Almost Avoiding Permutations

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Since Noonan [3], there has been considerable interest in counting permutations with a bounded number of copies of a pattern. The most general result to date — proved independently by Bóna [1] and Mansour and Vainshtein [2] — states that for any fixed \( r \), the number of permutations with at most \( r \) copies of 132 has an algebraic generating function; indeed, these generating functions lie in \( \mathbb{C}(x, \sqrt{1-4x}) \).

While these permutations can be said to “almost avoid” 132, there is a more natural formalization of this term. We say that \( \pi \) almost avoids \( \beta \) if one can remove a single entry from \( \pi \) to obtain a \( \beta \)-avoiding permutation (and thus it would be more accurate to say that \( \pi \) avoids or almost avoids \( \beta \)). Thus the permutations with at most 1 copy of 132 almost avoid 132 in this sense, but there are also permutations with arbitrarily many copies of 132 which almost avoid 132.

Using the Robinson-Schensted-Knuth correspondence, we give a formula for the number of permutations that almost avoid 123. In the case of 132 we are able to give a generating function via a lengthy case-by-case analysis.

References

