Comments to 'Logics of Public Communications'

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Abstract. This is a commentary on Jan Plaza's 'Logics of Public Communications', reprinted in this same issue.

Take your average publication on the dynamics of knowledge. In one of its first paragraphs you will probably encounter a phrase like "a logic of public announcements was first proposed by Plaza in 1989 (Plaza, 1989)." Tracking down this publication seems easy, because googling its title 'Logics of Public Communications' takes you straight to Jan Plaza's website where it is online available in the author's own version, including, on that page, very helpful and full bibliographic references to the proceedings in which it originally appeared. Those proceedings are then somewhat harder to find. In fact, I have never seen them. Unfortunately, for the research community, Plaza's work has never been followed up by a journal version. I am very grateful to the editor Wiebe van der Hoek of the journal 'Knowledge, Rationality, and Action' to correct this omission.

Plaza's work is reprinted as such, without an update encompassing more than fifteen additional years of research in this area. This commentary aims to provide some background to bridge that gap.

Public announcement In modal logics of knowledge or belief, information change can be expressed with dynamic modal operators. It is customary to introduce those modal operators in their $\Box$-version, or 'necessity'-version, such that $[\alpha]\psi$ stands for "after every execution of program α, formula ψ (holds). Instead of 'program', α is sometimes called event, action, update, ... The diamond version of that operation is then introduced by notational abbreviation as $\langle\alpha\rangle\psi$ which is by definition equivalent to $\neg[\alpha]\neg\psi$. Public announcements are a specific form of the epistemic programs α mentioned above: instead of $[\alpha]\psi$ we now have $[\varphi]\psi$, which stands for "after (every) announcement of φ, ψ." Its diamond form is therefore $\langle\varphi\rangle\psi$, which stands for "there is an execution of 'announce φ truthfully', and after that ψ is true," in other words "φ is true and after announcing φ, ψ is true." We are now very close to how Plaza models public announcement.

Plaza's public announcement What Plaza calls 'public communication' is now commonly named 'public announcement'. Plaza's primitive corresponds to the diamond form $\langle\varphi\rangle\psi$. In (Plaza, 1989), public announcement is modelled as a binary operation $+$, such that $\varphi + \psi$ is equivalent to $\langle\varphi\rangle\psi$, in other words, $\varphi+\psi$ also stands for "φ is true, and after announcing φ, ψ is true." The '+' operation reveals a viewpoint that is slightly different from the dynamic modal perspective. Plaza defines public announcement as a *binary* propositional connective—although it is rather different from propositional connectives such as $\wedge$ and $\vee$, because $+$ is not truth-functional. Instead, the common dynamic modal perspective is to see $\Box$ as a *unary* operator binding formulas that are postconditions of program execution, where $\Box$ is relative to a 'program' α (thus the notation $[\alpha]$); 'making a public announcement' is such a program, in which case the parameter can be identied with the formula of the announcement. The viewpoint of an announcement as a binary operation on formulas does not generalise to more complex programs α, as such programs are not straightforwardly identified with a single formula.

Before Plaza Jan Plaza was a PhD student of Melving Fitting, but his 'Logics of Public Communications' resulted from a collaboration also involving Rohit Parikh. Parikh's work forms part of a tradition that started in the early 1980s wherein computer scientists formalised communication, in particular how to achieve knowledge in a distributed environment. Influential work was by Halpern and Moses (Halpern and Moses, 1984), who used an interpreted systems approach to show that common knowledge cannot be achieved in many cases, by Parikh and Ramanujam (Parikh and Ramanujam, 1985), who introduced a history-based semantics to reason about the evolution of knowledge over runs of a protocol, and by Chandy and Misra (Chandy and Misra, 1985) who provided a characterisation of the minimum information flow necessary for a process to 'learn' specific facts about the system. Much of such 1980s work was later gathered and unified in the influential 'Reasoning about Knowledge' (Fagin et al., 1995) by Fagin et al.

Gerbrandy's public announcement A different public announcement logic was proposed, after but independently from Plaza, by Gerbrandy and Groeneveld in (Gerbrandy and Groeneveld, 1997), see also Gerbrandy's PhD (Gerbrandy, 1999). This work is based on a tradition in 'update semantics' and directly motivated by Veltman's (Veltman, 1996). The principal idea in update semantics was a dynamic interpretation for referents in sentences, as needed for predicate logical interpretations, see (Groenendijk and Stokhof, 1991). This was then reinterpreted as similar to the dynamics induced by PDL-style programs. The basic publication for this strand of dynamic epistemics, incidentally also (like (Plaza, 1989)) published in 1989, is Van Benthem's (van Benthem, 1989), wherein he proposed to model AGM-style belief revision (Alchourrón et al., 1985) with dynamic modal operators. This line of work includes Van Emde Boas, Groenendijk, and Stokhof (van Emde Boas et al., 1984), Landman (Landman, 1986), Groeneveld (Groeneveld, 1995), and Veltman (Veltman, 1996), and more PDL-motivated work by De Rijke (de Rijke, 1994), and Jaspars (Jaspars, 1994). Groeneveld's approach (Groeneveld, 1995) is typical for dynamic semantics in that is has formulas $[\varphi]_a\psi$ to express that after an update of agent a's information with φ, ψ is true. But note that there are no *epistemic* operators. De Rijke (de Rijke, 1994) defines theory change operators $[+\varphi]$ and $[*\varphi]$ with a dynamic interpretation corresponding to AGM-type belief expansion and belief contraction, respectively (Alchourrón et al., 1985). Gerbrandy's work (including the collaboration with Groeneveld) added an explicit epistemic dimension by combining epistemic modal operators and dynamic modal operators in the logical language. Their public announcements also figured in a context of weaker epistemic notions, such as introspective belief.

After Plaza We now describe developments after Plaza's 'Logics of Public Communications', that can be seen as various ways to expand such a public announcement logic. One issue was the addition of common knowledge to Plaza's framework, and the axiomatisation of that logic; another issue was how to model more complex actions than public announcements, for example private announcements to subgroups.

Common knowledge In public announcement logic without common knowledge, announcements are 'superfluous' in the sense that the logic with announcements is just as expressive as the 'normal' epistemic logic without announcements. One can eliminate all announcements from a given formula by 'rewrite rules' or (also known as) 'reduction axioms' such as $[\varphi]K_a\psi \leftrightarrow (\varphi \rightarrow K_a[\varphi]\psi)$ and (for atomic propositions p) $[\varphi]p \leftrightarrow (\varphi \rightarrow p)$. Reduction axioms are validities of the logic that are equivalences. In the first example reduction axiom, K_a stands for 'agent a knows'. It expresses "after announcement of φ, the agent knows ψ, if and only if (whenever φ is true, the agent knows, that after announcing that, ψ is true)." When one adds (only) common knowledge, one needs more than reduction axioms. There is only a 'one-directional' derivation rule relating common knowledge and announcement, so that we can no longer eliminate announcements from formulas. The axiomatisation of the public announcement logic *with* common knowledge has been established by Baltag, Moss, and Solecki (Baltag et al., 1998; Baltag and Moss, 2004). A simplified, direct completeness proof for public announcement logic with common knowledge, by Kooi, is presented in (van Ditmarsch et al., 2007b). Complexities of public announcement logic with and without common knowledge have been investigated by Lutz (Lutz, 2006). An endpoint of Baltag et al.'s extension of Plaza's work with common knowledge, from a theoretical perspective, is the concept of 'relativised common knowledge'. This is a generalisation of common knowledge with more appealing theoretical properties (van Benthem et al., 2006). Now, we have reduction *axioms* again.

From announcements to events In fact, Baltag et al.'s logic incorporating common knowledge into public announcement logic also provided a very general framework to model any sort of informative update, such as private announcements to subgroups; and their completeness results are for that more general logic of which public announcement logic is a special case. One issue with private announcements is that agents who are unaware of the announcement incorrectly (still) believe the knowledge conditions from before the announcement to persist: when a announces p to b only, c still believes, but now incorrectly, that a does not know p. This means that the actual information state is no longer considered a possibility: the corresponding accessibility relations to interpret epistemic operators therefore can no longer be equivalence relations, as in Plaza.

Baltag et al.'s work integrated Plaza's and Gerbrandy's achievements. Gerbrandy also proposed more complex updates (Gerbrandy, 1999). Other work rooted in the 1990s modelling more complex updates includes (Lomuscio and Ryan, 1998; van Ditmarsch, 2000).

In the meantime, many other aspects of dynamics were also modelled in the context of dynamic epistemic logics. Three directions of further extension are: (adding) assignments, arbitrary announcements, and degrees of belief.

Factual change Announcements 'merely' distribute information through a multiagent system but do not change the value of factual descriptions. Instead of such *information change*, one can also model *factual change* (i.e., assignment of other values to atomic variables), and even do both at the same time, such as players flipping coins or cards while being observed by other players. This has been addressed in, e.g., (van Ditmarsch et al., 2005; van Benthem et al., 2006). Factual change and its interaction with agent knowledge is also of particular interest to AI researchers (Scherl and Levesque, 2003; Demolombe et al., 2003). A typical example involves a gun being loaded and unloaded with agents possibly unaware of that (the 'Yale shooting problem'). This 'situation calculus' approach does not normally use dynamic modal operators.

Arbitrary public announcement One issue in public announcement logic is that formulas may become false when truthfully announced. An example is the Moore-sentence $p \wedge \neg K_a p$. It becomes false when announced to agent a, because after that the agent knows that p. For some applications, such as planning, sequences of announcements form plans that are intended to realise certain postconditions. Apparently these postconditions relate in non-trivial ways to the announcements realising them. Therefore, a perspective reasoning backwards from a postcondition ψ to 'some' announcement realising it may be fruitful. As we have seen, $\langle\varphi\rangle\psi$ stands for 'φ is true, and after announcing φ, ψ (is true).' Now consider modelling the existential quantification as well in 'there is a formula φ such that φ is true and after announcing φ, ψ is true,' in the modal announcement operator: let $\Diamond\psi$ stand for 'there is a φ such that $\langle\varphi\rangle\psi$.' Then operator $\Diamond$ stands for an 'arbitrary announcement'. Its logic is not (yet) known and under investigation in (Balbiani et al., 2006), that is strongly motivated by (Fine, 1970). It is non-trivial, e.g. $\Diamond(K_a p \vee K_a \neg p)$ is a validity, as the truth about a fact can always be made known by simply announcing it.

Dynamic operators for belief revision Another recent broadening of perspective is to model the interaction of plausible belief, knowledge, and change (Aucher, 2003; Liu, 2004; van Ditmarsch, 2005; Baltag and Smets, 2006; Laverny, 2006). Instead of one epistemic operator per agent, one may have many, expressing different degrees of belief ('plausibilities') in propositions, or conditional belief, and how these beliefs change due to new evidence. If one replaces 'plausibilities' with 'probabilities', the previous also relates to the interaction between probabilities, knowledge, and change (Fagin and Halpern, 1994; Kooi, 2003; Halpern, 2003).

This ends our elaboration on the different directions that public announcement logic has taken over the past fifteen years. We finish with two other ideas originating in 'Logics of Public Communications'.

Know-value operators Plaza presents a version of the announcement semantics for 'know-value'-operators binding non-rigid designators such that, e.g., $\mathsf{Kv}_a d$ stands for 'Agent a knows the *value* of the non-rigid designator d' (where d is, e.g., a number variable). The corresponding logic is not known to be complete. As far as we know, this is still an open question in 2006. Dynamic extensions to modal predicate logic or to a modal logic with quantification over propositional variables seem to be required to make this work (Fine, 1970; ten Cate, 2006; Renardel de Lavalette, 2004). One can easily conceive of systems with infinite domains wherein the formalisation of certain announcements appears to require infinitary formulas or quantification—the typical case handled by Plaza's 'know-value' operator. E.g., given the domain of natural numbers where an agent cannot see a number on its forehead, "I do not know my number" requires an infinitary conjunction. ("For every natural number, if it is on my forehead, I do not know that number.") To model dynamics in such systems, know-value operators, or similar, are essential.

Sum and Product Another 'first' is Plaza's modelling of the 'Sum-and-Product' riddle. This is an example of a highly proceduralised multi-agent system that benefits from an analysis in public announcement logic. Freudenthal's (1969) original version of the riddle (Freudenthal, 1969) became somewhat later widely known in AI circles by McCarthy's version (McCarthy, 1990) (Plaza's source), that dates from the 1970s and was only later published—incidentally, (McCarthy, 1990) also contains an interesting original definition of common knowledge semantics. The riddle recently resurfaced in (van Ditmarsch et al., 2007a). This provides a detailed discussion of Plaza's analysis of 'Sum and Product'.

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