

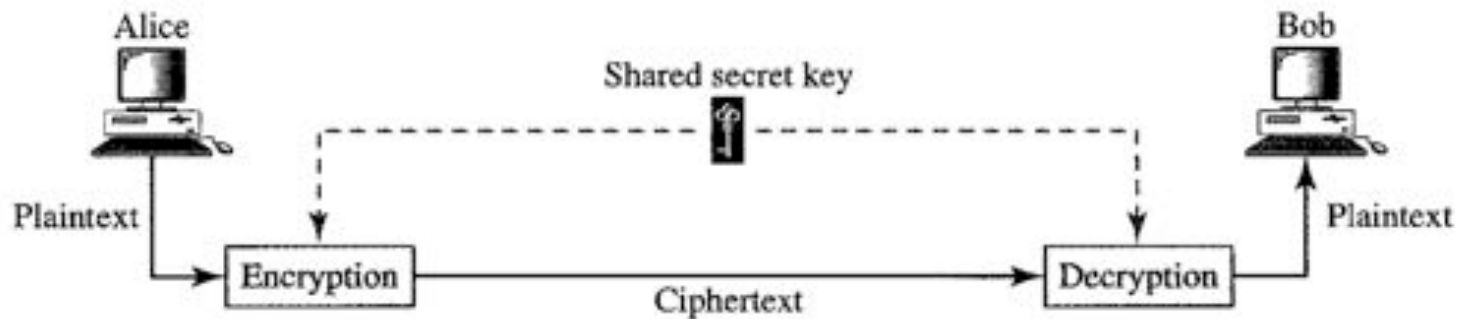
# Overview

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- Last Lecture
  - Data security 1 (Private key encryption)
- This Lecture
  - Data security 2 (Public key encryption)
  - Source: Sections 31.2-31.3
- Next Lecture
  - Introduction to networks
  - Source: Sections 2

# Private Key Encryption

- Symmetric-key: the same key (Secret key) is used by the sender and receiver. The key is shared.
  - Caesar cipher, Transposition ciphers, DES introduced in last lecture
  - Key is very important in these methods. The secrecy of the key must be protected
  - These methods are called private key encryption



# Private Key Encryption (cont.)

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- The best encryption method in the world is no good if the key cannot be kept secret.
- How does the sender communicate the key to the receiver?
  - Sender send the key: what if an unauthorized receiver gets it?
  - Encrypt the key: same problem?
  - Can we make the encryption key public while still making the encrypted message secure?

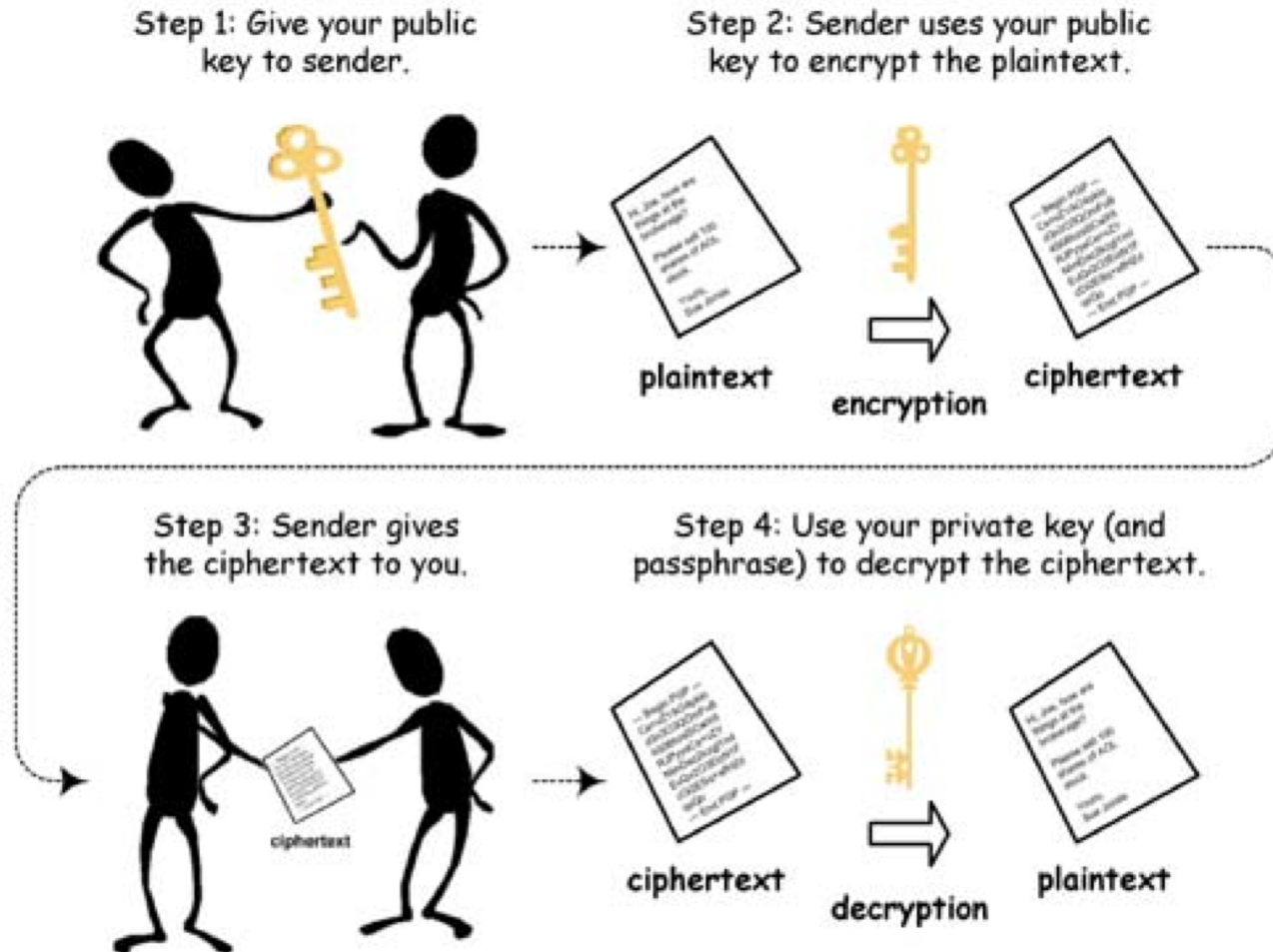
# Public Key Encryption

video1: to 1:23

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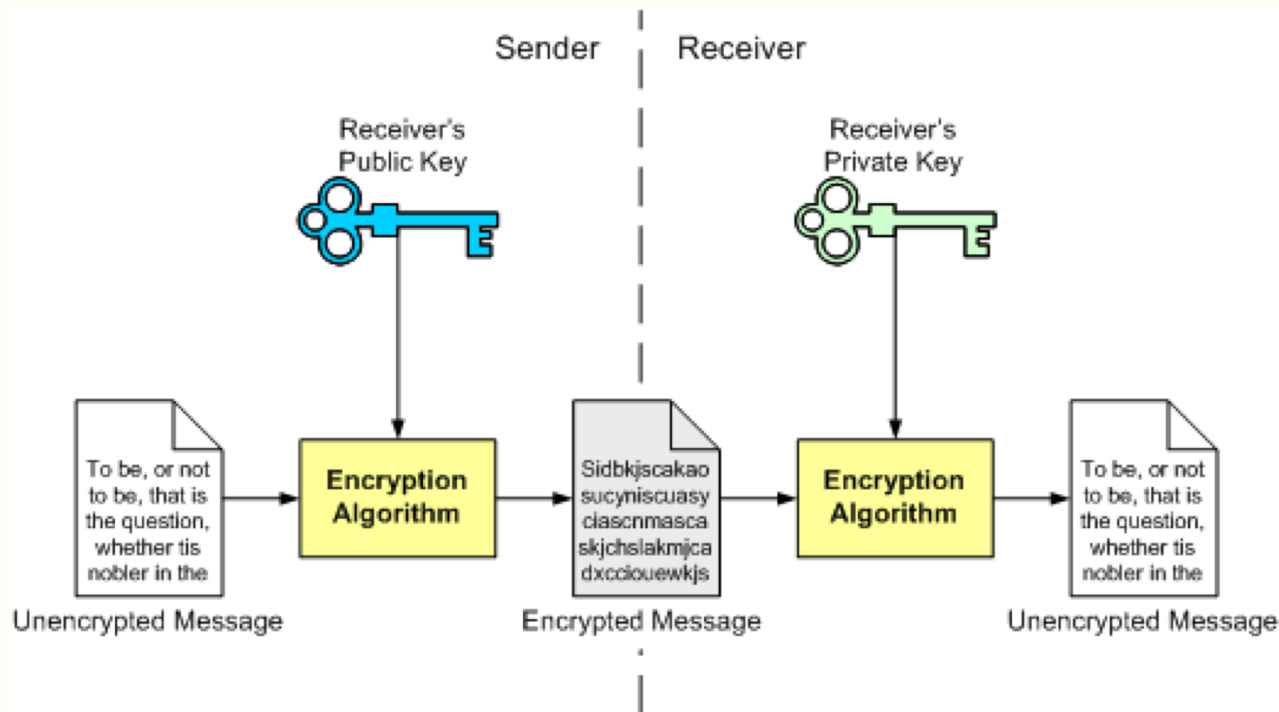
- Encryption and decryption functions are separated
- Encryption key cannot be used to decrypt a message
- The encryption key is public, but the decryption key is only known to the receiver.
- Used for banking/military use/electronic commerce

# Public Key Encryption (cont.)



# Public Key Encryption (cont.)

- Asymmetric-key: two separate keys instead of one
  - One key used for the sender to encrypt: Public key
  - One key used for the receiver to decrypt: Private key



# Public Key Cryptosystems (cont.)

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- The most popular is called RSA algorithm, (named after its inventors: Rivest, Shamir, Adelman. Invented in 1978, Turing Award in 2002, video2)



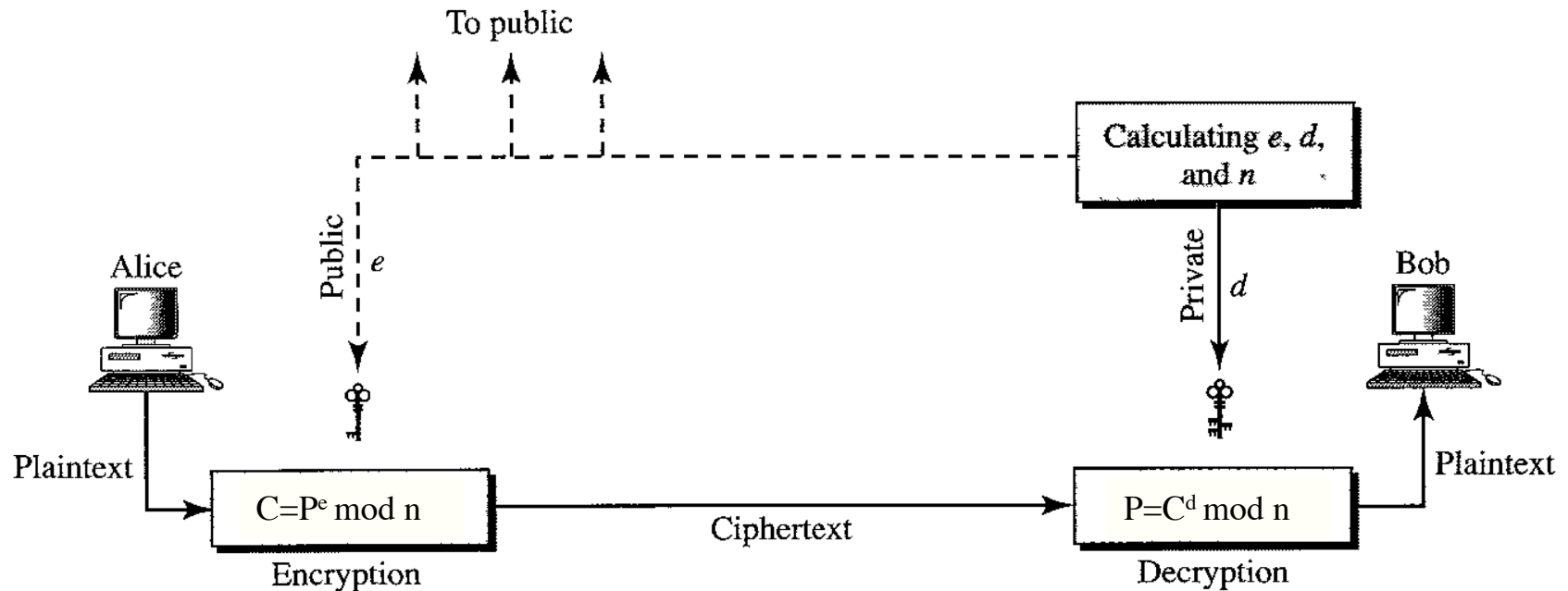
# Public Key Cryptosystems (cont.)

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- Terms used in RSA algorithm
  - prime number: number that can only be divided by 1 or itself
  - Greatest Common Divisor (GCD): largest number that two numbers can be divided by
  - Relatively Prime: when the GCD of two numbers is 1
  - Modulo: operation that gives the remainder of a division



# Public Key Cryptography algorithms



The two keys, public key  $e$  and private key  $d$ , have a special relationship to each other! (number theory)

# RSA Algorithm

## video 3

- Bob use the following steps to select private and public keys:
  - Choose two prime numbers, p and q.  $p = 7; q = 11$
  - $n = p \times q$   $n = 7 \times 11 = 77$
  - $m = (p-1) \times (q-1)$   $m = 6 \times 10 = 60$
  - Select e that is relatively prime to m. (That is, the greatest common divisor of e and m is 1.)  $e = 7$
  - Find d such that  $(e \times d - 1) \bmod m = 0$   $d = 43$ 
    - This means  $e \times d - 1$  is evenly divisible by m, or  $d \times e = k \times m + 1$ ,  $d = (k \times m + 1)/e$
- Public key = {e} (Bob announces  $e=7$  and  $n=77$  to the public)
- Private key = {d} (Bob keeps  $d=43$  and  $m=60$  secret)
- Encryption: Ciphertext,  $C = P^e \bmod n$ 
  - (Alice calculates using public key  $e=7$ ,  $C=8^7 \bmod 77 = 57$ )
- Decryption: Plaintext,  $P = C^d \bmod n$ 
  - (Bob calculates using private key  $d=43$ ,  $P=57^{43} \bmod 77 = 8$ )

# Example

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- Public key: 7, Private key: 43
- Suppose HELLO is to be sent (8, 5, 12, 12, 15)
- $8^7 \bmod 77 = 57$ ;  $5^7 \bmod 77 = 47$ ;  $12^7 \bmod 77 = 12$ ;  
 $15^7 \bmod 77 = 71$
- Send 57, 47, 12, 12, 71
- $57^{43} \bmod 77 = 8$ ;  $47^{43} \bmod 77 = 5$ ;  $12^{43} \bmod 77 = 12$ ;  $71^{43} \bmod 77 = 15$

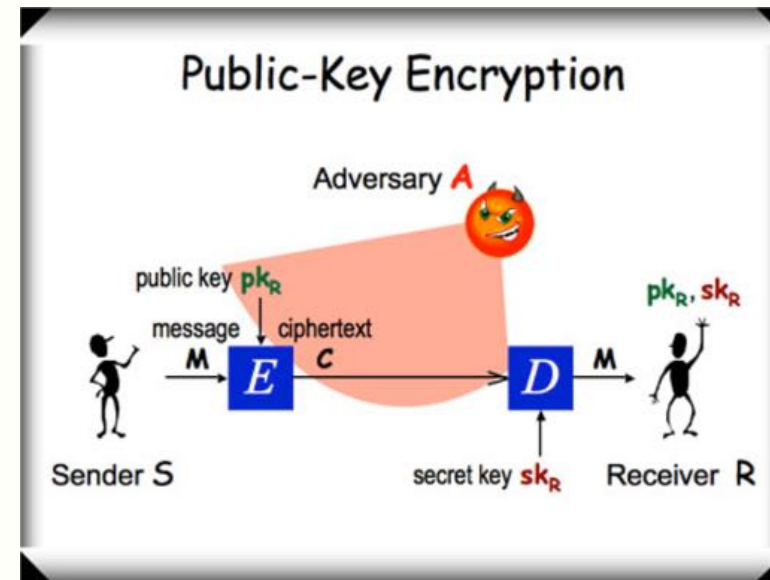
The encryption and decryption are surprisingly simple! – exponentiation and modular arithmetic

Online demo: <http://cobweb.cs.uga.edu/~dme/csci6300/Encryption/Crypto.html>

# RSA Algorithm (cont.)

- Justification of security
  - Suppose you know  $e$  and  $n$ , and intercept the encrypted message.
  - Since the encryption is not reversible, you cannot decrypt the message by a reverse operation.
  - You would need to find  $p$  and  $q$ , where  $p \times q = n$ , and then find  $d$ , where  $e \times d \bmod (p-1)(q-1) = 1$
  - If  $n$  is very large (e.g. 159,197), finding  $p$  and  $q$  that are prime numbers (factors of  $n$ ) is very time consuming.

The ciphertext is very difficult to break even when public key  $e$  and  $n$  are known.

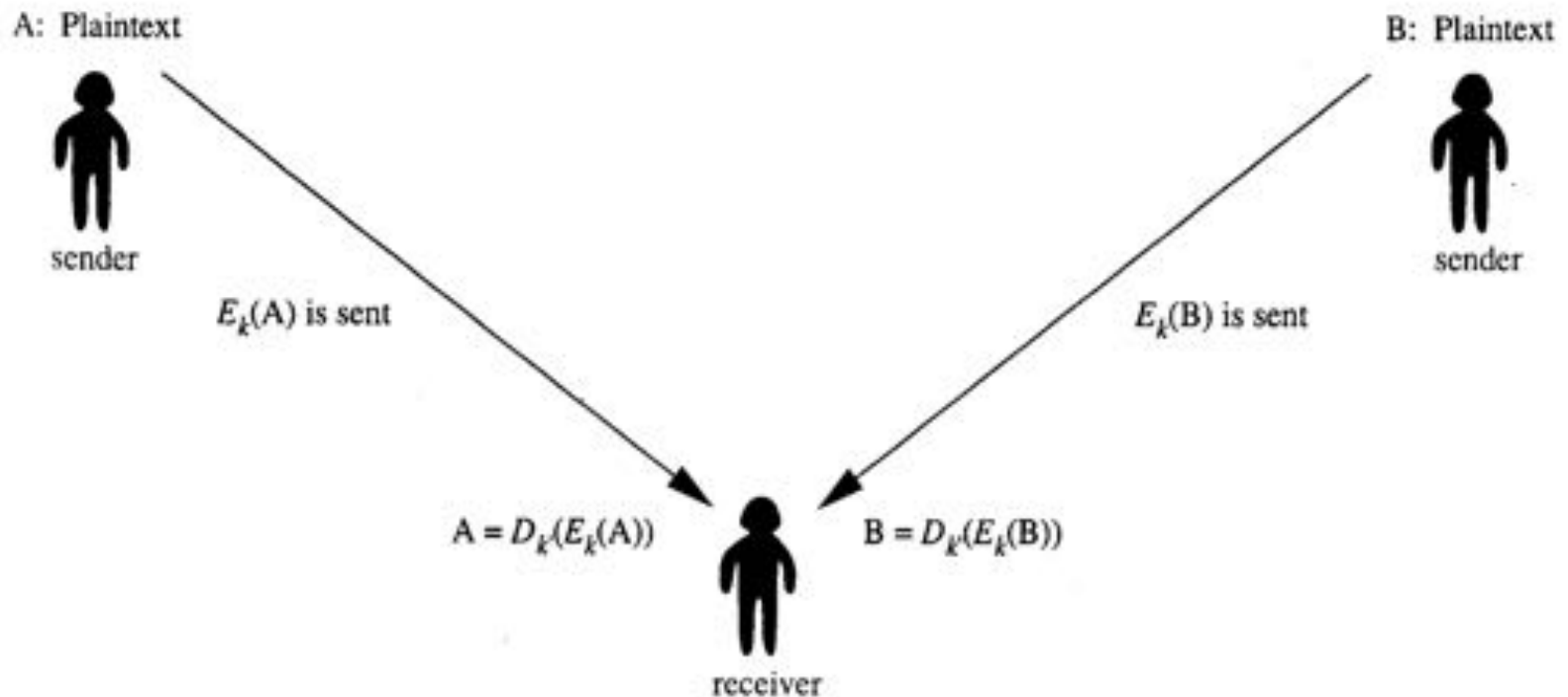


# Public Key Encryption (cont.)

## video 4

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**Figure 4.22** Multiple Senders Using the Same Encryption Method



# Public Key Encryption (cont.)

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- Problem: RSA is slow if the message is long
- RSA is useful for short messages, such as small message digest, a symmetric key etc.
- RSA Applications:
  - Authentication (Digital signature)
    - Signing the Message
    - Signing the Message Digest
  - PGP

# Authentication

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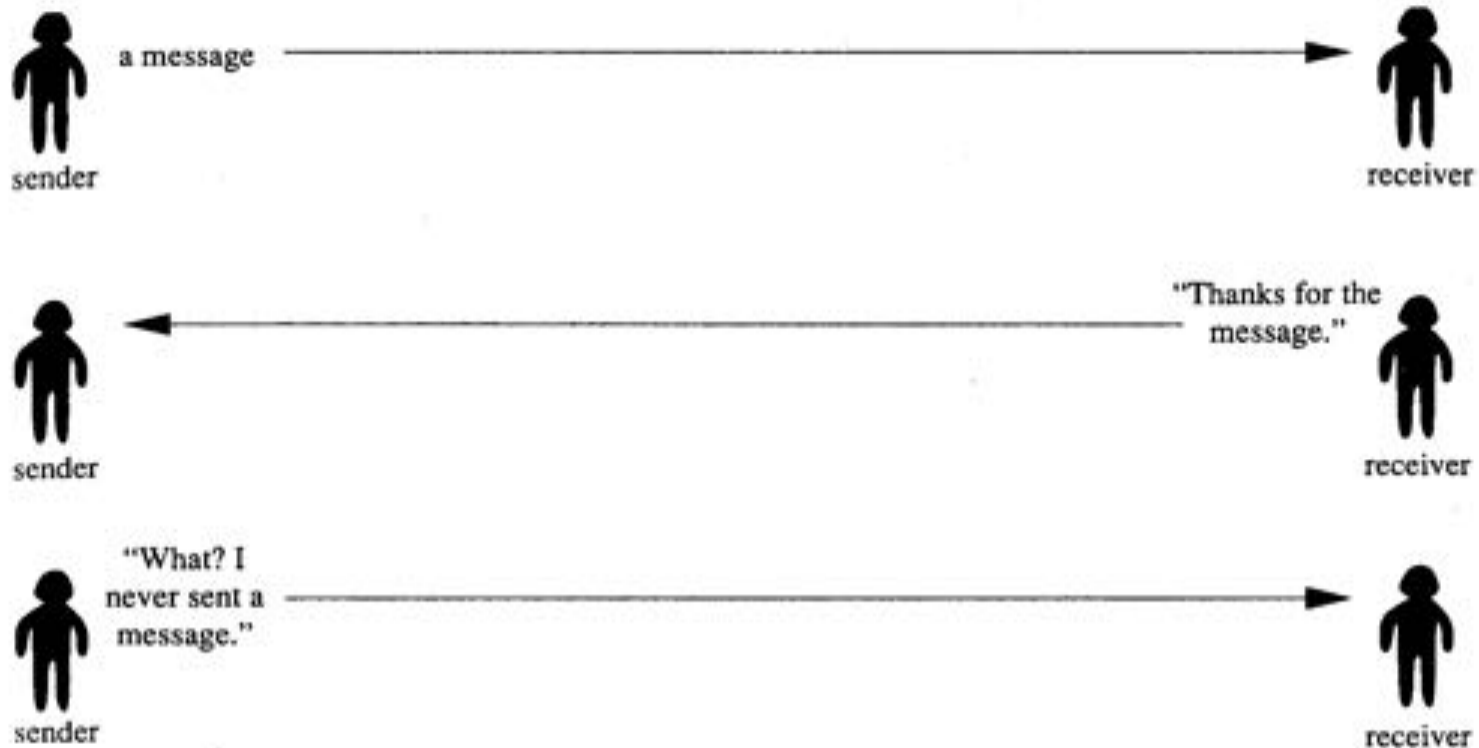


Figure 4.23 Sender Denying Sending a Message

# Authentication (cont.)

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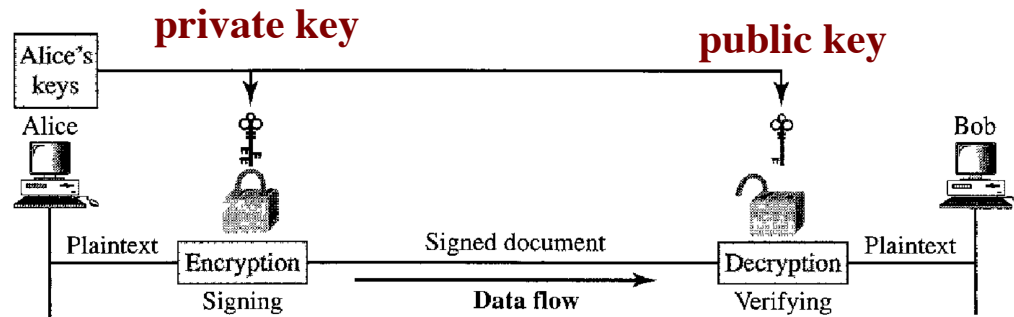
- Verifying the identity of a sender is called authentication
  - The sender may deny his/her message
  - Somebody may pretend to be the sender
- It is very important for banking system, e-commerce, digital contracts.



# Authentication (cont.)

- One method of authentication: digital signature
- Signing the Message (Document) using the private and public key of **the sender**, instead of the receiver

Figure 31.11 Signing the message itself in digital signature



- Since *only* the sender's public key can decrypt the digital signature (encrypted with the sender's *private* key), Alice is the only person who sent the message.

# Authentication (cont.)

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- Signing the Message Digest
  - Public key is inefficient to deal with long messages
  - Want to know that an electronic document or file has not be altered
    - Document: Message
    - Fingerprint: Message digest
    - hash function

# Authentication (cont.)

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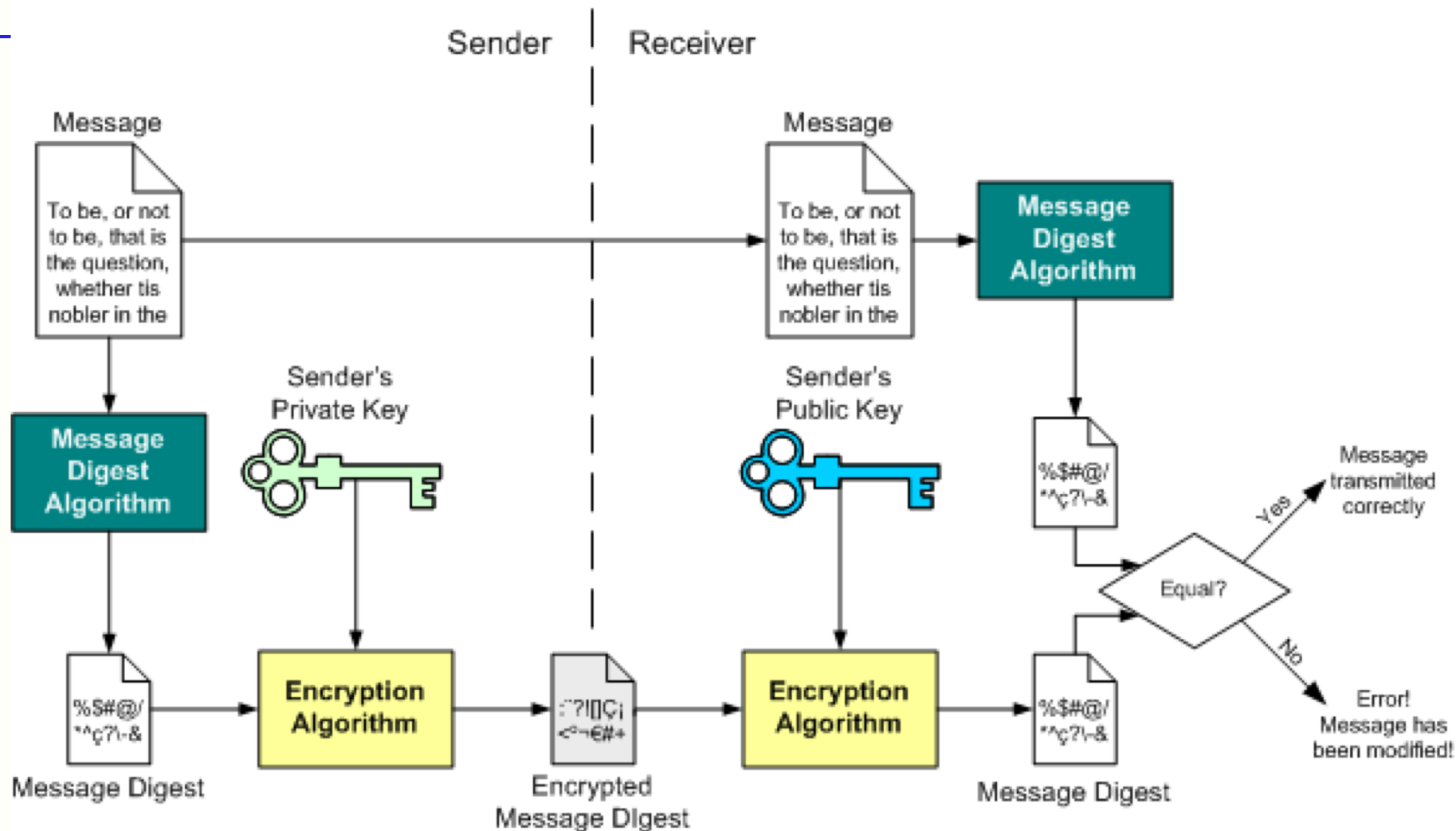
- Sender does the following:
  - Step 1: A *message digest* is generated. ('summary' of the message)
    - smaller than the message itself
    - slightest change in the message produces a different digest.
    - generated using a set of hashing algorithms
  - Step 2: The message digest is encrypted using the sender's *private* key. (The resulting encrypted message digest is the digital signature.)
  - Step 3: The digital signature is attached to the message, and sent to the receiver.

# Authentication (cont.)

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- The receiver then does the following:
  - Using the sender's public key, decrypts the digital signature to obtain the message digest generated by the sender.
  - Uses the same message digest algorithm used by the sender to generate a message digest of the received message.
  - Compares both message digests (the one sent by the sender as a digital signature, and the one generated by the receiver). If they are not *exactly the same*, the message has been tampered with by a third party

# Authentication (cont.)



# PGP

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- Pretty Good Privacy (PGP): encryption program developed by Phil Zimmerman in 1991. (free, email security)
- a combination of hashing, data compression, symmetric-key cryptography, and public-key cryptography
  - The message is encrypted using a symmetric-key encryption algorithm, which requires a symmetric key. (Each symmetric key is used only once and is also called a session key.)
  - The session key is encrypted using public-key encryption
  - The encrypted message along with the encrypted session key is sent to the receiver.
- using the strengths of one algorithm to compensate for the weaknesses of the other, PGP is one of the strongest and fastest encrypting algorithm.

# Summary

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- Symmetric-key and public-key
- Public key systems
  - RSA algorithm
  - Authentication methods
  - Digital signature
  - Hash-based authentication
- PGP