Estimating Image Manifold Dimension by Inversion Error

Shawn Martin and Alex Bäcker

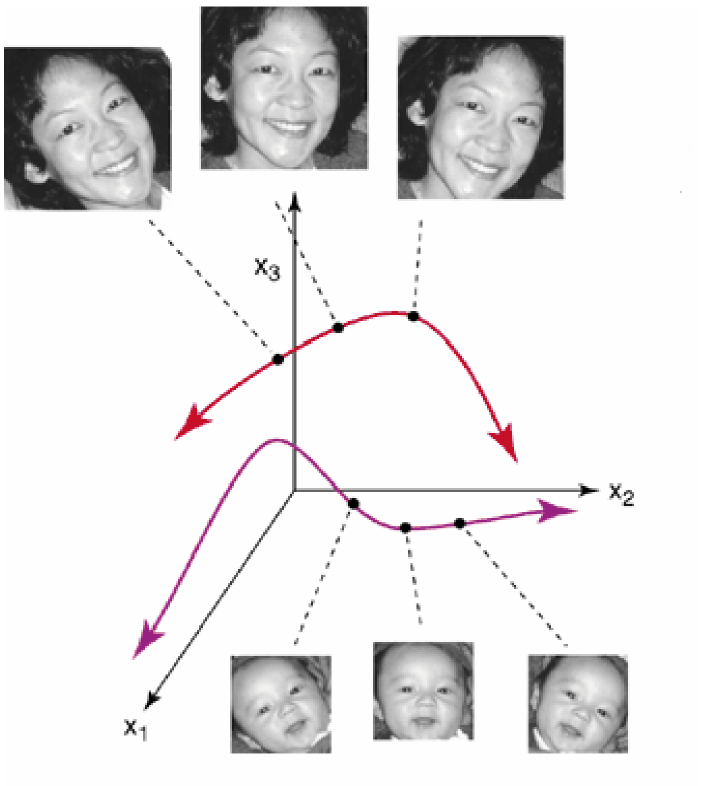
Sandia National Laboratories

Albuquerque, NM

3/7/2005

Outline of Talk

- Image Manifold
  - Definition
  - Dimension Reduction
- Inversion Error
  - Motivation
  - Algorithm
  - Using Locally Linear Embedding (LLE)
  - Using IsoMap
- Examples
  - Toy Examples
  - Image Datasets



(Seung & Lee, 2000)

Image Manifolds

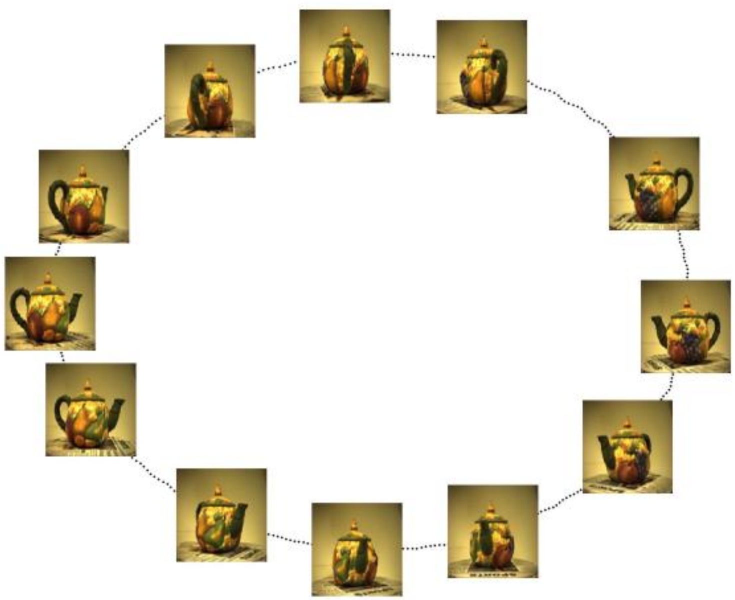
$$X = \{X_1, \ldots, X_N\} \subseteq M \subseteq \mathbb{R}^D$$

D = # of pixels,
N = # of images,
d = dimension of M

d = 2

IsoMap: (Tenenbaum et al., 2000)

d = 1



Semidefinite Embedding:
(Weinberger & Saul, 2004)

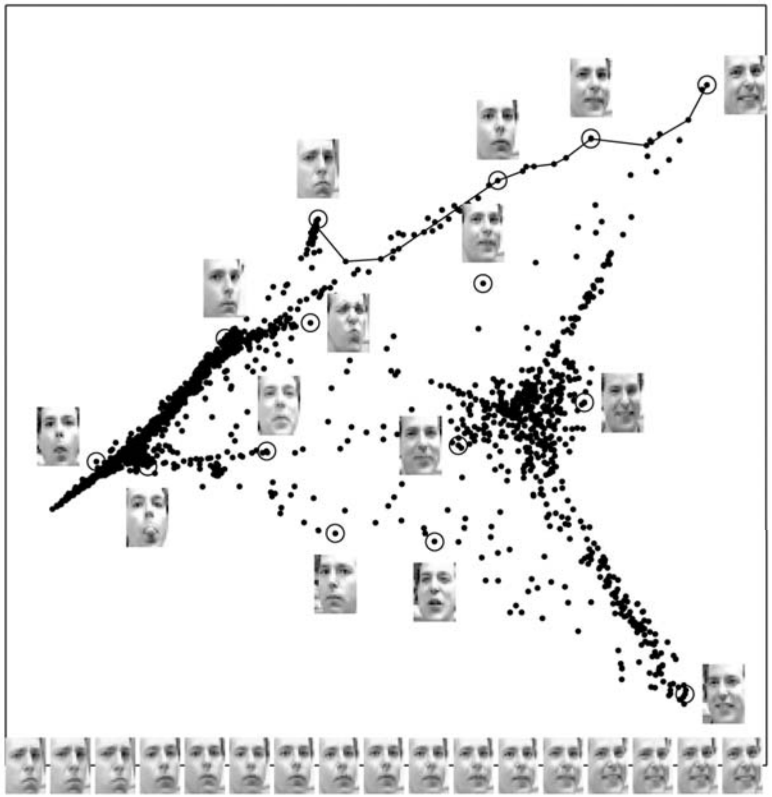
Dimension Reduction

Input: $X = \{X_1, \ldots, X_N\} \subseteq \mathbb{R}^D, d$

Output: $Y = \{Y_1, \ldots, Y_N\} \subseteq \mathbb{R}^d,$

$$f_d : X \to Y \text{ (implicit)}$$

Examples: Principal Component Analysis (PCA), Locally Linear Embedding (LLE), IsoMap, Semidefinite Embedding (SDE), …

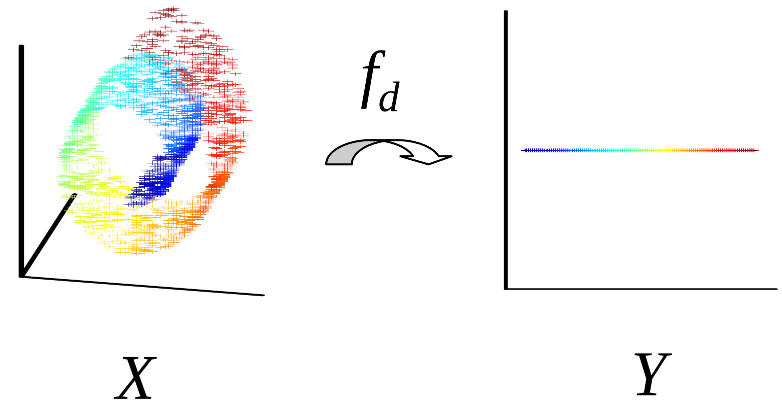


LLE: (Roweis & Saul, 2000)

Inversion Error – Motivation

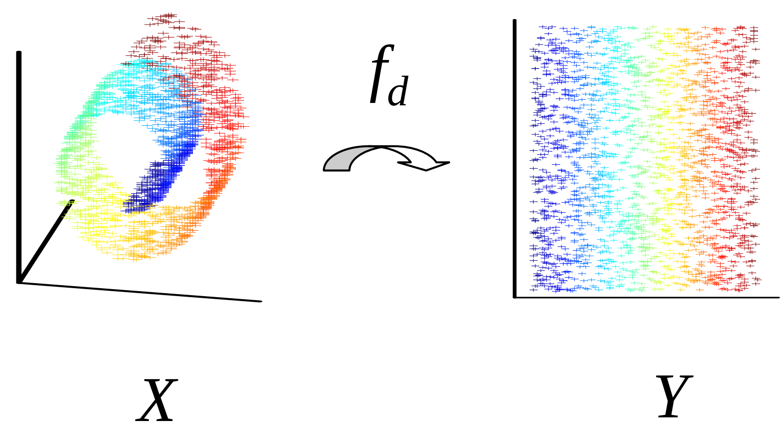
The map $f_d : X \to Y$ should be invertible.

d = 1:



Not Invertible

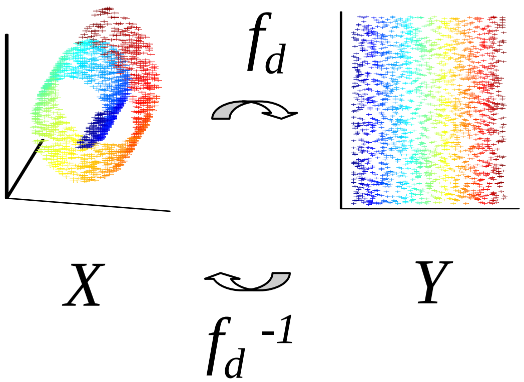
d = 2:



Invertible

Inversion Error – Algorithm

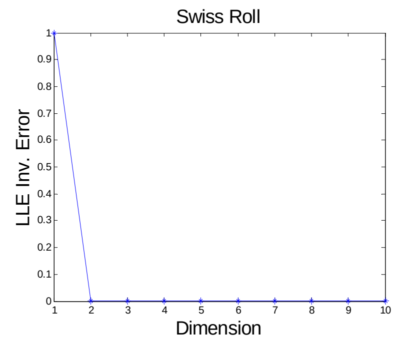
1. Compute $f_d : X \rightarrow Y$
   using *any* reduction algorithm.



2. Compute $f_d^{-1} : Y \rightarrow X$
   (again using any method).

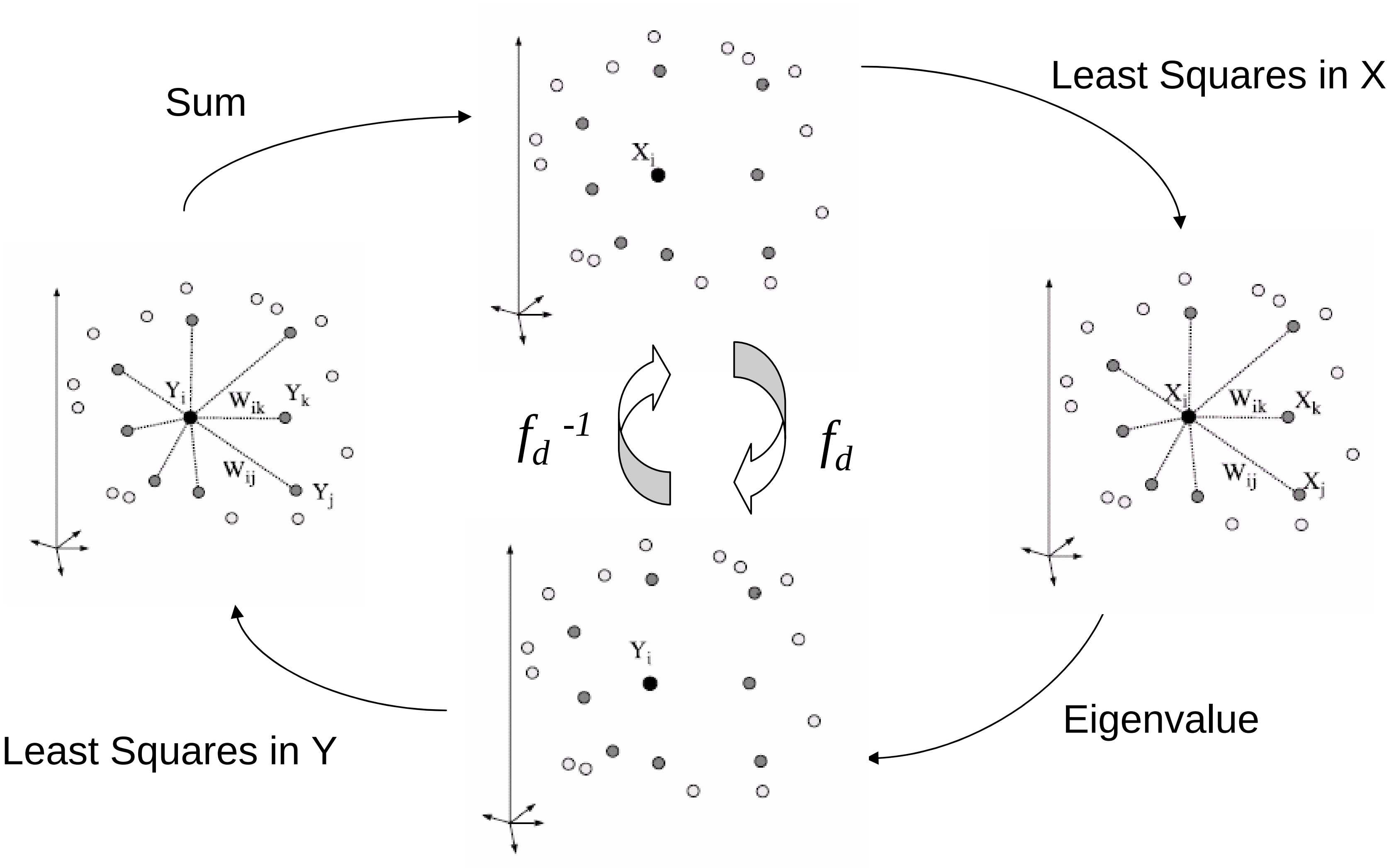
3. Compute $r_d = \sum_i \left\| X_i - f_d^{-1}\left(f_d\left(X_i \right)\right) \right\|^2$

4. Examine r_d vs. *d.*



Inversion Error – LLE

How do we compute $f_d^{-1} : Y \to X$?

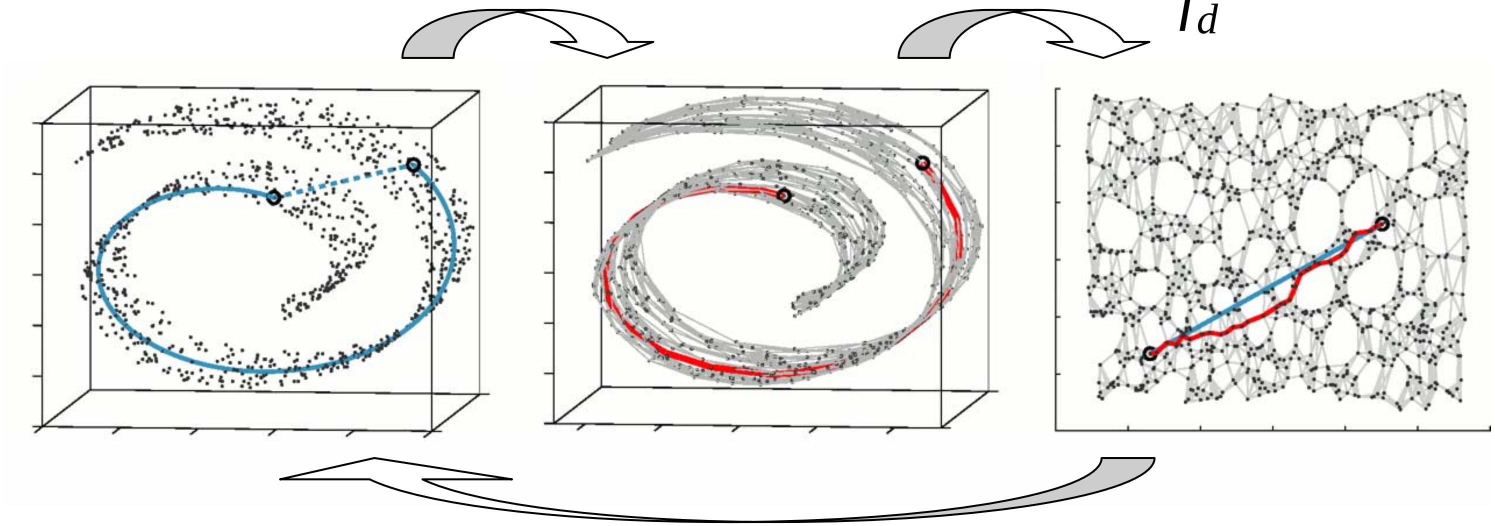


Inversion Error – IsoMap & SVR

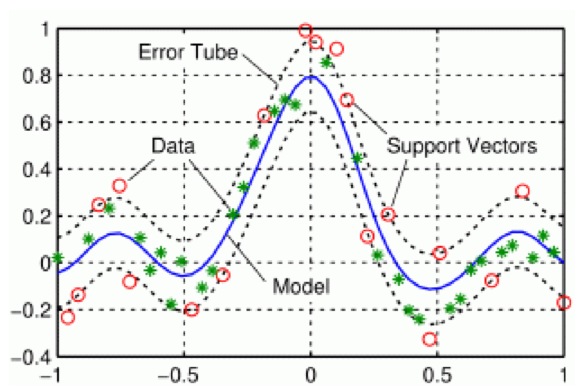
Compute geodesic distances

Use mutli-dimensional scaling (MDS)

f_d



$f_d^{\,-1}$



Support Vector Regression:
(Smola & Scholkopf, 1998)

Toy Examples

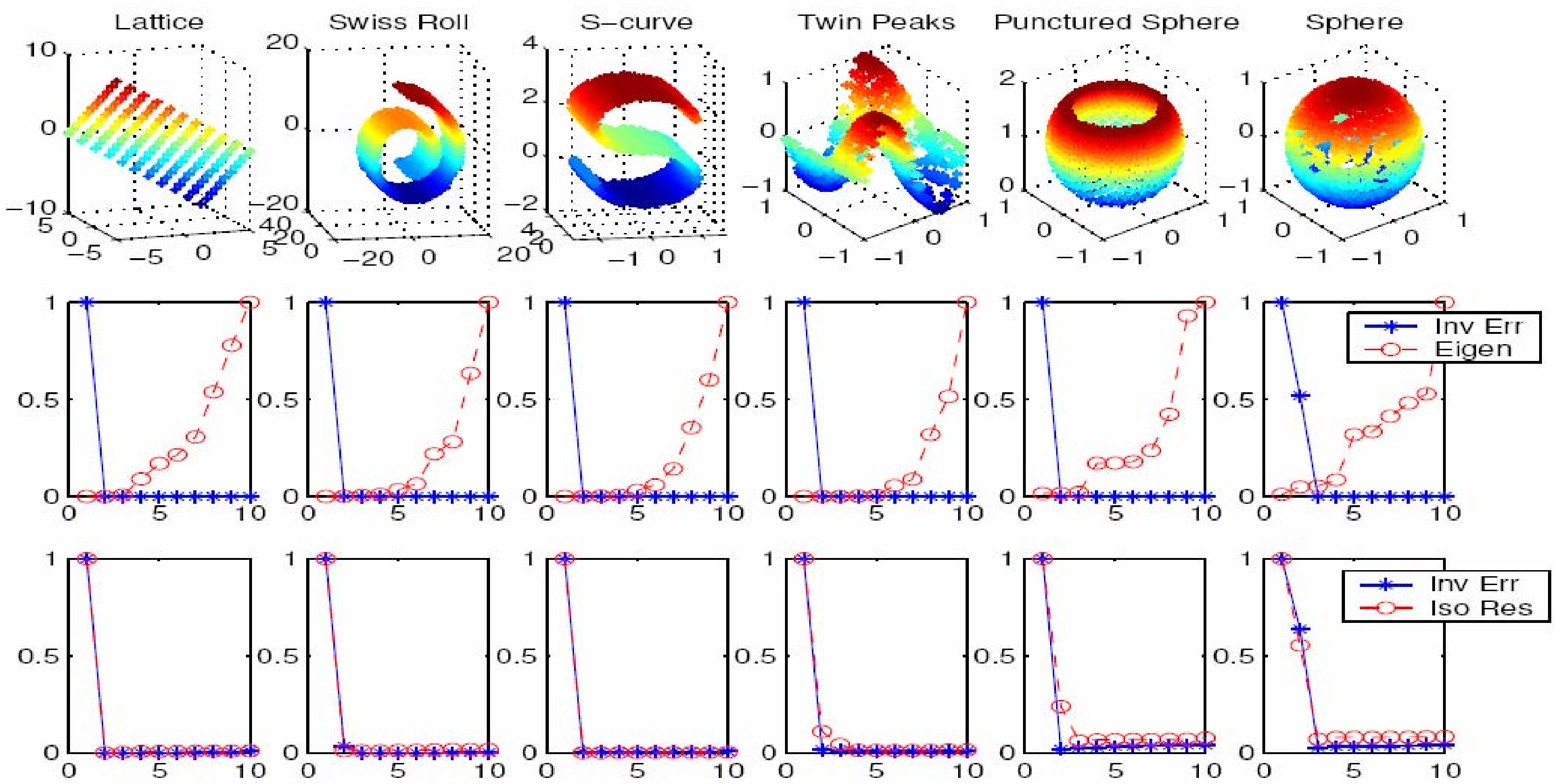
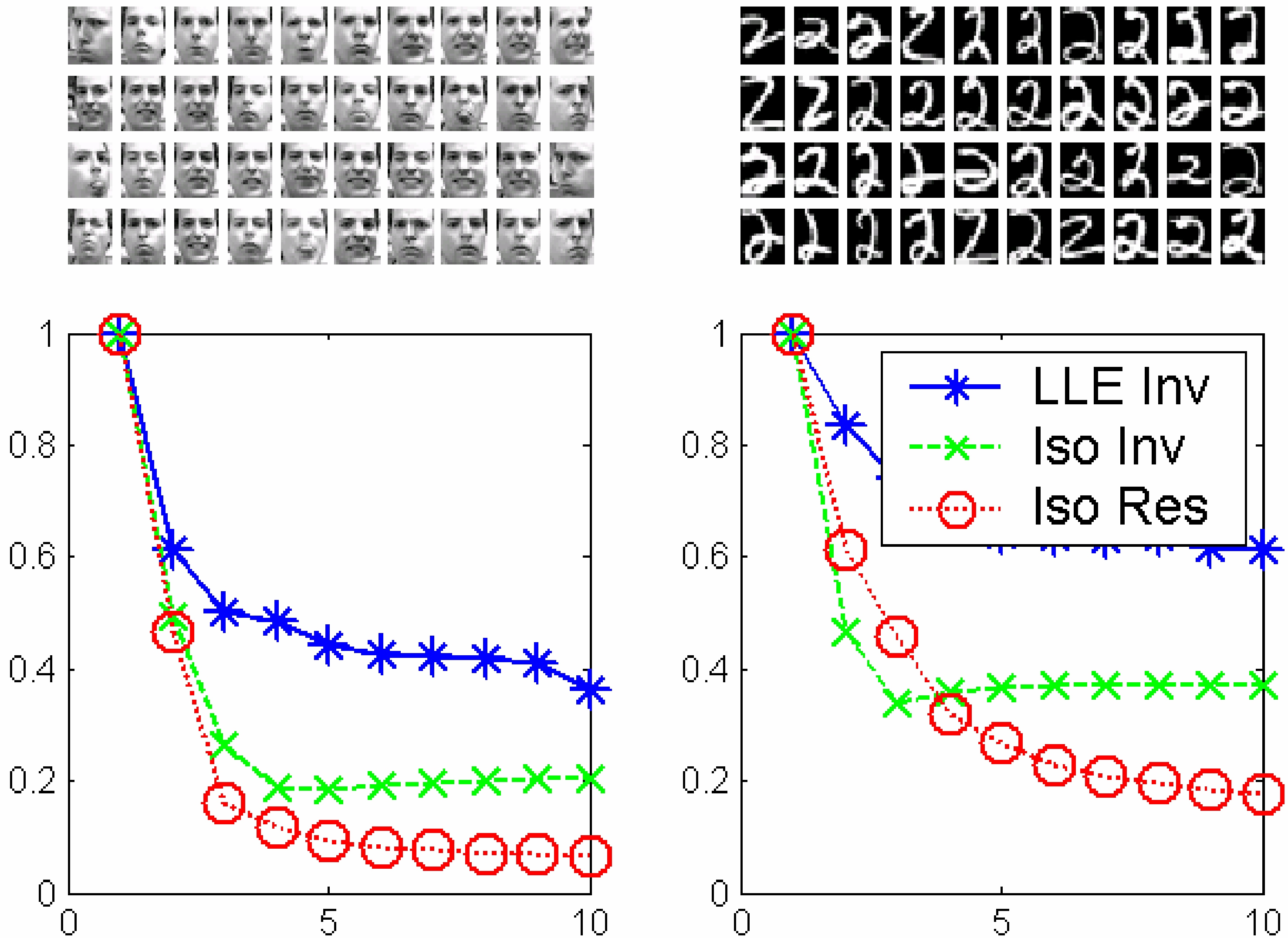


Figure 1: Six Examples. Here we show the results of our algorithm on six manifolds. In the first row (a) we show the manifolds; in the second row (b) we show a normalized version of the LLE inversion error, along with a normalized version of the first ten eigenvalues from LLE, both versus dimension; and in the third row (c) we show a normalized version of the Isomap inversion error, along with the Isomap residual error measure (also normalized).

Image Datasets



Conclusions

Advantages

- General: can be used with *any* reduction algorithm, and *any* method for function approximation.
- Can be used to evaluate the quality of a reduction algorithm.
- Can be used locally as well as globally.

Problems

- Results depend on accuracy of reduction algorithm.
- Learning f_d^{-1} is difficult, especially for large *D.*
- Interpreting results is not automatic.