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Filtered Noise and The Fourth Dimension

Filtered noise[1] has become an indispensable tool for creating texture and animation effects. It is best generated by convolving a filter function with a sparse grid of pseudo-random numbers (PRNs).[2] If the grid of PRNs is "poorly packed," the number of grid points falling within the filter radius can vary wildly depending on the location of the sample point. This causes undesirable axis-aligned features.

In this sense, a cubic grid in space is poorly packed. In a 3D grid sampled with a filter of radius 2 grid spacings, the number of grid points used varies from 28 to 56.

We have improved noise quality by using a grid that is more densely and evenly packed than a simple, axis-aligned cubical arrangement. This works particularly well in four dimensions.

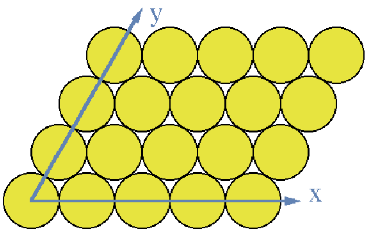

Figure 1
Ideal 2D packing

In two dimensions, the densest and "most even" packing of points is the triangular grid (Figure 1). This can be indexed using a skewed grid suggested by the blue lines.

Spheres do not pack well in 3D, but in 4D there is a very neat packing related to the 2D case. Figure 2 shows a 3D cubical grid of red spheres with an additional blue sphere at the centre of each cube. The blue spheres constitute a second, identical, interleaved grid. A grid in 3D space can be seen as a 3D slice of a corresponding grid in 4D. The blue spheres represent an adjacent slice, shifted along the red/blue axis. In Figure 2, the spheres are drawn undersize to show the structure. If the spheres have a unit diameter and the grid consists of unit cubes, then the (red) spheres touch their neighbors.

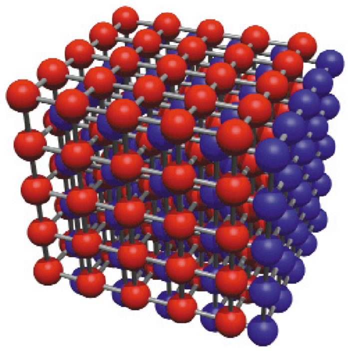

Figure2
Tight packing in 4D

The spheres are really 3D sections of 4D hyperspheres, and the blue hyperspheres in the next layer each touch eight of the adjacent red hyperspheres. Each hypersphere touches a total of 24 adjacent hyperspheres: eight in its own layer and eight at the corners of a cube in the 3D layer "before" and "after." Like the 2D packed grid, this grid can be indexed neatly by means of skewed axes.

Using this structure and a filter radius of 1.3, between 25 and 40 grid points affect any one noise value. We achieve a cheaper and better-quality 3D noise by taking a slice from a 4D structure than by working directly in 3D.

Is this structure the best possible? Surprisingly little has been proved about densest packing of hyperspheres. Conway and Sloane[3] have published a useful set of plausible hypotheses. Our structure fits with these.

An interesting application of 4D noise is the generation of a simulated underwater lighting effect. A 3D slice of the noise is used to apply lighting to every surface in the scene. The slice is then moved through the fourth dimension to produce the appearance of realistic caustics changing with time. Figure 3 shows a frame from "Dragon Gate."[4] The waterfall, mist, splashes, pebbles, and underwater lighting are modeled and animated using 4D noise.

Figure 3
"Dragon Gate:" Leaping Fish

Implementation details are available at:
atlas.otago.ac.nz:800/~geoff/graphics/Noise.html

References:

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2. Lewis, J.P. Algorithms For Solid Noise Synthesis, Computer Graphics (SIGGRAPH 89 Conference Proceedings), Vol. 23, No. 3, July 1989, 263-270
3. J.H. Conway and N.J.A. Sloane. Why Are All The Best Sphere Packings In Low Dimensions? Discrete and Computational Geometry, 13, 1995, 383-403
4. "Dragon Gate," Directed by Geoff Wyvill, University of Otago, 1999