Almost Avoiding Permutations

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Since Noonan [3], there has been considerable interest in counting permutations with a bounded number of copies of a pattern. The most general result to date — proved independently by Bóna [1] and Mansour and Vainshtein [2] — states that for any fixed r, the number of permutations with at most r copies of 132 has an algebraic generating function; indeed, these generating functions lie in $\mathbb{C}(x, \sqrt{1-4x})$.

While these permutations can be said to "almost avoid" 132, there is a more natural formalization of this term. We say that π *almost avoids* β if one can remove a single entry from π to obtain a β-avoiding permutation (and thus it would be more accurate to say that π avoids or almost avoids β). Thus the permutations with at most 1 copy of 132 almost avoid 132 in this sense, but there are also permutations with arbitrarily many copies of 132 which almost avoid 132.

Using the Robinson-Schensted-Knuth correspondence, we give a formula for the number of permutations that almost avoid 123. In the case of 132 we are able to give a generating function via a lengthy case-by-case analysis.

References

[1] Bóna, M. The number of permutations with exactly r 132-subsequences is P-recursive in the size! *Adv. in Appl. Math. 18*, 4 (1997), 510–522.

[2] Mansour, T., and Vainshtein, A. Restricted 132-avoiding permutations. *Adv. in Appl. Math. 26*, 3 (2001), 258–269.

[3] Noonan, J. The number of permutations containing exactly one increasing subsequence of length three. *Discrete Math. 152*, 1-3 (1996), 307–313.