Permutation tableaux and permutation patterns

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We introduce and study a class of tableaux which we call permutation tableaux; these tableaux are naturally in bijection with permutations, and they are a distinguished subset of the "Le-diagrams" of Alex Postnikov. The structure of these tableaux is in some ways more transparent than the structure of permutations; therefore we believe that permutation tableaux will be useful in furthering the understanding of permutations. We give two bijections from permutation tableaux to permutations. The first bijection carries tableaux statistics to permutation statistics based on relative sizes of pairs of letters in a permutation and their places. We call these statistics weak excedance statistics, because of their close relation to weak excedances. The second bijection carries tableaux statistics (via the weak excedance statistics) to statistics based on generalized permutation patterns. We then give enumerative applications of these bijections. One nice consequence of these results is that the polynomial enumerating permutation tableaux according to their content generalizes both Carlitz' q-analog of the Eulerian numbers and the more recent q-analog of the Eulerian numbers found by the second author.