1}^{3} \frac{\partial v^i}{\partial x^i} = \partial_I v^i$,

$\nabla \cdot \vec{D}$ divergence of $\vec{D}$ indicates the strength.

$\nabla \times \vec{E}$ curl of tensor field $\vec{E}$

The curl indicates the direction a wind vane would turn (time derivative of the motion).

Maxwell's equations for Electromagnetics

- for example the curl of the velocity is the circulation
- and the divergence is the rate of flow out of a small volume per unit volume.
- Divergence and curl are local (microscopic) properties of a vector field (dilation and rotation),
- They corresesond to macroscopic properties of the surface integral of the field over a closed surface (flux).

The Rendering Equation: Kajiya

- Doesn’t deal with time. No phase.
- No diffraction.
- Assumes constant index of refraction between surfaces.

The Eikonal Equation

$$z_x^2 + z_y^2 = h(x, y)$$

$$\sum_{i=1}^{n} \left(\frac{\partial u}{\partial x_i} \right)^2 = 1$$

$$(\nabla s)^2 = n^2$$

s is a scalar function of position,
n is the refractive index.

The Eikonal Equation

- From greek eikon ïmage”.
- The eikonal equation corresponds to a geometric description of the propagation of light.
- If the phase and amplitude are slowly varying functions of position.
- Determines the evolution of the phase
- The surfaces of constant phase, the wave fronts, define the shape of the radiation field.
- The normals to the wavefronts are rays.

BRDF

$$\rho(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{dL_i(\theta_r, \phi_r)}{dE_i(\theta_i, \phi_i)}$$

- E - irradiance,
- L - reflected radiance.
- ϕ_i, θ_i, incoming light angles.
- ϕ_r, θ_r, reflected light angles.
- Bidirectional Reflection Distribution Function

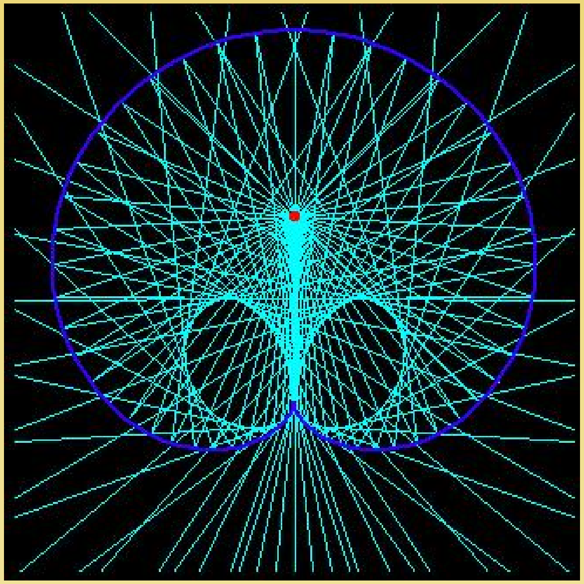
http://www1.cs.columbia.edu/CAVE/curet/html/sphere_meas.h

Caustics: Examples

- Concentrated light reflections.
- Light refracted through a transparent surface.
- Light bouncing off of a mirror.
- Light focussed by a lens.
- Light focussed by a liquid.
- A visual effect seen when light is reflected off a specular or reflective surface, or focused through a refractive surface, so that it indirectly illuminates other surfaces with focused light patterns.

Caustics: What are they

- (Mathworld) The curve which is the envelope of reflected (catacaustic) or refracted (diacaustic) rays of a given curve for a light source at a given point (known as the radiant point).
- The envelope of a family of curves is a curve which touches every member of the family. Actually is tanget to every member.



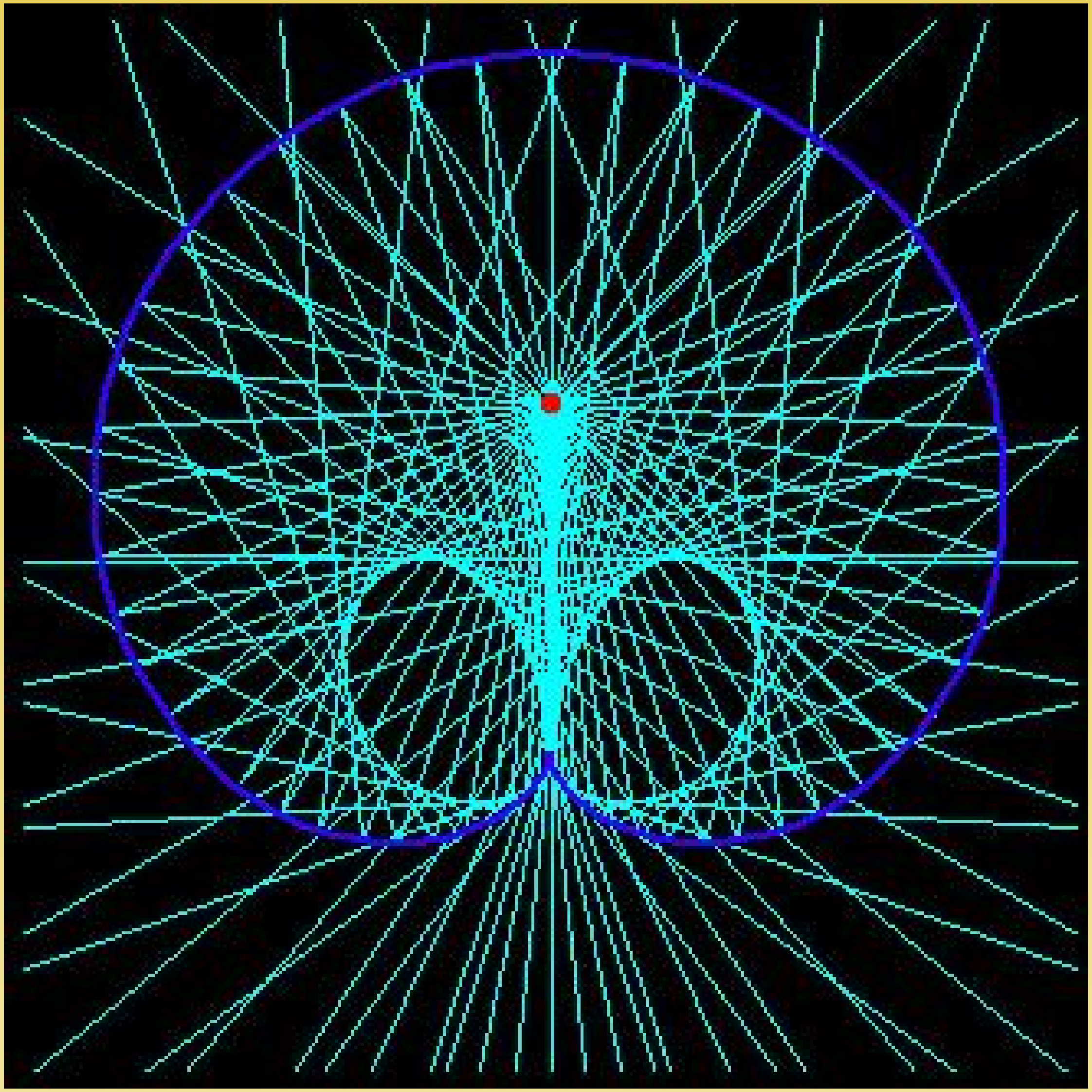
Caustics: What are they

- If light from some source (radiant) reflects off of a curve, then the envelope of the reflected rays are a type of caustic known as a catacaustic. (cata - from the greek to signify down, or opposed to.)
- When light refracts off of a curve, the resulting envelope is a diacaustic (dia - from the greek, through or across)

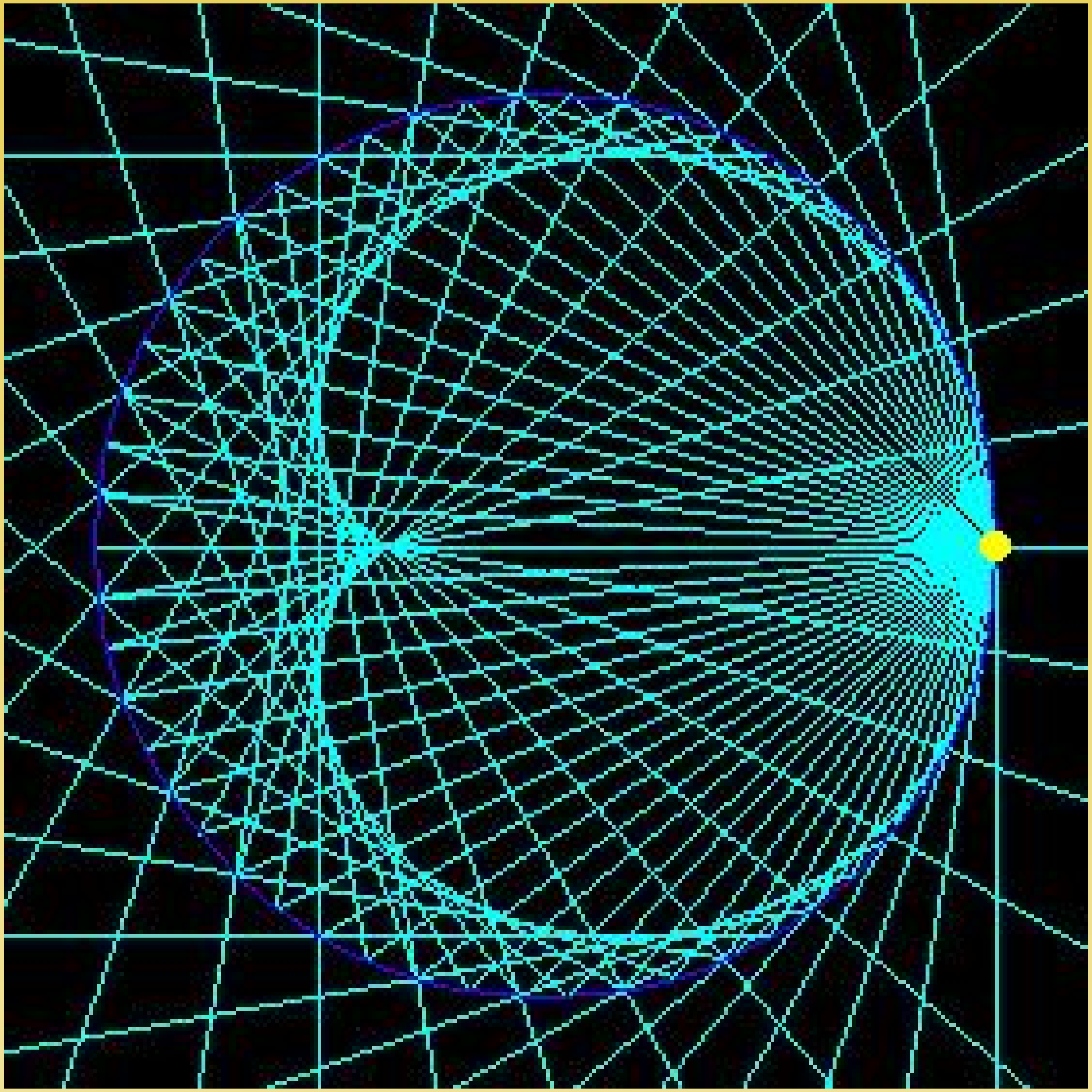
Caustics: What are they

- Caustic is a method of deriving a new curve based on a given curve and a point. A curve derived this way may also be called caustic. Note light rays may also be parallel, as when the light source is at infinity.
- Caustics were first studied by Huygens and Tschirnhaus around 1678. As well, Johann Bernoulli, Jacob Bernoulli, de l'Hpital and Lagrange studied caustic curves.
- http://www.cacr.caltech.edu/ roy/Caustic/

Catacaustic of a cardioid

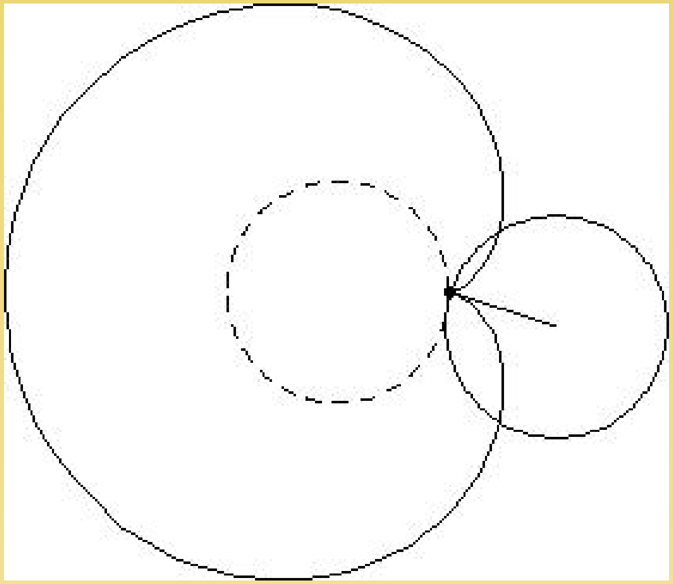
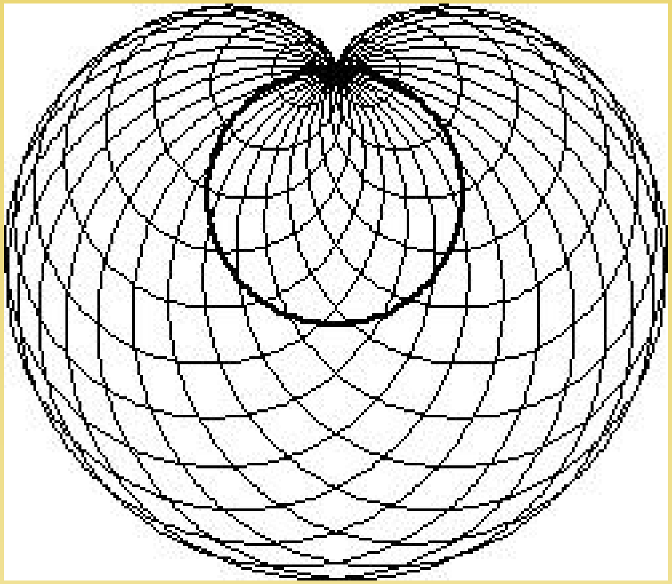
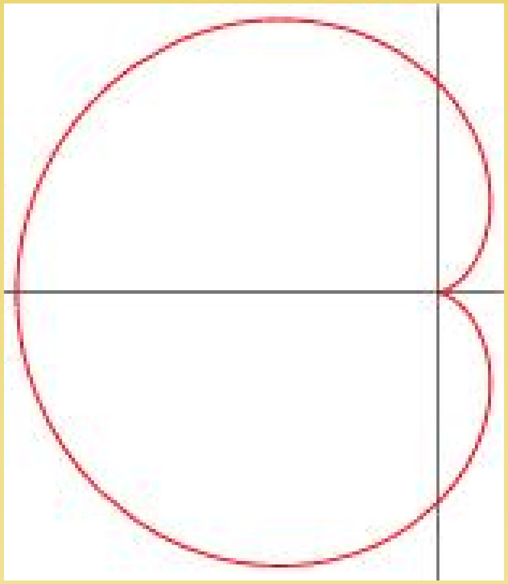


Catacaustic of a circle is a cardioid



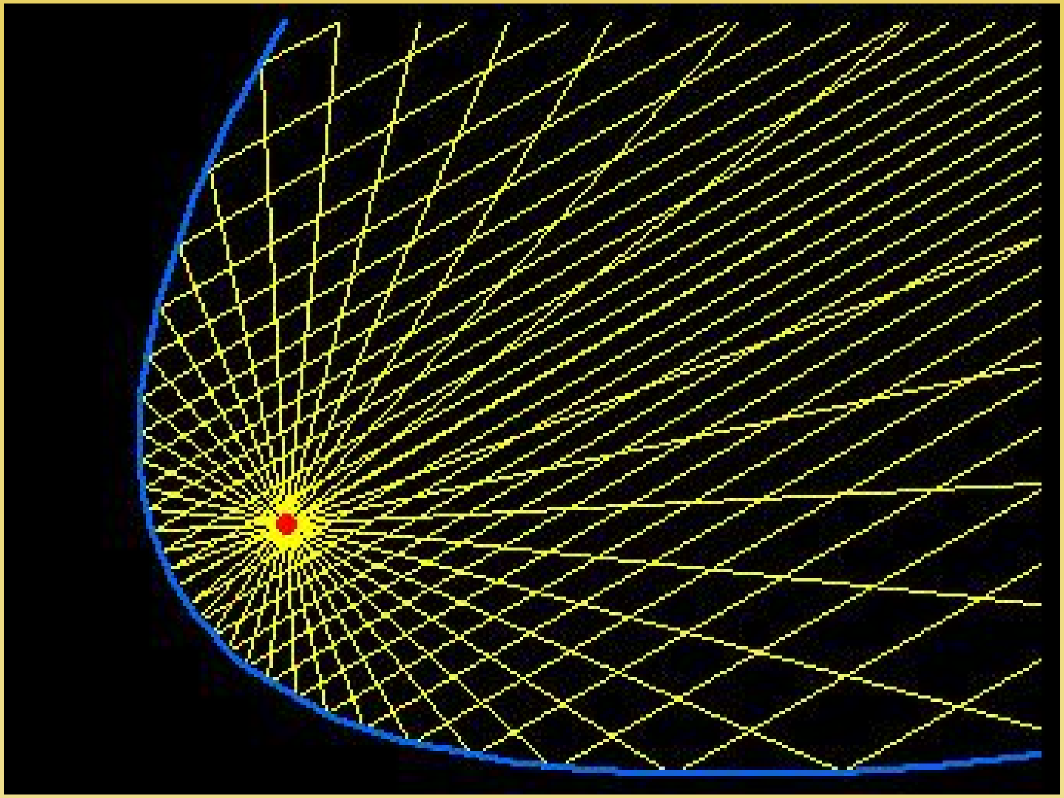
What is a Cardioid

- heart-shaped.
- a cardioid is the catacaustic of a circle with the light source on the circle.

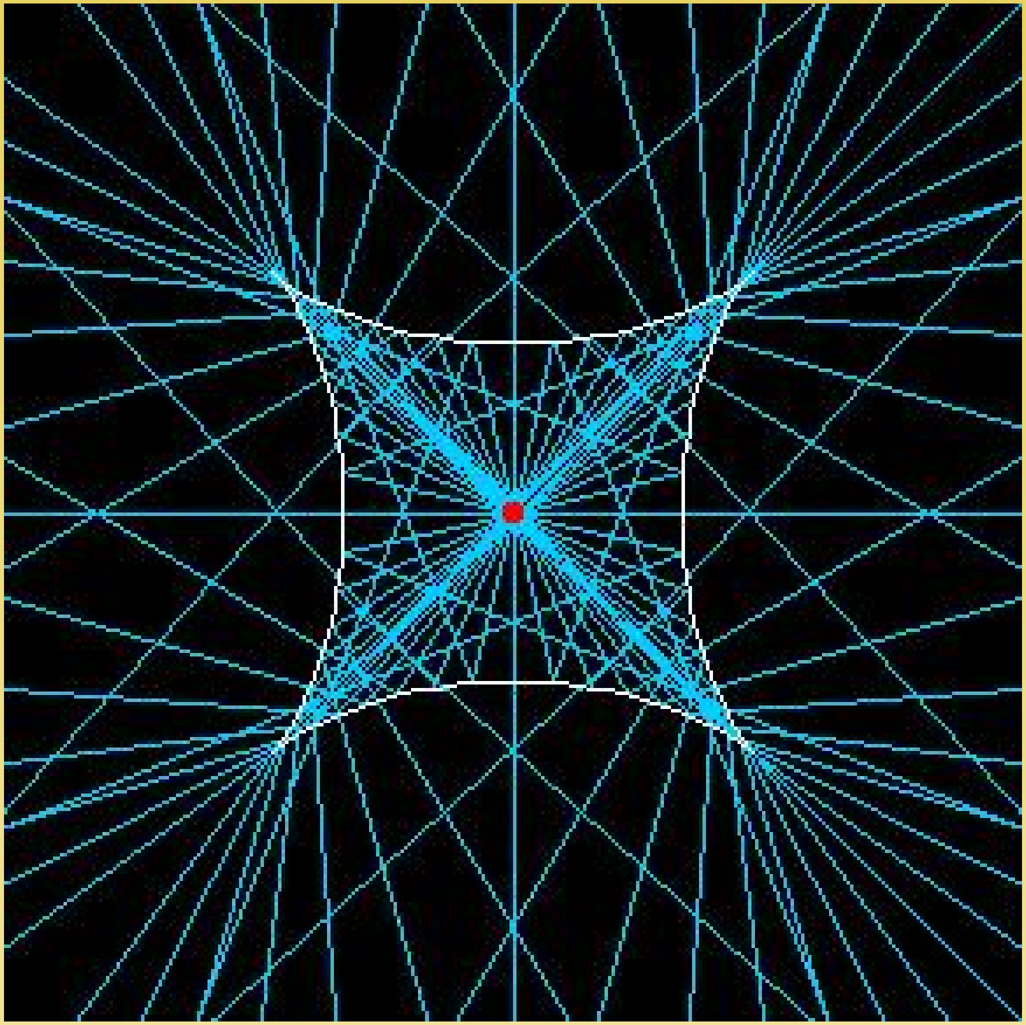
Caustics aren't always a curve.

Caustics do not always generate a curve, as in light rays reflecting from the focus of a parabola.



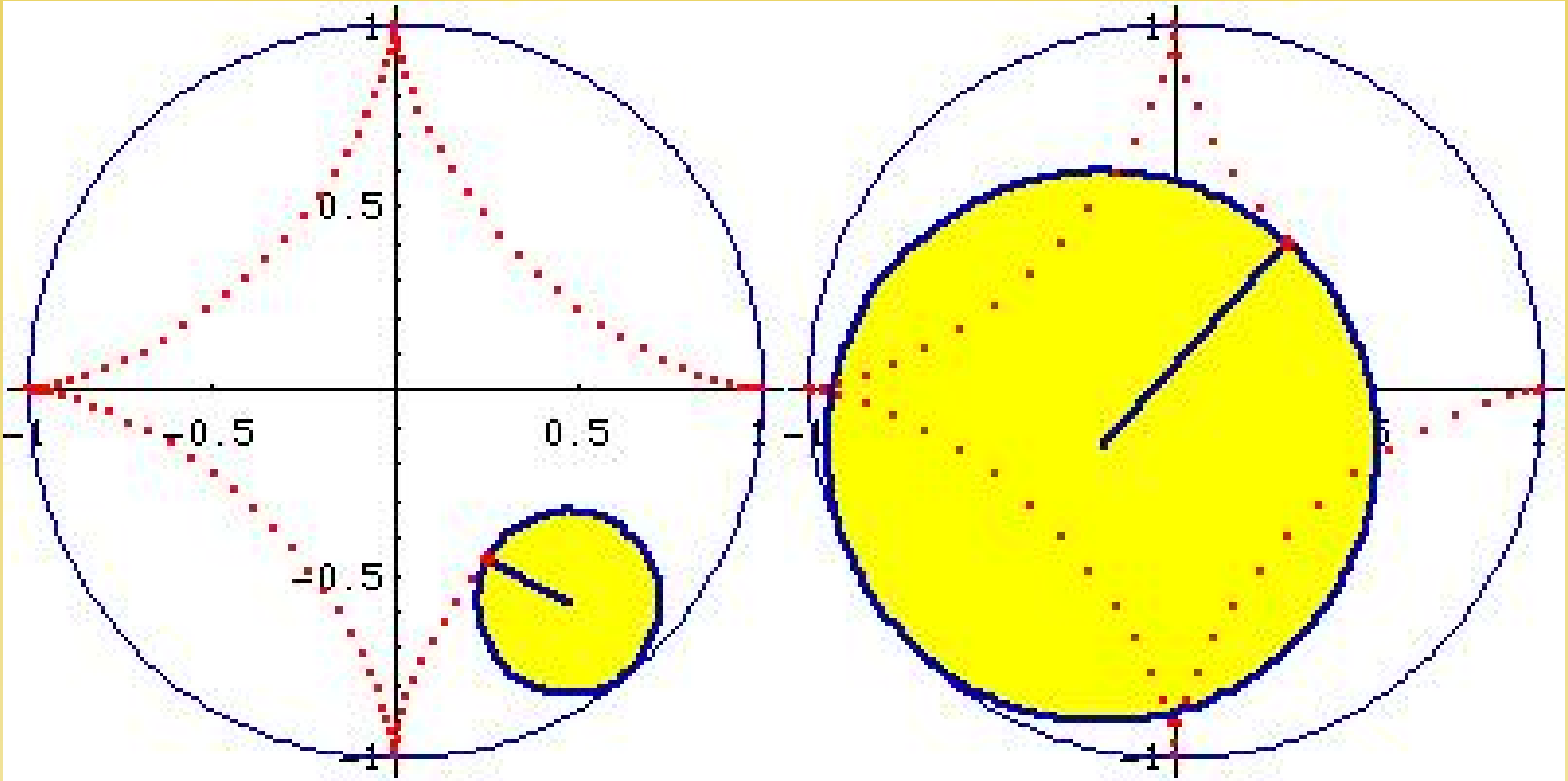
Caustics aren’t always a curve.

The catacaustic of an astroid.



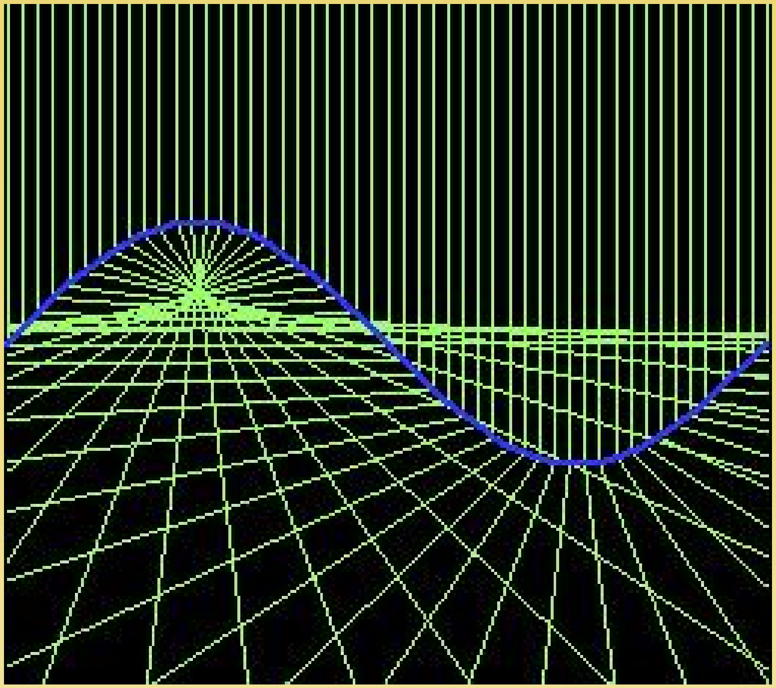
What is an Astroid?

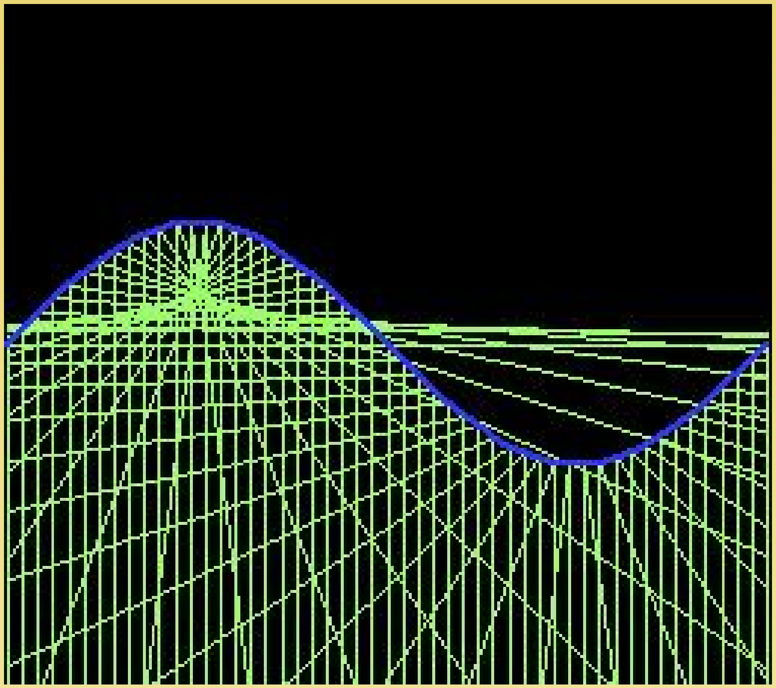
An astroid is the trace of a point on a circle rolling along the inside of a circle with radius $4r$ or $\frac{4}{3}r$.



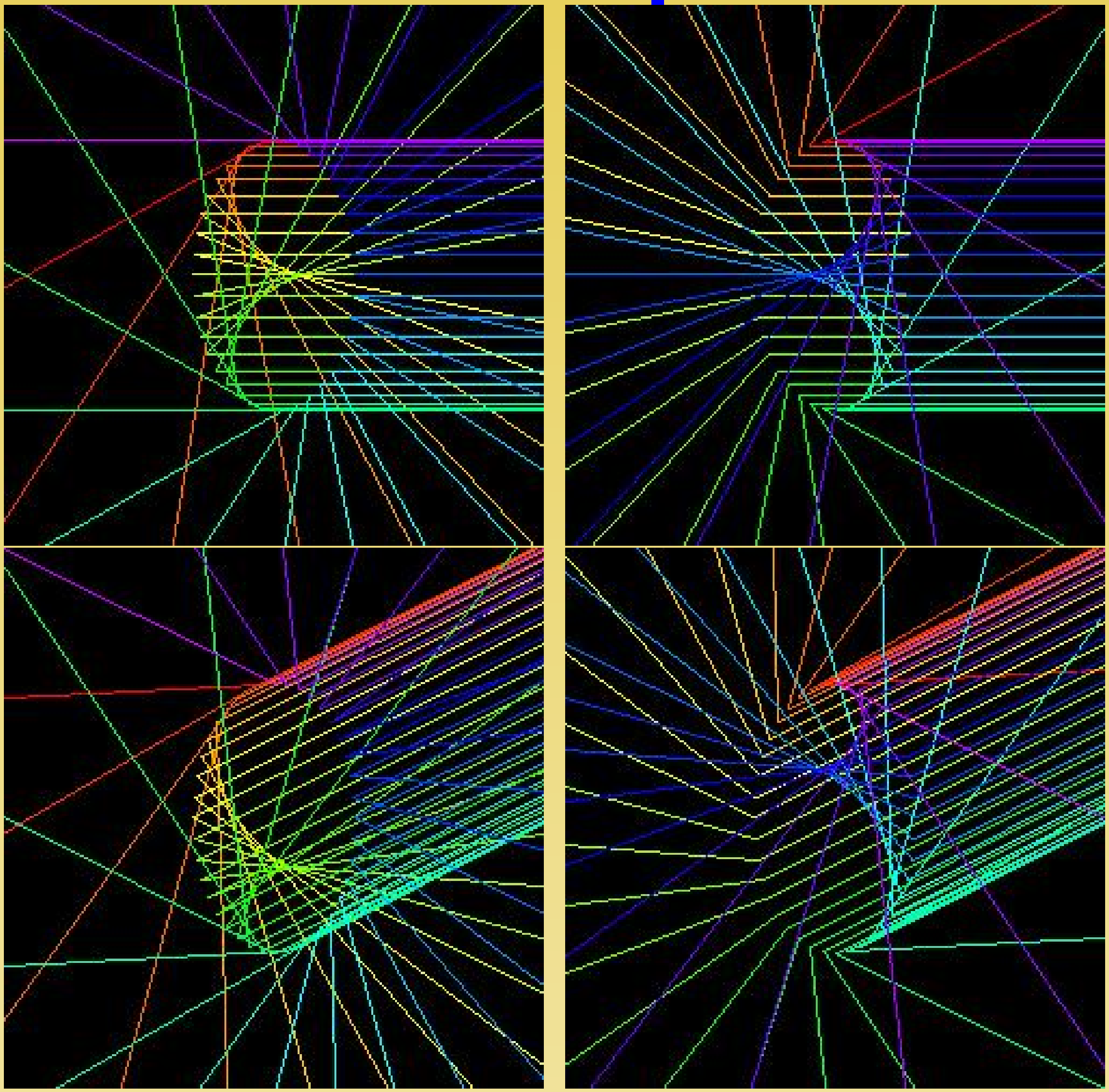
Caustic Properties.

- Catacaustic of a curve C with parallel rays from one direction generate a curve that is also the diacaustic of C with parallel rays from the opposite direction. For example the catacaustic and diacaustic of sinusoid.



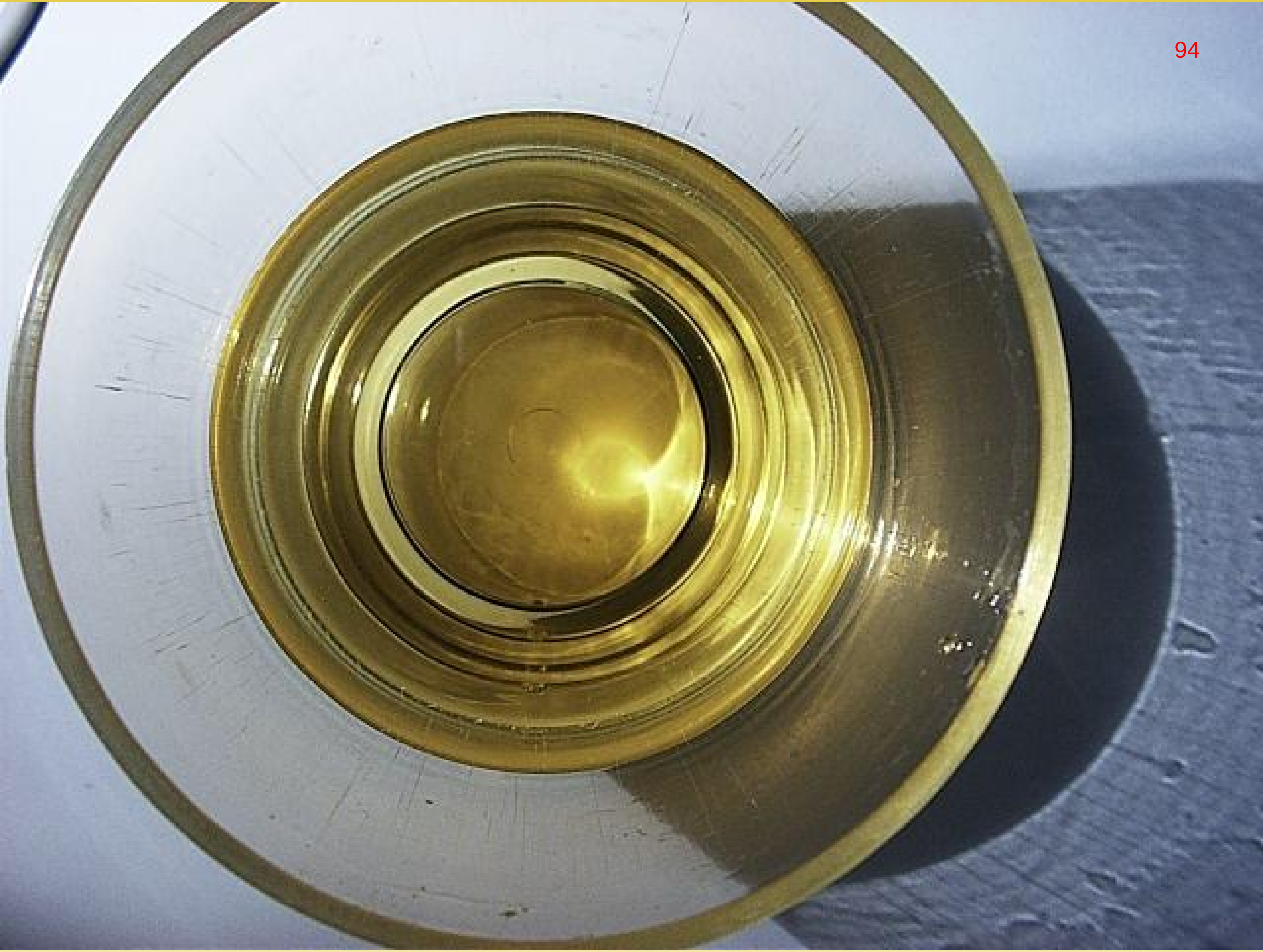


Caustic Properties.

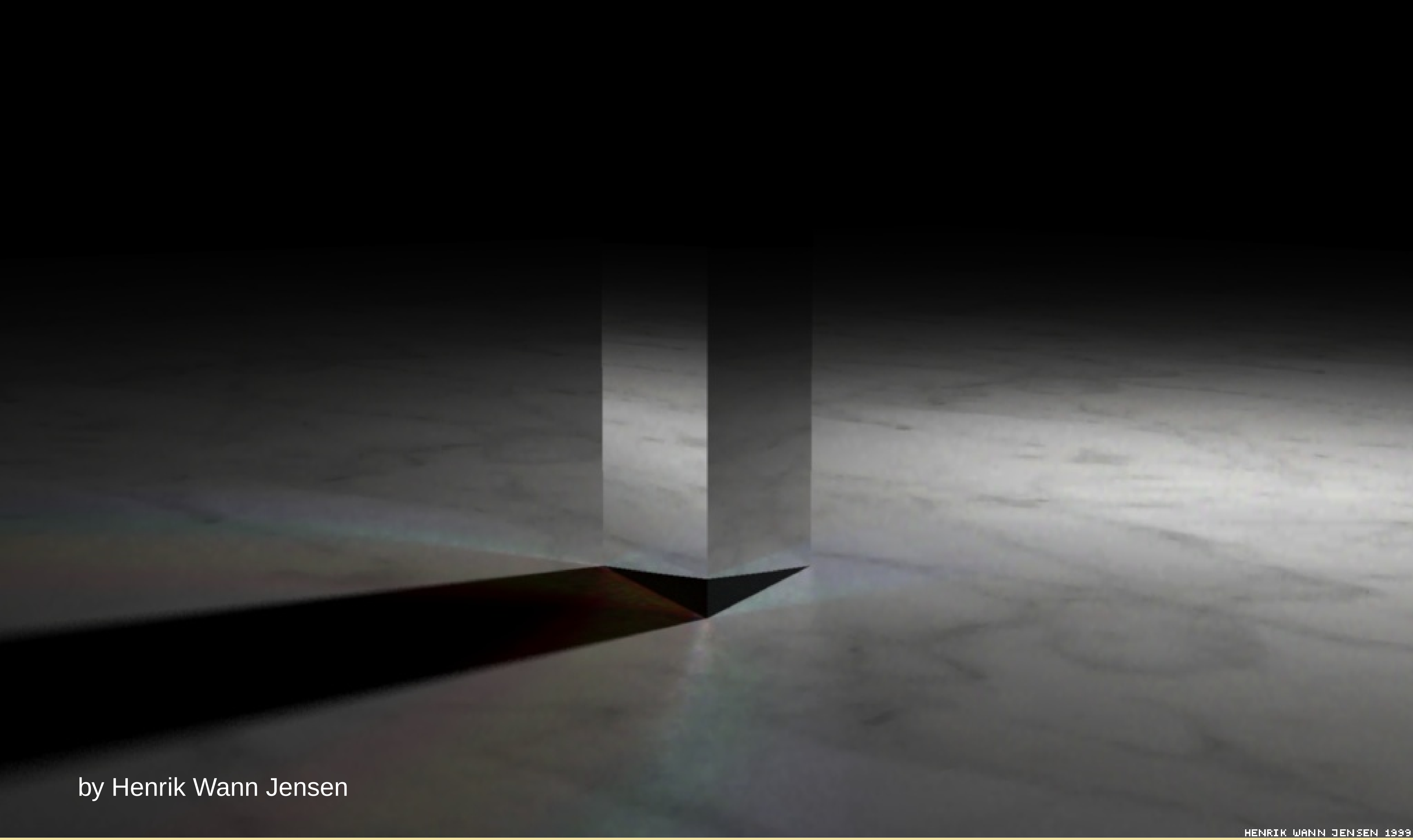


Caustics

Base Curve	Light Source	Catacaustic
circle	on curve	cardioid
circle	not on curve	limacon of Pascal
circle	Infinity	nephroid
parabola	$\perp$ to directrix	Tschirnhausen's cubic
Tschirnhausen's cubic	focus	semicubic parabola
cissoid of Diocles	focus	cardioid
cardioid	cusp	nephroid
quadrifolium	center	astroid
deltoid	Infinity	astroid
equiangular spiral	pole equiangular	spiral
cycloid	$\perp$ line thru cusps	cycloid 1/2
$y == E^x$	rays $\perp$ y-axis	catenary

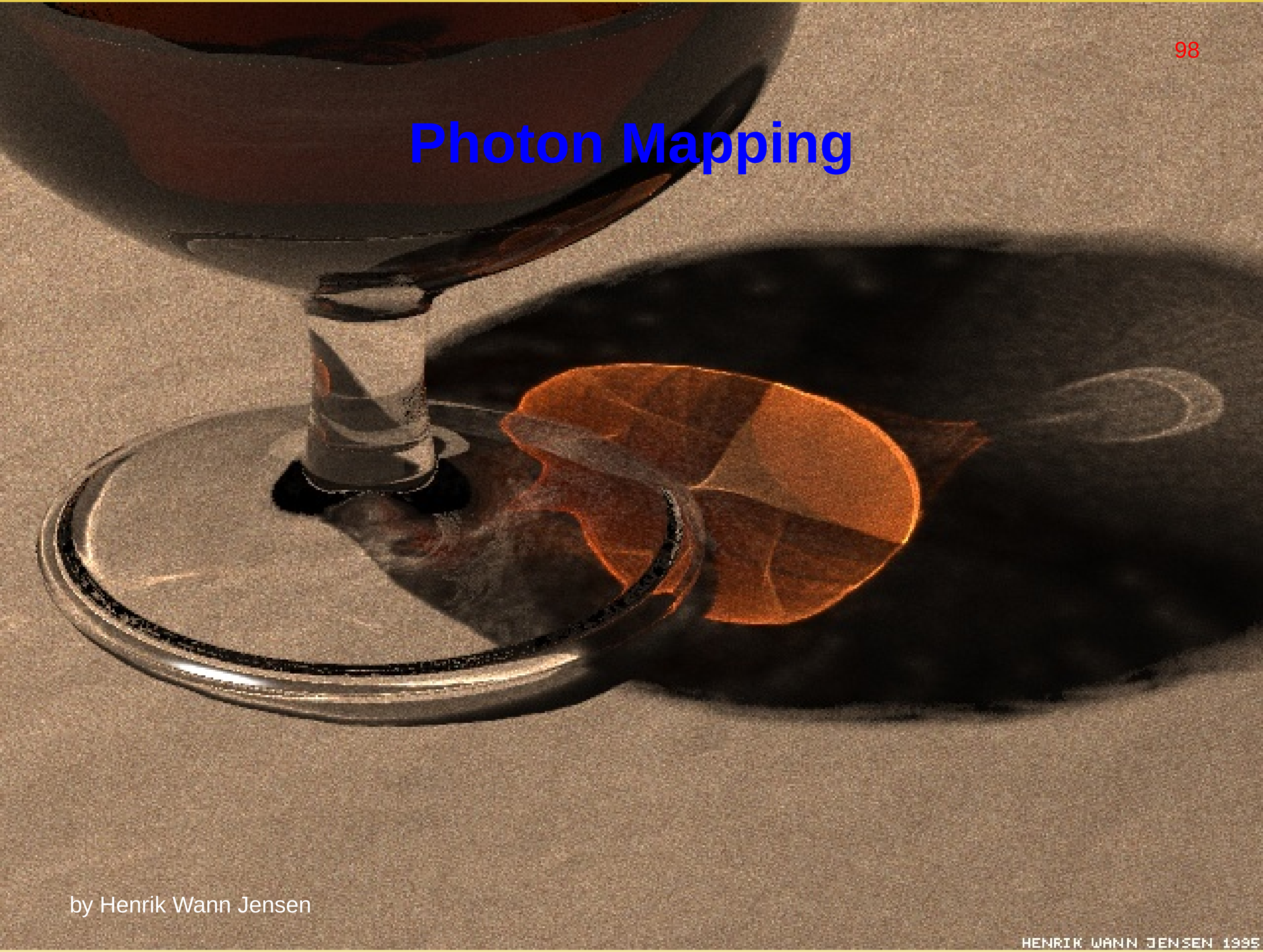


photon mapping



by Henrik Wann Jensen

Photon Mapping



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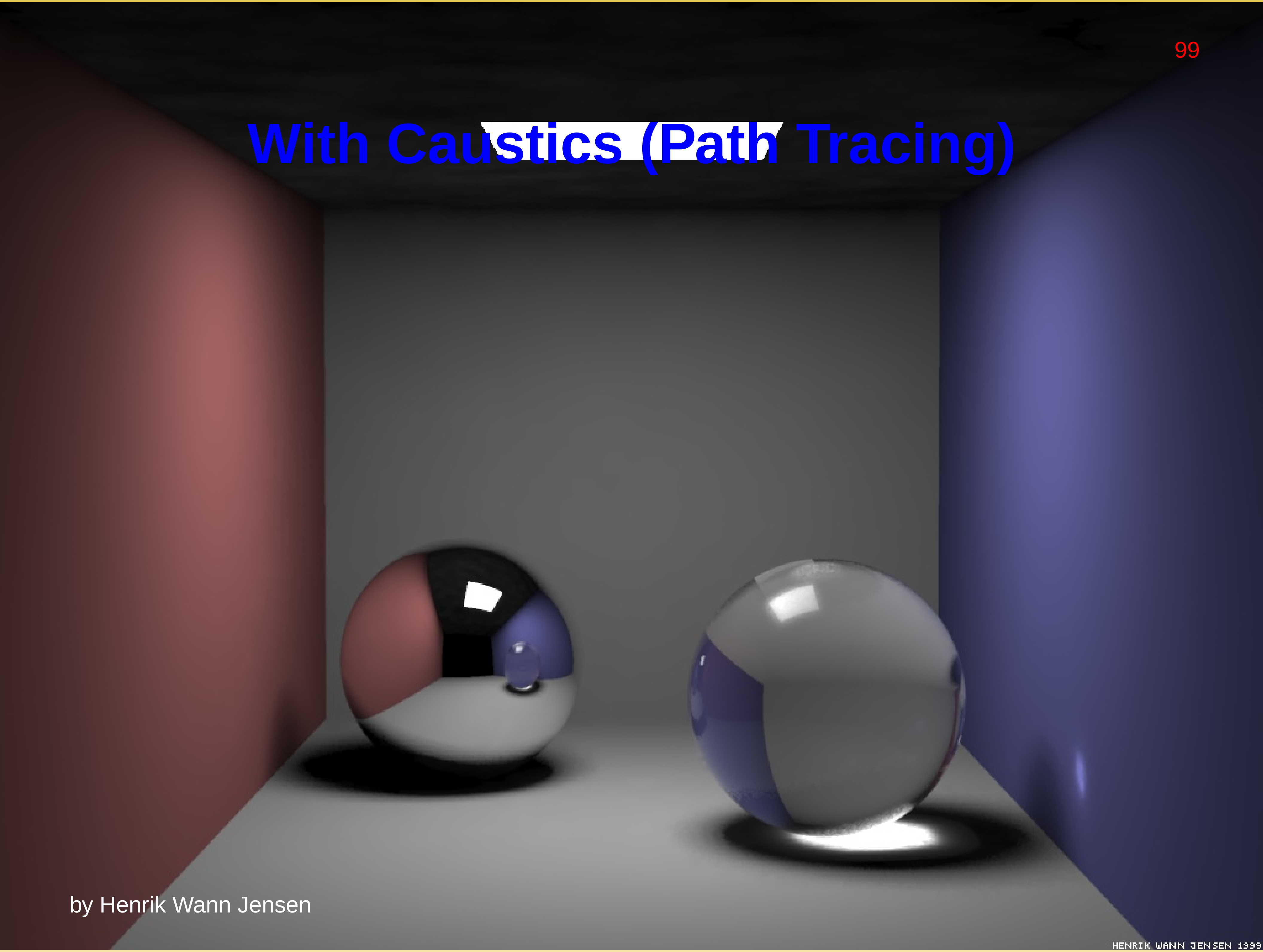
With Caustics (Path Tracing)



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Diffraction

- Diffraction - Change in the directions and intensities of a group of waves after passing by an obstacle or through an aperture whose size is approximately the same as the wavelength of the waves.
- Remarked by Grimaldi (1665)
- When the waves go through the object or around an object the waves scatter.
- When waves scatter, they will get out of phase, they can then combine constructively or destructively.

Diffraction

- This is because the light waves can have different path lengths due to do the scattering.
- This happens when distance between atoms is comparable to the wavelength of the light.
- http://www.journey.sunysb.edu/ProjectJava/Bragg/home.html

Diffraction Experiment

- Hold the two pencils vertically, side by side, the tape will make a thin slit just below the tape. Hold the pencils close to an eye and look at the candle.
- Make sure the facets of the pencils line up.
- Squeeze the pencils and release, changing the width of the slit.
- Rotate the pencils so they are horizontal.

Diffraction Experiment

- What is happening?
- What happens as you rotate the pencils?
- What happens as the slit changes width?
- What color are the blobs of light?
- Now use a hair.
- Try a piece of cloth.

What does this show?

- The black bands between the blobs of light show that there is a wave associated with the light. The light waves that go through the slit spread out, overlap to cancel or add together.
- The angle at which the light bends is proportional to the wavelength of the light. Red light, for instance, has a longer wavelength than blue light, so it bends more than blue light does, this is why there are diffe-rent colors on the edges.

What does this show?

- The narrower the slit, the more the light spreads out. In fact, the angle between two adjacent dark bands in the diffraction pattern is inversely proportional to the width of the slit.
- Thin objects, such as a strand of hair, also diffract light. Light that passes around the hair spreads out, overlaps, and produces a diffraction pattern.