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Short Communication: Calculating the 2D Motion of Lumbar Vertebrae using Splines

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Abstract

In this study we investigate the use of splines and the ICP method (Besl and McKay, 1992) for calculating the transformation parameters for a rigid body undergoing planar motion parallel to the image plane. We demonstrate the efficacy of the method by estimating the finite centre of rotation and angle of rotation from lateral flexion/extension radiographs of the lumbar spine. In an *in vitro* error study, the method displayed an average error of rotation of 0.44 ± 0.45 degrees, and an average error in FCR calculation of 7.6 ± 8.5 mm. The method was shown to be superior to that of Crisco et al. (1995) and Brinckmann et al. (1994) for the tests performed here. In general we believe the use of splines to represent planar shapes to be superior to using digitised curves or landmarks for several reasons. First, with appropriate software, splines require less effort to define and are a compact representation, with most vertebra outlines using less than 30 control points. Second, splines are inherently sub-pixel representations of curves, even if the control points are limited to pixel resolutions. Third, there is a well defined method (the ICP algorithm) for registering shapes represented as splines. Finally, like digitised curves, splines are able to represent a large class of shapes with little effort, but reduce potential segmentation errors from two dimensions (parallel and perpendicular to the image gradient) to just one (parallel to the image gradient). We have developed an application for performing all the necessary computations which can be downloaded from http://www.claritysmart.com.

Key words: shape representation, planar motion, splines

Nomenclature

p_k k^{th} control point of a spline

n Number of control points in spline

$P(u)$ Parametric form of a spline with spline parameter u

u Spline parameter, $0 \leq u < n$

t Alternative spline parameter, $0 \leq t < 1$, $t = u/n$

s $s = 0.5$ - used in definition of spline

FCR Finite centre of rotation

ICP Iterative closest point algorithm

Introduction

Calculating the two-dimensional rigid motion parameters between two distinct poses is a popular technique for assessing joint function (Petit et al., 2004; Sakamaki et al., 2002). In two dimensional applications it is typical to either define feature points (landmarks) on the object of interest (Brinckmann et al., 1994; Frobin et al., 1996, 2002) or to define feature lines (Quinnell and Stockdale, 1983; Van Akkerveeken et al., 1979; Bogduk et al., 1995; Stokes and Frymoyer, 1987). In either case, the points or lines can be placed by an expert operator or algorithmically by computer as in (Frobin et al., 1996, 2002). However such placement can be subject to errors from either inter-expert variability or inter-session variability in the case of manual placement, or by susceptibility to noise in the case of algorithmic placement. In this paper we focus on the lumbar spine and therefore take the method of Brinckmann et al. (1994) to be the current state-of-the-art method as this protocol has been extensively tested (Frobin et al., 1997; Leivseth et al., 1998) and has been used by other authors (Muggleton and Allen, 1998; Pfeiffer and Geisel, 2003).

The major disadvantage of algorithmic landmark techniques is that a different algorithm needs to be defined for each class of shape (Frobin et al., 2002). Further, any algorithm is liable to suffer from errors due to noise in image formation and contour extraction - the effects of which are usually mitigated by least squares techniques (Challis, 2001; Spiegelman and Woo, 1987; Crisco et al., 1994; Spoor and Veldpaus, 1980; Woltring et al., 1985; Veldpaus et al., 1988; McCane et al., 2005). Of course, least-squares techniques become more accurate as the number of (reliably extracted) landmarks increase, but defining a large set of such points is difficult for human vertebrae (Frobin et al., 1997; Pfeiffer and Geisel, 2003).

The closest method to the one developed here is the method of Crisco et al. (1995) which performs registration based on curvature estimates along a digitised curve. Their method is global and does not easily allow for sub-pixel registration. Further it has not been applied to the problem of estimating motion parameters in the lumbar vertebrae. We include their method for comparison with the method described here.

Materials and Methods

In this paper we focus on estimating the planar transformation of a rigid body where the image plane is parallel to the plane of transformation. We use the Finite Centre of Rotation (FCR) and angle of rotation to define the transformation as they are often used for assessing joint function (Bogduk et al., 1995; Amevo et al., 1992; Petit et al., 2004). We use the least squares technique of McCane et al. (2005) to calculate the angle of rotation and the FCR, which we note is equivalent to the method of Challis (2001).

The shapes to be matched are represented as Catmull-Rom splines (Hearn and Baker, 1994). These are interpolating splines which can be closely matched to a large class of smooth shapes. In our application these splines are specified interactively by a user marking up the vertebral body silhouettes from digital scans of radiographs of the lumbar spine (in full flexion/extension, see Figure 1). Pedicles and posterior elements are ignored as their appearance on lateral radiographs can be inconsistent and affected by x-ray dose and resolution.

A Catmull-Rom spline is defined by a set of control points (Hearn and Baker, 1994):

$$p_k = (x_k, y_k), \quad k = 0, 1, 2, ..., n-1. \tag{1}$$

We use closed splines, so $p_n = p_0$. A point on the spline is specified by the parameter $0 \leq u < n$:

$$\begin{aligned} P(u) = p_{k-1}(-su^3 + 2su^2 - su) + p_k[(2-s)u^3 + (s-3)u^2 + 1] + \\ p_{k+1}[(s-2)u^3 + (3-2s)u^2 + su] + p_{k+2}(su^3 - su^2) \end{aligned} \tag{2}$$

where $s = 0.5$, $k = \lfloor u \rfloor$ the floor of u, and p_k are the control points. For lumbar vertebrae, we have used on the order of 30 control points to adequately describe the shape.

Splines would typically be defined independently in each image of interest (in our application in flexion and extension images). It is likely that the two splines will have slightly different shapes and almost certainly different parameterisations (i.e. different control points - a problem largely ignored when

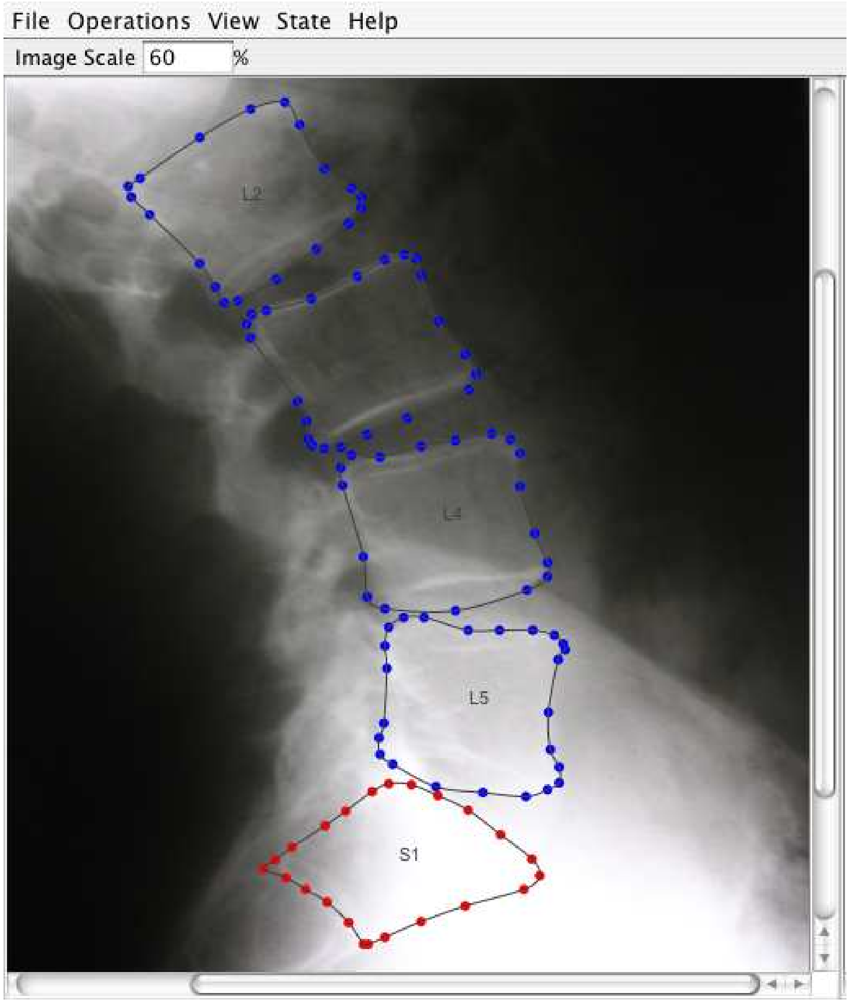


Fig. 1. An example radiograph with several splines.

digitised curves are used). As such, there is no straightforward way of directly establishing a correspondence between the two splines. In this paper, we use the well known iterative closest point (ICP) algorithm of (Besl and McKay, 1992). This algorithm is monotonically convergent (i.e. it always converges to a local minima) and finds the locally best correspondence between the two contours in a least squares sense. The method has always converged on the global minimum in our experiments. The algorithm is shown in Figure 2.

The `closest_point(P, p)` routine calculates the nearest point on a parametric entity `P` to the point `p` which can be found using Newton's minimisation

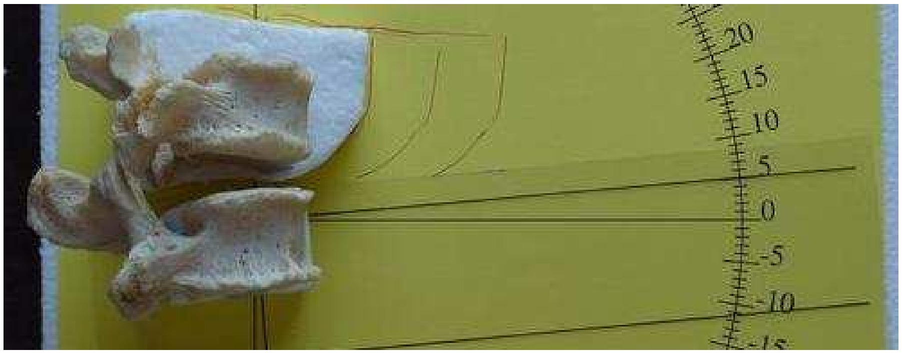
```
Input: P_1, P_2: two parametric contours with coincident centroids
       N: number of sample points to use (N = 100)
Output: θ: angle of rotation to register contours
θ = 0
while (not converged)
    t_1 = 0
    for i = 0 to N do
        p1[i] = P_1(t_1)
        p1 = closest_point(P_2, p1)
        t_1 = t_1 + 1.0/N
    endfor
    φ = find_best_rotation(p1,p2)
    P_2 = rotate(P_2, φ)
    θ = θ + φ
endwhile
```

Fig. 2. The ICP algorithm.

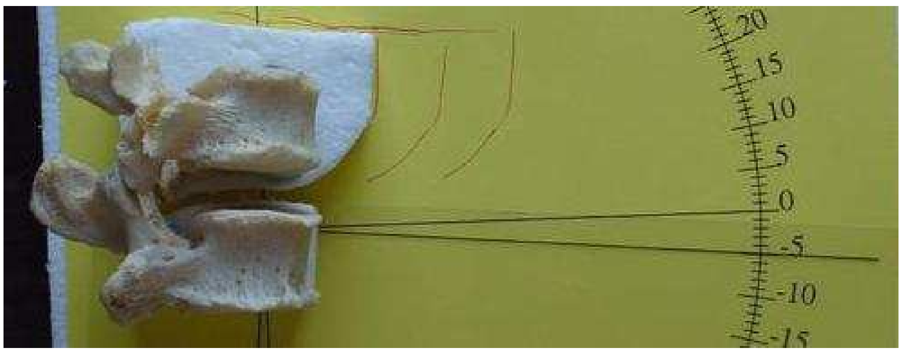
method (Besl and McKay, 1992). Spline centroids are calculated by digitising the spline to image resolution, summing all points in the digitised spline, and dividing by the number of points (that is, a numerical estimate of the true centroid, which can't be calculated analytically). This method gives the best estimate of the relative translation between splines in a least-squares sense (Challis, 2001), if those splines are related by a combination of translation, rotation, and isotropic scale transformations. Therefore, the method is valid for closed splines and open splines which terminate at common anatomical points in both images. The method will not work for open splines without common termination points, for which partial matching techniques would be needed.

Four methods are compared: the ICP method using Catmull-Rom splines (ICP); the method of Crisco et al. (1995) by extracting the curvature at 100 equally spaced points along the spline (Crisco); the method of Brinckmann et al. (1994) using ventral and dorsal corner points extracted from the spline (Brinck Cor.); and the method of Brinckmann et al. (1994) using ventral and dorsal midpoints (Brinck Mid.). We have performed two experiments. The first involved an *in vitro* error study with known angle of rotation and FCR. Two human vertebrae (L4 and L5) were mounted on polystyrene bases. One of the vertebrae was then fixed onto a base which had a printed protractor attached to it. The second was fixed to a transparent sheet which was in turn fixed to the base at a single fixed point (the FCR) with a metal pin. Radiographs were taken for six different angular rotations and three different locations of the FCR resulting in 18 total radiographs. Angles were read off the protractor as the radiograph was taken (accuracy $\pm 0.1°$) and the metal pin appears on the radiograph. Figure 3 shows a picture of the experimental setup. The pin appeared as a line on the radiograph as it was not parallel to the x-ray beam. The ends of the pin were located to within 1 pixel (0.17 mm), and the FCR estimated as a fixed ratio along the length of the pin line corresponding to the ratio of the pin above and below the surface of the base (accuracy ± 0.29 mm). Each radiograph was matched with every other radiograph with the same FCR position, resulting in 45 flexion/extension pairs to be analysed with angle of rotation varying from 3 degrees to 15 degrees. The radiographs were digitised on a consumer grade flat-bed scanner (maximum distortion 0.5mm) at 150 dpi.

The second experiment is an *in vivo* intra-operator consistency study where



(a) Angle = $5°$



(b) Angle = $-5°$

Fig. 3. Two positions of the vertebrae in the error analysis. The angle is read directly off the protractor as the radiograph is taken. The metal pin (obscured by a vertebra) shows up directly on the radiograph).

the FCR and angle of rotation are measured on two different occasions for the same operator. Thirty radiographs of different patients in full flexion and extension were used.

Results

In the *in vitro* error study, the ICP method produced an FCR error of 7.3 ± 8.3 mm and an angle error of 0.44 ± 0.45 degrees (table 1). These results are

Measure	ICP	Crisco	Brinck Mid	Brinck Cor.
FCR error (mm)	7.3 ± 8.3	11.6 ± 15.5	31.7 ± 64.5	33.4 ± 45.6
angle error (degrees)	0.44 ± 0.45	1.00 ± 0.63	1.48 ± 0.97	2.27 ± 1.57

Table 1

In vitro true error (criterion-related validity). Mean and standard deviation of the errors for calculating the FCR and the angle of rotation for each of the methods tested.

Comparison	Angle		FCR	
	Wilcox test	F test	Wilcox test	F test
ICP vs Crisco	5.4×10^{-6}	0.017	0.00010	3.1×10^{-5}
ICP vs Brinck Mid	1.5×10^{-8}	7.3×10^{-7}	1.3×10^{-8}	$p < 2.2 \times 10^{-16}$
ICP vs Brinck Cor	9.0×10^{-10}	6.6×10^{-14}	5.1×10^{-11}	$p < 2.2 \times 10^{-16}$

Table 2

p-values for *in vitro* error study statistical tests.

consistent with previous studies (Crisco et al., 1994; Harvey and Hukins, 1998; Challis, 2001). We used a paired Wilcoxon signed rank test to compare the errors of the ICP method with each of the other methods, and an F test to compare variances. With one exception it was found that the new method was superior in both error and variance to an extremely significant level ($p < 0.001$). The angle variance of the ICP method was superior to the method of Crisco et al. (1995) to a significant level only ($p = 0.017$). Actual p-values are shown in Table 2.

In the *in vivo* intra-operator consistency study, the ICP method produced

Statistic	ICP method	Crisco	Brinck Mid.	Brinck Cor.
FCR distance (mm)	12.4 ± 7.4	12.8 ± 8.1	48.0 ± 159.2	15.0 ± 10.4
angle difference (degrees)	1.3 ± 1.1	23.2 ± 54.2	2.3 ± 1.9	2.4 ± 2.0

Table 3

Intra-operator consistence (precision). Mean and standard deviation of the differences between the FCR and the angle of rotation calculated on two separate occasions. The FCR error is only reported for cases where the angle of rotation is greater than 5 degrees as it is known to result in large errors for small angles.

an FCR distance (RMS) of 12.4 ± 7.4 mm and an angle difference (RMS) of 1.3 ± 1.1 degrees (table 3). Again we have used the paired Wilcoxon signed rank test, and F test to compare methods. The ICP method was found to produce lower angle differences and variances to an extremely significant level ($p < 0.001$). The ICP method produced lower FCR distances and variances to a highly significant level ($p < 0.01$) in all but one case for which the result was significant ($p < 0.05$). Table 4 shows the actual p-values for each of the tests.

Discussion

We have tested a new method for determining the plane transformation parameters for the lumbar spine from lateral flexion/extension radiographs. We have found this method to be superior to previously used methods including the method of Crisco et al. (1995). However, the method of Crisco et al. (1995) is a global search method and therefore suffers from potentially large errors when the shapes exhibit some rotational symmetries - this was the major rea-

Comparison	Angle		FCR	
	Wilcox test	F test	Wilcox test	F test
ICP vs Crisco	6.0×10^{-15}	$p < 2.2 \times 10^{-16}$	0.0020	$p < 2.2 \times 10^{-16}$
ICP vs Brinck Mid	1.3×10^{-7}	1.4×10^{-8}	0.0022	0.023
ICP vs Brinck Cor	6.2×10^{-9}	6.9×10^{10}	0.0025	$p < 2.2 \times 10^{-16}$

Table 4

p-values for *in vivo* consistency study statistical tests. FCR errors for small angles are included here - some results are not significant when small angles are eliminated (Crisco FCR Wilcox and FCR F test, and Brinck Cor FCR Wilcox).

son for large errors in the *in vivo* study. The ICP method on the other hand is a local search method and hence tends to favour smaller rotations - these rotations can still be quite large, but are generally limited to the smallest rotation for which some rotational symmetry appears in the matching shapes (90° in the case of vertebrae). This limitation of the ICP method can be somewhat alleviated by using several starting rotations for the search. Similarly, the method of Crisco et al. (1995) could be improved in this case by limiting the size of the rotations or by increasing the number of match points. We have also shown that the ICP method is superior to the method of Brinckmann et al. (1994) and has several advantages to that technique. Most notably errors are reduced by using a large number of matching points which do not have to be explicitly defined on a per-shape basis.

The main limitation of our study is that we did not explicitly test the method for errors involving axial rotation or lateral tilt, however the studies by Brinckmann et al. (1994); Frobin et al. (1997); Leivseth et al. (1998); Shaffer et al.

(1990) demonstrate that moderate magnitudes of rotation or tilt (up to 10°) introduce only modest amounts of error. This is a potential area for future investigation, perhaps using a method similar to that of Harvey and Hukins (1998). We note that this is also a limitation of many other studies investigating two-dimensional rigid body kinematics (Crisco et al., 1995; Challis, 2001; Bogduk et al., 1995).

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