

## Tree-like permutations

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Given a permutation  $\pi \in S_n$ , construct a graph  $G_\pi$  on the vertex set  $\{1, 2, \dots, n\}$  by joining  $i$  to  $j$  if (i)  $i < j$  and  $\pi(i) < \pi(j)$  and (ii) there is no  $k$  such that  $i < k < j$  and  $\pi(i) < \pi(k) < \pi(j)$ . We say that  $\pi$  is forest-like if  $G_\pi$  is a forest. We first characterize forest-like permutations in terms of pattern avoidance, and then by a certain linear map being onto. Thanks to recent results of Woo and Yong, this shows that forest-like permutations characterize Schubert varieties which are locally factorial. Thus forest-like permutations generalize smooth permutations (corresponding to smooth Schubert varieties).

We compute the generating function of forest-like permutations. As in the smooth case, it turns out to be algebraic. We then adapt our method to count permutations for which  $G_\pi$  is a tree, or a path, and recover the known generating function of smooth permutations.