Abstract—Modern multi-core computers often adopt a Multi-socket Multi-core architecture with shared caches in each socket. However, traditional task-stealing schedulers tend to pollute the shared cache and incur more cache misses due to their random scheduling. To relieve this problem, this paper proposes a Cache Aware Bi-tier (CAB) task-stealing scheduler, which improves the performance of memory-bound applications by reducing memory footprint and cache misses of tasks running inside the same CPU socket. CAB uses an automatic partitioning method to divide an execution Directed Acyclic Graph (DAG) into the inter-socket tier and the intra-socket tier. Tasks generated in the inter-socket tier are scheduled across sockets, while tasks generated in the intra-socket tier are scheduled within the same socket. Experimental results show that CAB can improve the performance of memory-bound applications up to 55% compared with the traditional task-stealing.

Index Terms—Multi-socket Multi-core architecture, Cache aware, Task-stealing, Work-stealing, Cilk

I. INTRODUCTION

Multi-core processors have become mainstream as chip manufacturers like AMD and Intel keep producing new CPU chips with more cores. Modern multi-core computers often use a Multi-Socket Multi-Core (MSMC) architecture in order to obtain more computing power. In the MSMC architecture, multiple multi-core chips share the same memory (RAM), while the cores in the same CPU chip (also referred as CPU socket in this paper) share the L2 or L3 caches. This architecture is popular now and will continue to be a dominating architecture for high performance computing in future.

Despite the rapid development of the multi-core technology, a lot of software are yet to be parallelized to utilize the power of multi-core computers. This need has promoted the development of parallel programming environments.

There are many parallel programming environments that are popular nowadays. They can be classified into two groups in terms of their task scheduling. The first group is based on manual task scheduling. Pthread[1], MPI[2] and Maotai[3] are examples of this group. Programmers need to manually arrange tasks for every thread/process through careful programming in order to achieve optimal load balance. This manual task scheduling is often burdensome for developing applications that recursively generate tasks.

The second group is based on automatic task scheduling. In these programming environments, programmers can specify and generate tasks at runtime. Parallelism in programs is mostly expressed as tasks that are scheduled automatically among the executing threads. Examples of this group are Cilk[4], Cilk++[5], TBB[6], OpenMP[7], and Java’s fork-join framework[8]. This feature of automatic task scheduling enables convenient expression of dynamic tasks and automatic load balancing.

In programming environments with automatic task scheduling, the execution of a parallel program can be represented by a task graph, which is a Directed Acyclic Graph (DAG) \( G = (V, E) \), where \( V \) is a set of nodes, and \( E \) is a set of directed edges [9]. A node \( n_i \) in a DAG represents a task (i.e., a set of instructions) that must be executed sequentially without preemption. The edges in a DAG, denoted by \((n_j, n_k)\), correspond to the dependence relationship among the nodes (tasks).

Most DAG-based automatic task scheduling algorithms such as task-stealing (also known as work-stealing[10]) and task-sharing [7] schedule tasks onto processors randomly. This randomness in task scheduling causes Task Relocation Incurred Cache Yeastiness (TRICY) syndrome in the MSMC architecture, which is depicted as follows.

Suppose there are three tasks \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) to be executed in the MSMC architecture. \( \gamma_1 \) and \( \gamma_2 \) share data, but they share nothing with \( \gamma_3 \). If \( \gamma_1 \) and \( \gamma_2 \) are scheduled to the cores of the same CPU socket, the shared data are loaded into the shared caches (e.g., L3) only once but can be accessed by both tasks through the caches. However, this data sharing is not respected by traditional task scheduling algorithms due to their randomness in selecting cores for the tasks. Most often the task schedulers would move \( \gamma_1 \) or \( \gamma_2 \) to a core in a different socket, where \( \gamma_3 \) is being executed.

The above random scheduling causes two problems. First, it increases cache misses. Suppose \( \gamma_2 \) is scheduled to the socket of \( \gamma_3 \). \( \gamma_2 \) cannot use the data already loaded into the caches by \( \gamma_1 \). Instead, it needs to read data from the main memory since it cannot find its data in the caches of the new socket. Second, the random scheduling enlarges the memory footprint of the sockets. Since \( \gamma_2 \) and \( \gamma_3 \) share nothing but run in the same

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†We use “task-stealing” in this paper for the consistency of terms.
socket, the memory footprint of the socket directly increases. One immediate consequence of large memory footprint is the increase of chances for more cache misses, since $\gamma_2$ may pollute the cache entries for $\gamma_3$ due to conflicts or limited cache capacity. We call the above performance degrading problem, which is caused by the random task scheduling in the MSMC architecture, as the TRICY syndrome.

The TRICY syndrome damages the performance of memory-bound applications dramatically, since most of their execution time is spent on accessing memory. If we can relieve the TRICY syndrome by scheduling tasks with shared data to the same socket, the performance of memory-bound applications will be greatly improved in the MSMC architecture.

Among traditional task scheduling algorithms, task-stealing, which is introduced in Cilk[4], is increasingly popular due to its high performance. In task-stealing, each worker (i.e. thread) has a task pool to store tasks. Whenever a worker finishes its current task, the worker try to get a new task from its own task pool first. If its task pool is empty, the worker will choose a victim worker randomly to steal a task from the victim’s task pool. If succeeded, the worker will execute the stolen task; otherwise, the worker will keep trying to steal a task from another randomly-chosen victim. Unfortunately, task-stealing unexpectedly suffers from the TRICY syndrome due to its random stealing.

In order to relieve the TRICY syndrome, we propose a Cache Aware Bi-tier (CAB) task-stealing scheduler. CAB addresses the syndrome by scheduling tasks that share data onto the cores in the same socket in order for them to share data in caches. It divides the execution DAG of a program into two tiers: inter-socket tier and intra-socket tier. Tasks in the inter-socket tier, which are called inter-socket tasks, are scheduled across the sockets, while tasks in the intra-socket tier, called intra-socket tasks, are scheduled within the same socket. Moreover, CAB can automatically and optimally partition the execution DAG into the two tiers according to the data input size of an application, data cache size, the number of sockets, and etc.

The contributions of this paper are three-fold.

- The CAB task-stealing significantly relieves the TRICY syndrome by scheduling tasks with shared data onto cores of the same socket.
- CAB presents an automatic partitioning method to divide a DAG into two tiers so that tasks in different tiers are generated and scheduled in different ways.
- The experiment shows that CAB can significantly achieve a performance gain of up to 55% for memory-bound applications.

The rest of this paper is organized as follows. Section II introduces the background and motivation of CAB. Section III presents the DAG partitioning method and the CAB task-stealing algorithm, followed with a discussion of the theoretical time and space bounds of CAB. Section IV gives the implementation details of CAB. Section V shows the experimental results and evaluates the performance. Section VI discusses related work. Finally, Section VII draws conclusions and sheds light on future work.

II. BACKGROUND AND MOTIVATION OF CAB

There are two main classes of automatic task scheduling algorithms: task-sharing and task-stealing. In task-sharing, workers push new tasks into a central task pool when they are generated. Tasks are popped out from the task pool when workers are free to execute them. The push and pop operations need to lock the central task pool, which often causes serious lock contention.

Task-stealing, on the other hand, uses a task pool for each worker. Most often each worker pushes and pops tasks to its own task pool without locking. Only when a worker has no tasks in its task pool, should it try to steal tasks from other workers with locking. Since there are multiple task pools for stealing, the lock contention is much lower than task-sharing even at task steals. Therefore, task-stealing performs even better than task-sharing when the number of workers is increasing.

However, as mentioned before, task-stealing still suffers from the TRICY syndrome. Let us take the five-point heat program as an example, which simulates the heat distribution of a metal plate. In the program, the metal plate is divided into points of a two-dimensional grid. At each simulation step, the points in row $r$ are computed based on the points in rows $r$, $r-1$ and $r+1$ of the previous step.

Given a $10 \times 10$ matrix $M$ as the input data with the data type double (rows 0, 9 and columns 0, 9 are boundary data, and the real grid to be computed is a $8 \times 8$ matrix). In the parallel heat program, the heat procedure recursively generates two subtasks until the data set for each task is small enough. Fig. 1 shows the execution DAG of the heat program. The input data is recursively divided into two parts until each of the leaf tasks in the DAG only processes two rows.

![Execution DAG of five-point heat program](image)

According to the dependence relationship, tasks in the DAG can be divided into levels. If a task $\gamma$ in level $i$ generates a task $\beta$, then $\beta$ is in level $i+1$. The task that executes the “main” procedure is in level 0 and it is the initial task in the DAG.

Note that, in Fig. 1, only the leaf tasks (i.e., $T_4$, $T_5$, $T_6$, $T_7$) touch data, while all the other tasks in levels 0, 1, and 2 only divide the input data into two parts recursively.

Suppose this parallel heat program is executed on a dual-socket dual-core architecture with a hypothetical shared cache...
size of 480 bytes\(^2\) in each socket.

![Diagram](image)

Fig. 2. Two possible scheduling of tasks of heat running on dual-socket dual-core system. The first scheduling can gain performance improvement from cache-sharing and reduction of memory footprint, because \(T_4\) and \(T_5\), \(T_6\) and \(T_7\) have shared data.

In the above scenario, if the leaf tasks \(T_4, T_5, T_6\) and \(T_7\) are ideally scheduled in the way as shown in Fig. 2(a), data in the shared cache can be re-used and thus cache misses can be reduced. In Fig. 2(a), tasks running on the cores of the same socket (e.g. \(T_4\) and \(T_5\)) share two rows of input data. The two tasks in each socket only need to read six rows into the shared cache from the main memory altogether, i.e., \(6 \times 10 \times 8 = 480\) bytes. This data set size can fit into the shared cache of a socket. The overall memory footprint of the system is \(2 \times 480 = 960\) bytes.

However, for traditional task-stealing, since it distributes tasks randomly, the leaf tasks \(T_4, T_5, T_6\) and \(T_7\) are very likely scheduled in the way as shown in Fig. 2(b). Note that, in Fig. 2(b), tasks running on the cores of the same socket (e.g., \(T_4\) and \(T_6\)) do not share any data. In this situation, every task needs to access the main memory and reads four rows of the matrix into the cache. Because the two tasks in each socket need to read \(2 \times 4 \times 10 \times 8 = 680\) bytes, the data size exceeds the capacity of the shared cache of each socket, which leads to more capacity cache misses and increases the chances for conflict cache misses. Furthermore, the overall memory footprint of the system is \(2 \times 680 = 1280\) bytes, which leads to more compulsory cache misses.

In order to relieve the TRICY syndrome and schedule tasks in the same way as in Fig. 2(a), we propose the CAB task-stealing, which partitions the execution DAG into two tiers: inter-socket tier and intra-socket tier. Tasks in the inter-socket tier are scheduled across sockets, while tasks in the intra-socket tier are scheduled within the same socket. For example, in Fig. 1, tasks in levels 0-2 are in the inter-socket tier and tasks in level 3 are in the intra-socket tier. The tasks in level 2, the boundary of the two tiers, are called leaf inter-socket tasks. In CAB, the intra-socket tasks such as \(T_4\) and \(T_5\) are bound to the same socket. Since the intra-socket tasks generated by the same leaf inter-socket task often share data in real applications, their binding to the same socket in CAB can enforce the scheduling in Fig. 2(a) and results in fewer cache misses.

In the next section, we will present the details of the CAB task-stealing scheduler such as how the two tiers can be optimally partitioned so that there are enough intra-socket tasks for stealing in the same socket, while there should be sufficient inter-socket tasks for balancing the workload among sockets.

### III. Cache Aware Bi-tier Task-stealing Scheduler

This section presents CAB, a Cache Aware Bi-tier task-stealing scheduler. First, we give an overview of CAB. Then we introduce an automatic partitioning method for dividing the execution DAG into two tiers. Third, we present the CAB task-stealing algorithm. Finally, we give the time and space bounds of the parallel execution based on the CAB task-stealing.

#### A. Overview of CAB

CAB divides the workers into squads corresponding to the MSMC architecture. A squad is a group of workers running in the same socket. Each squad has a head worker. Supposing in an MSMC architecture there are \(M\) sockets with \(N\) cores each, CAB launches \(M \times N\) workers (i.e. threads) to work on the DAG in parallel. The workers are divided into \(M\) squads with \(N\) workers in each squad. A worker is affiliated with a hardware core, while a squad is affiliated with a socket. Fig. 3 depicts the relationship among cores, sockets, workers, and squads.

![Diagram](image)

Fig. 3. Relationship among cores, sockets, workers, and squads in an MSMC architecture. Workers that run on the cores of the same socket are grouped into the same squad. Each squad has one inter-socket task pool, which is used by the head worker of the squad to store inter-socket tasks. Each worker has one intra-socket task pool, which is used to store intra-socket tasks.

CAB adopts two types of task pools: inter-socket task pool and intra-socket task pool. A task pool is a double-ended queue (deque) that is used to store tasks. The inter-socket task pool is used to store tasks from the inter-socket tier of the DAG, and the intra-socket task pool stores tasks from the intra-socket tier. Each squad has an inter-socket task pool, and each worker has an intra-socket task pool, as shown in Fig. 3.

Suppose a new task \(\gamma\) is generated by a worker \(w\) of a squad \(\rho\). If \(\gamma\) is an intra-socket task, it is pushed into the intra-socket task pool of \(w\). If \(\gamma\) is an inter-socket task, it is pushed into the inter-socket task pool of \(\rho\).

When CAB starts to execute a parallel program in an MSMC architecture, CAB uses an automatic partitioning method (to be described shortly) to divide the execution DAG of the program into the inter-socket tier and the intra-socket tier.
tier. After the partitioning, CAB starts to execute the tasks by scheduling the inter-socket tasks and the intra-socket tasks based on the following stealing protocol.

A free worker in CAB first tries to obtain a task from its intra-socket task pool. If the task pool is empty, it tries to steal a task from the intra-socket task pools of other workers in the same squad. If all the workers of the squad have empty task pools, the head worker of the squad tries to obtain a task from its own inter-socket task pool. If its inter-socket task pool is empty, the head worker of the squad tries to steal an inter-socket task from other squads.

The above protocol only allows the head worker of a squad to steal inter-socket tasks so that the lock contention of the inter-socket task pools is reduced. Also a squad is not allowed to execute more than one inter-socket task at the same time, because the data of different inter-socket tasks may pollute the shared caches if multiple inter-socket tasks are executed at the same time in the same squad, which leads to more cache misses.

B. Automatic DAG partitioning method

As mentioned before, tasks in a DAG are divided into inter-socket tasks and intra-socket tasks according to their levels in the DAG. We compute a boundary level $BL$ that partitions the DAG into the inter-socket tier (the upper tier) and the intra-socket tier (the lower tier). Tasks in the boundary level $BL$ are called leaf inter-socket tasks. Since intra-socket tasks are scheduled within a squad, all the child tasks of a leaf inter-socket task are executed in the same socket.

However, finding the proper boundary level $BL$ to partition the DAG optimally is challenging. If the intra-socket tier is too thick, the involved data for a squad can be too large to fit into the shared caches of the socket of the squad. On the other hand, if the intra-socket tier is too thin, the workload of a squad can be too small to get better balanced among the workers of the squad.

Therefore, we require that the DAG partitioning method satisfy three constraints. The first constraint is that there should be enough leaf inter-socket tasks to be distributed to the sockets. The second constraint is that the involved data size of a leaf inter-socket task is small enough to fit into the shared caches of a socket. The third constraint is that a leaf inter-socket task should be large enough to enable a squad to have sufficient intra-socket tasks. After careful study, we model these constraints as follows using the following parameters: the input data size of the application, the number of sockets of the MSMMC architecture, the shared cache size of each socket, and the branching degree of the DAG.

Note that in the following model we assume that the program directly generates the task of the recursive divide-and-conquer procedure in the $main$ procedure, which is the case for all our benchmarks. For example, in Fig. 1, the $main$ procedure directly spawns the $heat$ procedure that recursively spawns tasks executing itself until a cut-off point. However, if the recursive procedure is not directly generated by $main$, we need either manually adjust the $BL$ value, or compiler support to adjust $BL$ automatically. We will discuss compiler support in Section IV.

In the model, we suppose an $M$-socket $N$-core system has a shared cache size $S_c$ for each socket and a recursive divide-and-conquer program has an input data size $S_d$. We assume the program divides the input data into $B$ parts each time sub-tasks are generated, i.e., the branching degree of the DAG of the recursive procedure is $B$. In this scenario, the boundary level $BL$ should have $B^{BL-1}$ leaf inter-socket tasks, since each task generates $B$ sub-tasks for the next level and this is repeated $BL-1$ times until the boundary level, assuming levels are numbered from 0 and the level 0 starts with $main$.

Since there are $M$ squads, in order to balance workload among squads, we should ensure that there are at least $M$ leaf inter-socket tasks. Therefore, $BL$ needs to satisfy Eq. 1 in order to fulfill the aforementioned first constraint.

$$B^{BL-1} \geq M$$

(1)

In order to fulfill the second constraint, we should ensure the data size for each leaf inter-socket task to fit into the shared cache of a socket. Since the input data are often divided evenly among the leaf inter-socket tasks, the second constraint can be expressed with Eq. 2.

$$S_d/B^{BL-1} \leq S_c$$

(2)

From Eq. 1 and 2, we can deduce two conditions for selecting an appropriate value for $BL$, as shown in Eq. 3.

$$\begin{cases} 
BL \geq \log_B M + 1 \\
BL \geq \log_B (S_d/S_c) + 1 
\end{cases}$$

(3)

From Eq. 3, we can select any $BL$ that is large enough to satisfy the two inequations. But, unfortunately, if $BL$ is too large, the number of the intra-socket tasks generated by a leaf inter-socket task will be too small, which leads to poor load balance within a squad. Therefore, we set $BL$ to be the smallest value that satisfies both inequations, as shown in Eq. 4.

$$BL = \max\{[\log_B M + 1], [\log_B (S_d/S_c) + 1]\}$$

(4)

In our current implementation, we use a semi-automatic method to acquire parameters $B$, $M$, $S_d$, and $S_c$ and then computes $BL$ according to Eq. 4. Parameters $M$ and $S_c$ are automatically acquired from /proc/cpuinfo by the runtime system, but $S_d$ and $B$ are provided through command line arguments. In Section IV, we will discuss how to automatically acquire these parameters by the compiler through program analysis.

In summary, CAB chooses $BL$ to be the smallest value while ensuring that the data set of the leaf inter-socket tasks can fit into the shared cache and that the number of leaf inter-socket tasks is large enough so that there is at least one inter-socket task for each and every squad. Experimental results in section V show that our automatic DAG partitioning method can find the optimal boundary level that enables the highest performance of the CAB scheduler.
Tasks in the inter-socket tier and the intra-socket tier are generated with different policies in CAB. There are generally two policies for task generation: child-first and parent-first. In the child-first policy, a worker executes the child task immediately once it is generated, leaving the parent task for later execution or for stealing by other workers. For example, the MIT Cilk uses the child-first policy, which is called work-first in [4]. However, in the parent-first policy, a worker executes the parent task continually after spawning a child task, pushing the child task into the task pool. One such example is the the help-first policy proposed in [11].

Both policies have their advantages in different situations. The child-first policy works better than the parent-first policy when the execution DAG is deep. However, the parent-first policy works better when the initial DAG is shallow and the steals are frequent, since it can quickly produce enough tasks for free workers [11].

Because there are more steals needed in the inter-socket tier where the execution DAG is expanding initially, CAB adopts the parent-first policy in the inter-socket tier in order to distribute the leaf inter-socket tasks to squads as soon as possible. After a squad gets a leaf inter-socket task, it uses the child-first policy to generate the intra-socket tasks. Since the number of workers is small and the steals are not frequent in a squad, the child-first policy is more suitable for intra-socket tasks. Also the child-first policy is more space efficient, which will be discussed shortly in Section III-E.

D. CAB task-stealing

As mentioned before, a free worker follows the stealing protocol in Section III-A to obtain or steal tasks. According to the stealing protocol, a squad is not allowed to execute more than one inter-socket task at the same time. In order to implement this requirement, CAB uses a boolean variable busy state for each squad. busy state indicates whether there is an inter-socket task running in the squad right now. When there is an inter-socket task running in a squad, busy state of the squad is true. When a squad finishes its inter-socket task, its busy state is set false. Only when busy state is false, can the squad obtain or steal another inter-socket task.

Algorithm I shows the detailed task-stealing algorithm that implements the stealing protocol.

E. Theoretical time and space bounds

We model the execution of a parallel program as an execution DAG $E$. Each node in $E$ represents a unit task, and each edge represents a dependence between tasks. Our following discussion is based on the time and space bounds of task-stealing proved in [10].

1) Time bound: For a DAG $E$, the work $T_1(E)$ of $E$ is the number of nodes in $E$, and the critical-path length $T_\infty(E)$ is the number of nodes along the longest path from the start node to the end node.

Since CAB divides a DAG into two tiers and executes them differently, we need to divide a DAG into sub-DAGs using the leaf inter-socket tasks. Given a leaf inter-socket task $\gamma$, we use the notation $E_\gamma$ to represent the subgraph rooted with $\gamma$, which includes the set of tasks that are generated from $\gamma$. Therefore, the total work of $E$ is divided as in Eq. 5, where $E_{inter}$ represents the subgraph of the inter-socket tier and $K$ is the total number of the leaf inter-socket tasks at the boundary level $BL$.

$$T_1(E) = T_1(E_{inter}) + \sum_{i=1}^{K} T_1(E_{\gamma_i}) \quad (5)$$

The execution time of $E$ in an $M$-socket $N$-core architecture, $T_{M+N}(E)$, can be divided into two parts: the execution time of the inter-socket tier $T_{M+N}(E_{inter})$ and the execution time of the intra-socket tier $T_{M+N}(E_{intra})$. Even though the two parts can be overlapped, we use their sum to get the worst bound of $T_{M+N}(E)$ as shown in Eq. 6.

$$T_{M+N}(E) = T_{M+N}(E_{inter}) + T_{M+N}(E_{intra}) \quad (6)$$

Since the inter-socket tier is executed by $M$ head workers using task-stealing, according to the proof of [10], the execution time of $E_{inter}$ is bounded by Eq. 7.

$$T_{M+N}(E_{inter}) \leq \frac{T_1(E_{inter})}{M} + T_\infty(E_{inter}) \quad (7)$$

For the execution of the intra-socket tier, each $E_{\gamma_i}$ is executed by $N$ workers within a squad using task-stealing. Therefore, the execution time of $E_{\gamma_i}$ is bounded by Eq. 8.

$$T_N(E_{\gamma_i}) \leq \frac{T_1(E_{\gamma_i})}{N} + T_\infty(E_{\gamma_i}) \quad (8)$$
Since $K$ leaf inter-socket tasks are scheduled among $M$ squads using task-stealing, the execution time of the intra-socket tier is bounded by Eq. 9.

$$T_{M*N}(E_{\text{intra}}) \leq \sum_{i=1}^{K} \frac{T_1(E_\gamma_i)}{M*N} + T_\infty(E_{\text{intra}})$$ (9)

Deducing from Eq. 8 and 9, we can get Eq. 10.

$$T_{M*N}(E_{\text{intra}}) \leq \sum_{i=1}^{K} \frac{T_1(E_\gamma_i)}{M*N} + \sum_{i=1}^{K} T_\infty(E_\gamma_i)$$ (10)

From Eq. 6, 7 and 10, $T_{M*N}(E)$ can be bounded as in Eq. 11.

$$T_{M*N}(E) \leq \frac{T_1(E_{\text{inter}})}{M} + T_\infty(E_{\text{inter}}) + \frac{\sum_{i=1}^{K} T_1(E_\gamma_i)}{M*N} + \frac{\sum_{i=1}^{K} T_\infty(E_\gamma_i)}{M} + T_\infty(E_{\text{intra}}$$ (11)

After further tidying Eq. 11 up, we have Eq. 12.

$$T_{M*N}(E) \leq \frac{T_1(E_{\text{inter}})}{M} + \frac{\sum_{i=1}^{K} T_1(E_\gamma_i)}{M*N} + \frac{\sum_{i=1}^{K} T_\infty(E_\gamma_i)}{M} + T_\infty(E)$$ (12)

According to Eq. 4, $K$ is at most several times of $M$. Therefore, the third item in Eq. 12 can be merged with the fourth item. Finally, we have the time bound of $E$ in an $M$-socket $N$-core architecture as shown in Eq. 13.

$$T_{M*N}(E) = O\left(\frac{T_1(E_{\text{inter}})}{M} + \frac{T_1(E_{\text{intra}})}{M*N} + T_\infty(E)\right)$$ (13)

According to Eq. 13, the inter-socket tier is executed by only $M$ head workers. However, in most recursively divide-and-conquer applications, only the leaf tasks in the DAG process input data, while the higher level tasks only divide the input data into smaller parts. Therefore, for a divide-and-conquer application, the main part of the execution time is spent by the leaf tasks, i.e., the intra-socket tasks. Our experiments show that the execution time of the inter-socket tier is often less than 5% of the overall execution time. Therefore, the time bound of Eq. 13 is very close to the traditional task-stealing schedulers such as Cilk for many divide-and-conquer applications.

2) Space bound analysis: According to the proof of [10], the space used by $E$ in an $M$-socket $N$-core architecture is bounded by Eq. 14, where $S_1(E)$ denotes the space used by the serial execution of the program.

$$S_{M*N}(E) \leq M*N*S_1(E)$$ (14)

Eq. 14 supposes there are at most $M*N$ workers expanding the DAG using the child-first policy. However, since CAB uses the parent-first policy to expand the inter-socket tier quickly, each of the leaf inter-socket tasks may use $S_1$ space in the worst case. Therefore, the space used by the CAB scheduler $S_{M*N}(E)$, can be bounded as in Eq. 15.

$$S_{M*N}(E) \leq \max\{K*S_1(E), M*N*S_1(E)\}$$ (15)

Again, according to Eq. 4, the number of leaf inter-socket tasks, $K$, is not much larger than $M$, so the space bound has the same O-notation as the traditional task-stealing schedulers.

IV. IMPLEMENTATION OF CAB

In this section, we present the implementation of CAB. First, we briefly introduce the MIT Cilk in which CAB is implemented. Then, we present the compiler support implemented for CAB, followed with the implementation of the CAB runtime system. Finally, we discuss issues related to the implementation. Note that Cilk programs can run in our current implementation without any modification.

A. Overview of MIT Cilk

MIT Cilk is one of the earliest parallel programming environments that implement task stealing [12]. It extends C with three keywords: cilk, spawn and sync to declare parallelism in the program. cilk identifies a procedure as a Cilk procedure, spawn is used to generate a child task, and sync waits for all the child tasks that are generated by the current task to return. Only Cilk procedures can be invoked with spawn.

MIT Cilk consists of two main parts: compiler and scheduler. Cilk compiler, named as cilk2c, is a source-to-source translator that transforms a Cilk source into a C program. cilk2c generates a fast clone and a slow clone for every Cilk procedure. The slow clone is executed if the task of the procedure is stolen; otherwise, the fast clone is executed instead. In addition, cilk2c uses a task frame data structure for every Cilk procedure. Once a task is generated, a task frame is created to store the information needed by the task and the scheduler.

B. Compiler support of CAB

We have modified cilk2c to support two types of spawns for the inter-socket and intra-socket tasks respectively. At each spawn, we compare the DAG level of the current task with the boundary level $BL$. If the level is smaller than $BL$, we spawn the child task as an inter-socket task and follow the parent-first policy. Otherwise, we spawn the child task as in intra-socket task and follow the child-first policy.

We also modified cilk2c to support two types of sync for the inter-socket and intra-socket tasks. This is because we use the child-first policy to generate the intra-socket tasks but use the parent-first policy to generate the inter-socket tasks. We use the different syncs to manipulate different return behaviors of the tasks.

We add into each task frame three variables: level, parent and inter_counter. level represents the level of the task in the execution DAG, parent is a pointer to the parent frame, and inter_counter is the number of astounding child inter-socket tasks spawned by the task. For example, when a task $\gamma$ generates a child inter-socket task $\gamma_1$, the inter_counter in the task frame of $\gamma$ is increased by 1. When $\gamma_1$ returns, through the
parent pointer in its task frame, $\gamma_1$ decreases the inter_counter in the task frame of $\gamma$ by 1. If $\gamma$’s inter_counter equals 0, that means all the inter-socket tasks generated by $\gamma$ have finished and the sync can be passed through.

The intra-socket tasks are handled similarly by an original Cilk data structure called closure, which includes the information of the above task frame but is more complicated than necessary for us to handle the inter-socket tasks.

C. CAB runtime system

For an M-socket N-core architecture, CAB launches $M \times N$ workers, and affiliates each worker to one individual core. The ID of each worker is the same as the ID of the core on which the worker is running. CAB groups workers into squads according to their IDs. If the core $i$ is in the socket $j$, the worker $i$ is grouped into the squad $j$. In each squad, the worker with the smallest ID is the head worker.

CAB executes a parallel program following the runtime algorithm in Algorithm II. Note that in the algorithm BL is set to 0 when there is only one socket in the architecture so that all tasks are generated as intra-socket tasks, which is the same as MIT Cilk.

ALGORITHM II
CAB RUNTIME ALGORITHM

**Assumption:** Suppose there is an M-socket and N-core architecture and a worker $w$ belongs to a squad $p$.

**Global initiation:**

**Step 1:** CAB launches $M \times N$ workers and affiliates them to the corresponding cores.

**Step 2:** CAB calculates BL. If $M$ equals 1, CAB sets BL to 0. Otherwise, CAB calculates $BL$ according to Eq. 4.

**Step 3:** Worker 0 begins to execute the initial task, while all other workers are trying to steal tasks.

**Task scheduling:** Assume worker $w$ is executing task $\gamma$.

(a) $\gamma$ generates $\gamma_1$: $\gamma$ computes the level of $\gamma_1$. If $\gamma_1$ is in the inter-socket tier, it is generated as an inter-socket task. Then $w$ pushes $\gamma_1$ into the inter-socket task pool of $p$ and continues to execute $\gamma$. Otherwise, if $\gamma_1$ is in the intra-socket tier, it is generated as an intra-socket task, which is pushed into the task pool of $w$ and to be executed by $w$ immediately.

(b) $\gamma$ suspends: $w$ tries to get a task according to Algorithm I.

(c) $\gamma$ returns: $w$ returns the results of $\gamma$ and sets busy_state of $p$ to false if $\gamma$ is an inter-socket task. Then $w$ tries to get a task according to Algorithm I.

**Termination:** If all the tasks have finished, CAB terminates.

D. Discussion

CAB does not require modification of the existing Cilk programs. Our current implementation uses a semi-automatic method to acquire the parameters for the calculation of BL. $M$ and $S_c$ are acquired automatically from the system, while the branching degree $B$ and the input data size $S_d$ of the recursive procedure are provided through command line.

An alternative method to use the CAB scheduler is through a new keyword inter_spawn which generates a task as an inter-socket task. This method enables the programmer to explicitly inform the scheduler about the type of tasks. Through the new keyword, the programmer can control the scheduling behavior and fine-tune the performance according to the program requirements. However, this method requires the programmer to modify the existing Cilk programs and more programming effort is required. According to our experiments, our semi-automatic method can achieve similar performance comparable to the well-tuned programs using this manual method.

It would be preferable to fully automate the acquisition of the parameters used for the calculation of BL. Compiler support can be useful to automatically acquire the branching degree $B$ and the input data size $S_d$ of the procedure. $B$ can be found by analyzing the source code based on the pattern of task generation, e.g. the keyword spawn in Cilk. However, to track the input data size $S_d$ of a procedure can be challenging. The compiler needs to track the calling chains and arguments to the procedure. This compiler support is an interesting research issue. Fortunately, it is relatively easy to track the data size of arguments in many strongly typed languages other than C.

Another issue in our current implementation is the calculation of BL. Currently we assume the main procedure directly calls a divide-and-conquer procedure that recursively spawns tasks. Even though this assumption is valid in all our benchmarks, there are real applications that may call multiple divide-and-conquer procedures indirectly through low level tasks. In such situations, we need to associate a $BL$ with each divide-and-conquer procedure and calculate the $BL$ on-the-fly when the procedure is initially spawned as a task. Though it is feasible to implement it with compiler support, however, this is not an issue we address in this paper.

To use the CAB scheduler for all applications with complete automation, the compiler should tell if an application is memory-bound or CPU-bound. A simple rule of thumb for the compiler to make the decision is that if the application has a large data size, it is memory-bound; otherwise, it is CPU-bound. For CPU-bound applications, CAB sets BL to 0 so that their tasks are scheduled as the traditional task-stealing.

Even though compiler support is an obvious way to transparently utilize the CAB scheduler, there may be other ways to use the scheduler without programming effort. For example, heuristic information may also be acquired at runtime to inform the scheduler about the inter-socket and intra-socket tasks. Such information can flexibly guide the CAB scheduler to schedule tasks within a socket or not. This is another interesting issue for future research.

V. PERFORMANCE EVALUATION

In the performance evaluation, we use a Dell 16-core computer which has four AMD Quad-core Opteron 8380 processors (codenamed “Shanghai”) running at 2.5 GHz. Each core has a 512K L2 cache for each core and a 6M L3 cache shared by all four cores. The computer has 16GB RAM and runs Linux 2.6.29. Accordingly CAB sets up four squads with four workers each.

Table III lists the benchmarks used in our experiments.

The Cilk benchmarks run with CAB without any modification. All benchmarks are compiled with “cilkc -O2”, which is
TABLE III
BENCHMARKS USED IN THE EXPERIMENTS

<table>
<thead>
<tr>
<th>Name</th>
<th>Type (bound)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>queens(20)</td>
<td>CPU</td>
<td>N queens problem</td>
</tr>
<tr>
<td>fft</td>
<td>CPU</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>ck</td>
<td>CPU</td>
<td>Rudimentary checkers</td>
</tr>
<tr>
<td>cholesky</td>
<td>CPU</td>
<td>Cholesky decomposition</td>
</tr>
<tr>
<td>Heat</td>
<td>Memory</td>
<td>Five-point heat</td>
</tr>
<tr>
<td>Mergesort</td>
<td>Memory</td>
<td>Merge sort on 1024*1024 numbers</td>
</tr>
<tr>
<td>SOR</td>
<td>Memory</td>
<td>Red/Black 2D Successive Over-Relaxation</td>
</tr>
<tr>
<td>GE</td>
<td>Memory</td>
<td>Gaussian elimination algorithm</td>
</tr>
</tbody>
</table>

Based on gcc 4.4. For each test, every benchmark is run ten times. The average execution time is used in the results.

A. Performance of memory-bound applications

Fig. 4 shows the performance of four memory-bound applications with a 1024 * 1024 matrix as input data. From the figure, we can see that CAB can significantly improve the performance of memory-bound applications, with the performance gain ranging between 10% and 55%. For example, Mergesort has achieved up to 55% performance gain with CAB compared with Cilk.

The performance gain of CAB is resulted from the relieved TRICY syndrome. According to Table IV, the L2 and L3 cache misses are prominently reduced in CAB compared with Cilk. Since the data set used by a squad is often shared by the workers of the squad and can fit into the L3 cache according to CAB, the number of L3 cache misses is much smaller than Cilk whose random scheduling often causes larger memory footprint and thus more cache misses for workers inside the same socket. Likewise, in CAB, the small memory footprint and the likely data sharing of workers in a squad help reduce the L2 cache misses as well.

B. Scalability of CAB

Input data sizes can affect the performance of CAB. If an input data is very large, the performance gain of CAB tends to be smaller. Due to limited space, we only use heat and SOR to discuss the scalability of CAB, though other benchmarks show similar results.

Fig. 5 shows the performance of heat and SOR with different input data sizes. According to the results, the performance gain of CAB is 55% when the input data is small (512*512), but drops to 5% when the input data is large (4k*8k).

One reason for the diminishing gain is that, with the increasing input data sizes, the shared data set between intra-socket tasks relatively becomes smaller, which increases the proportion of non-shared data and the cache misses in the leaf inter-socket tasks. Fig. 6 shows the L2 and L3 cache misses of heat and SOR. When the input data is small, CAB can reduce nearly 60% L3 cache misses and 80% L2 misses compared to Cilk. When the input data size is large, CAB can only reduce about 27% L3 cache misses and 59% L2 misses.

Another reason for the diminishing gain is that, when the input data is large, the granularity of the leaf tasks becomes large, which is not good for load balance within a squad. In order to relieve this problem, we have modified the the cut-off point in the heat program. Instead of using the fixed 128 rows of data as the data set for the leaf tasks, we use 64 rows and 32 rows respectively. According to Fig 7, when the number of rows for each leaf task is 32, the performance gain is increased to 10% compared with Cilk. Interestingly, the performance of Cilk has also increased significantly when the number of rows is 32 due to smaller grain of parallelism.

The above results show that when the input data is large, we should adapt the cut-off point accordingly in the program, so that the data size of each leaf inter-socket task can fit into
the shared cache while there are sufficient parallel tasks (intra-socket tasks) within the socket. In this way, CAB can maintain the performance gain of 10% when the input data sizes become large.

C. Effectiveness of automatic DAG partitioning method

In Section III-B, we have proposed a model to calculate the boundary level $BL$ in order to partition the DAG. The model uses four parameters: $B$, $M$, $S_d$, and $S_v$, as shown in Eq. 4. In this section, we use heat to evaluate the effectiveness of the model, though we have verified that the model works for other applications as well.

We evaluate the performance of heat with all possible $BL$ values. Since the heat program divides the input data into two parts each time sub-tasks are generated until the data size becomes 128 rows, there are fewer possible $BL$ values when the input data sizes are small.

Fig. 8 shows the performance of heat with different input data sizes and all possible $BL$ values. For example, for a $3k \times 2k$ matrix of double, there are 7 levels (0-6) in the execution DAG and the overall input data size is $3072 \times 2048 \times 8 = 48MB$. According to Eq. 4, CAB calculates $BL$ as $\max\{\left\lceil \log_2 4 + 1 \right\rceil, \left\lceil \log_2 (48MB/6MB) + 1 \right\rceil \} = 4$. From Fig. 8, we see that heat gains the best performance for data size $3k \times 2k$ when $BL$ is 4. The $BL$ values calculated for other data sizes are the ones with the best performance as well according to Fig. 8. This proves the effectiveness of Eq. 4 and our automatic DAG partitioning method.

Note that, for larger data sizes, when $BL$ is smaller than 3, the performance of CAB is worse than Cilk. This is because, when $BL$ is small, there is only a small number of leaf inter-socket tasks. In this situation, workload is not balanced well in CAB, because CAB may not utilize all the sockets due to the lack of inter-socket tasks. One such extreme case is when $BL = 1$, there is only one leaf inter-socket task, and thus only one squad can get the task.

On the other hand, if $BL$ is too large (e.g., $>4$), each leaf inter-socket task only contains a small number of intra-socket tasks. In such a situation, the workload within a squad cannot be balanced well. For example, for $BL = 6$ in the case of $3k \times 2k$, leaf inter-socket tasks are in level 6 and do not generate any intra-socket tasks. In this case, there is only one worker contributing to the performance of every squad.

D. Performance of CPU-bound applications

Since CAB is proposed to relieve the TRICY syndrome of memory-bound applications, CPU-bound applications cannot achieve better performance in CAB compared to the traditional task-stealing. Therefore, CAB schedules the tasks of CPU-bound applications as the traditional task-stealing by setting $BL$ to be 0.

Fig. 9 shows the performance of CPU-bound benchmarks listed in Table III. For most cases, the extra overhead added into the applications by CAB is around 1-2%. For fft, the extra overhead caused by manipulating the variable level in the task frames is less than 5%, though optimizations are possible to further reduce this overhead.

VI. RELATED WORK

Task-stealing is increasingly popular for automatic task scheduling. There have been a lot of research work on its adaptation and improvement [13], [14], [15], [16], [17].
There are generally two policies for task scheduling based on task-stealing: child-first and parent-first. In [11], the performance of the two policies were compared. Both child-first and parent-first policies have their strengths and are used pervasively in task-stealing schedulers. For example, MIT Cilk [4], Cilk++ [5], and Intel TBB [6] use the child-first policy, while Java’s fork-join framework [8] and Task Parallel Library (TPL) [18] use the parent-first policy. Also there are some task-stealing schedulers that adopt both policies, e.g., SLAW [19]. In SLAW, tasks are generated following either the child-first policy or the parent-first policy according to the stack pressure and task-stealing conditions. Although SLAW uses both policies as in our CAB scheduler, it does not associate the policies to the DAG level of tasks as we do in CAB. We adopt the parent-first policy to quickly generate the tasks in the inter-socket tier, but use the child-first policy to prevent the excessive task proliferation in the intra-socket tier.

Reducing the overhead of task-stealing has been a popular research issue. The overhead of task-stealing mainly includes task generating overhead, large number of unnecessary steals and etc. In [15], an adaptive task generation strategy, called AdaptiveTC, was proposed. AdaptiveTC can adaptively generate tasks to keep all workers busy most of the time while reducing the number of tasks generated. In [16], a non-blocking steal-half algorithm was introduced for a worker to steal half of the tasks from the victim worker, which can reduce the number of steals. In [17], an idempotent task-stealing was introduced and several algorithms were proposed to exploit the relaxed semantics of task execution in order to achieve a better performance. The relaxed semantics guarantee that each task is eventually executed at least one time, instead of exactly one time. These work could be applied to our CAB scheduler to further reduce task-stealing overhead.

There were some works on extending task-stealing to asymmetric multi-processors and distributed memory systems. In [20], Cilk was extended to run on asymmetric multi-processors and asymmetric multi-core processors. It presented a model in which each processor maintains an estimation of its speed. The model allows a fast processor to grab tasks from a slow processor when all the task pools are empty. In [21], a runtime system was proposed for supporting task-stealing on 8,192 processing cores on a cluster computer with distributed memory. In contrast to these architectures, our CAB scheduler is dedicated to the popular MSMC architecture.

Agrawal et al. in [22] proposed “helper locks” in task-stealing to execute large parallel critical sections which are processed serially in Cilk. Helper locks allow programs with large parallel critical sections, called parallel regions, to execute more efficiently by asking processors that might otherwise be waiting on the helper lock to aid in the execution of the parallel region. The notion of parallel region is somewhat similar to our tiers in DAG, but CAB treats the TRICY syndrome while the helper lock tries to accelerate the execution of a large parallel critical section.

Cache awareness is another interesting issue in task-stealing. In [23] a theoretical bound on the number of cache misses for random task-stealing was presented and a locality-guided task-stealing algorithm was implemented on a single-core SMP. In [24] cache behaviors of task-stealing and a parallel depth-first scheduler were compared and analyzed on a multi-core simulator that has shared L2 caches between cores. It proposed to promote constructive cache sharing through controlling task granularity. However, the above researches did not take the MSMC architecture into consideration.

To the best of our knowledge, CAB is the first cache-aware task-stealing scheduler that relieves the TRICY syndrome in the MSMC architecture.

VII. CONCLUSIONS AND FUTURE WORK

The CAB scheduler can effectively relieve the TRICY syndrome caused by the random task-stealing in the MSMC architecture. For memory-bound applications, CAB can achieve a performance gain up to 55% thanks to the large reduction of cache misses. CAB partitions the execution DAG into the inter-socket tier and the intra-socket tier using an automatic partitioning method. The method models the calculation of the partitioning boundary with four parameters. From our experimental results, this method can effectively find the optimal boundary level that enables CAB to achieve the best performance. Moreover, the extra overhead introduced by CAB is very small. For most CPU-bound applications, for which CAB cannot improve performance, the overhead is only around 1-2%.

Apart from the encouraging results, interesting issues have been discussed in the paper. One issue is the scalability of CAB when the input data size increases. Since the relative proportion of shared data set becomes small when the input data is large, the performance gain from reduced cache misses is getting small. However, according to our experimental results, CAB can still achieve 10% performance gain for large input data sizes.

Another interesting issue is compiler support for automatically acquiring the parameters such as the input data size and the branching degree to calculate the boundary between the inter-socket tier and the intra-socket tier. Though programmers can manually provide these parameters through command line in our current implementation, it is preferable to acquire them through program analysis of compiler. This work opens a door for program analysis to provide useful information for runtime optimization of task-stealing in parallel programming environments.

Future research includes a more flexible DAG partitioning method that can decide inter-socket and intra-socket tasks with heuristic information and compiler support instead of a single boundary level. Prefetching techniques with helper thread as in [3] can also be applied to CAB to further improve the performance of CAB when input data sizes are large but the data of each leaf inter-socket task can fit into the shared cache.

REFERENCES