Using Graph Layout to Generalise Focus+Context Image Magnification

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Image Magnification

There are three main methods used for image magnification.

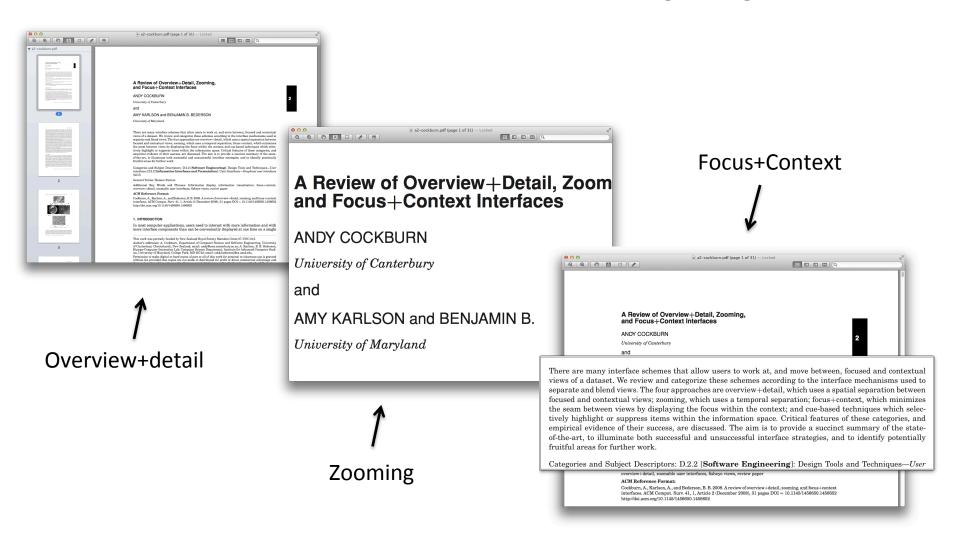
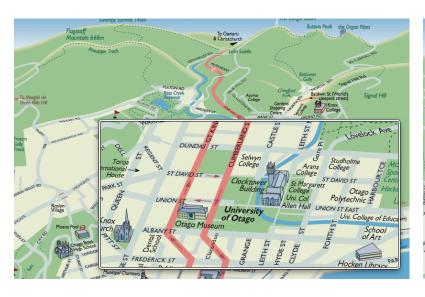
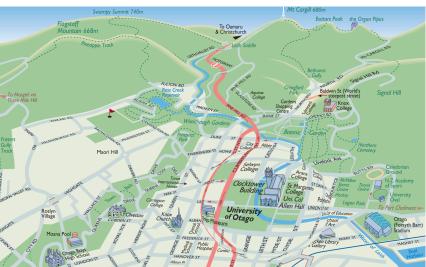


Image Distortion

 Because focus+context image magnification can obscure the context, distortion-based methods have been proposed, e.g. fish-eye lenses.





- Our goal is to improve image distortion for magnification.
 - Can we create an *image-aware* magnifier?

The Image Graph

Underlying every image is a graph.



A fish-eye lens is a distortion of the graph.



Spectral Graph Layout

- Spectral graph layout can be used to distort the image graph.
- Given a graph G = (V, E) with edge weights w_{ij} , define the Laplacian of G:

$$L_{ij} = \begin{cases} \sum_{k} w_{ik} & i = j \\ -w_{ij} & i \neq j \end{cases}$$

• A one-dimensional layout of G can be obtained by solving:

$$\min_{\mathbf{x}} E(\mathbf{x}) = \sum_{ij} w_{ij} (x_i - x_j)^2 = \mathbf{x}^T L \mathbf{x}$$

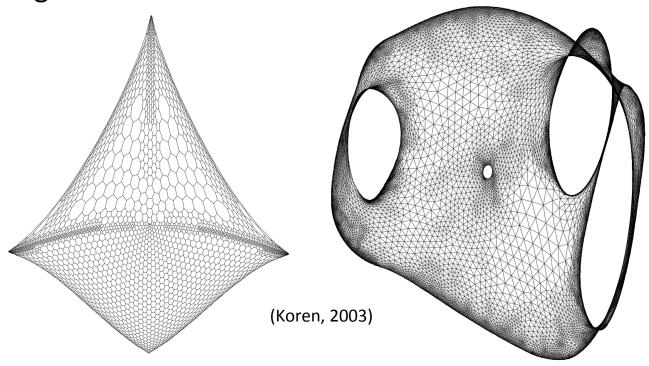
s.t.
$$\mathbf{x}^T \mathbf{x} = 1$$

$$\mathbf{x}^T \mathbf{1}_n = 0$$

- This problem is solved by taking the 2^{nd} smallest eigenvector of L.
- A two-dimensional layout is obtained by taking the 2^{nd} and 3^{rd} smallest eigenvectors of L.

Spectral Graph Layout Example

 Spectral graph layout is fast and yields aesthetically pleasing drawings ...



 ... however, the drawings are not (in general) rectangular, so they don't work well for images.

Constrained Graph Layout

 We can modify the standard spectral layout algorithm to fix the edges of the graph to lie on a rectangle.

$$\begin{array}{cccc}
\min_{\mathbf{x}} & \mathbf{x}^T L \mathbf{x} \\
\text{s.t.} & \mathbf{x}^T \mathbf{x} = 1 \\
\mathbf{x}^T \mathbf{1}_n = 0
\end{array}
\qquad \begin{array}{cccc}
\min_{\mathbf{x}} & \mathbf{x}^T L \mathbf{x} \\
\text{s.t.} & A \mathbf{x} = \mathbf{b}
\end{array}$$

- Replace the scaling $(\mathbf{x}^T\mathbf{x} = 1)$ and translation $(\mathbf{x}^T\mathbf{1}_n = 0)$ constraints by constraints which directly restrict the edge coordinates $(\mathbf{A}\mathbf{x} = \mathbf{b})$.
- This is a quadratic programming problem with linear constraints, so it's a linear problem.

Constrained Graph Layout Example

• By supplying appropriate weights, we can use our layout algorithm to obtain a fish eye lens.

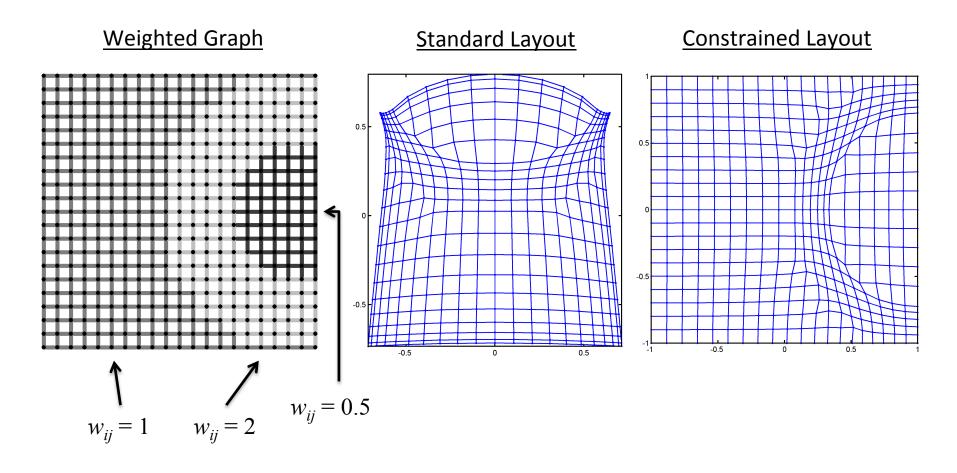


Image Magnification Example

 An example of a fish-eye lens using our constraint layout algorithm.





New Zealand House of Representatives, circa 1900-1902.

Image-aware Distortion

 We can modify the image graph weights according to pixel value to get image-aware magnification.



Here we weight according to 1 - pixel intensity.

Image-aware Distortion

Another example using pixel intensity.

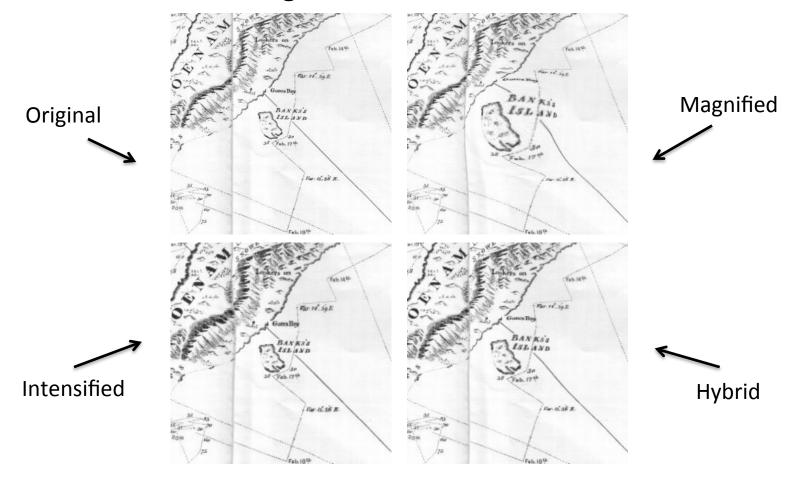
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• Fur trader's ledger, 1876-1885.

An Algebra of Distortions

We can combine weight functions.



Cook's map of Bank's island (actually a peninsula).

Conclusions

- Constrained graph layout can be used to magnify and distort images in useful ways.
- Future Work (Technical)
 - We need to investigate/invent useful weighting functions.
 - What is the best way to combine weighting functions?
 - A weighting function can produce a distortion. Can we solve for a weighting function if given a distortion?
 - More generally, can we use constrained graph layout as a means of universal image distortion?
- Future Work (User Interface)
 - Is this type of image magnification/distortion useful to an average person?
 - What is the best way to present this technology to a user?
 - How can users select/combine weighting functions in an intuitive manner?