Ray Tracing Basics

COSC342

Lecture 11
4 April 2017

Ray Tracing Algorithm

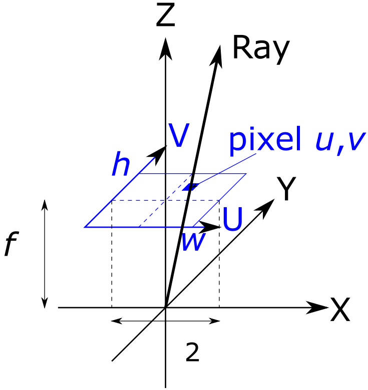
The basic ray tracing algorithm:

- Set up a camera – a projection point and an image plane
- For each pixel in the plane:
  - Cast a ray from the projection point through the pixel
  - **Determine the first object hit by that ray**
  - Cast additional ray(s) to determine lighting
  - Additional rays can be used for reflection, refraction, etc.

The Primary Ray

The ray through pixel (u, v) is

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} -1 + \left(u + \frac{1}{2}\right) \frac{2}{w} \\ \frac{-h}{w} + \left(v + \frac{1}{2}\right) \frac{2}{w} \\ f \\ 0 \end{bmatrix}$$



More generally rays have the form

$$\mathbf{p} = \mathbf{p_0} + \lambda \mathbf{d},$$

where the ray starts at $\mathbf{p}_0$ and goes in direction $\mathbf{d}$

Spheres

Spheres have a simple mathematical form:

- Given a point $\mathbf{p} = (x, y, z)$
- Its squared distance from the origin is

$$\|\mathbf{p}\|^2 = \mathbf{p} \cdot \mathbf{p} = x^2 + y^2 + z^2$$

- So a unit sphere at the origin is defined by

$$\mathbf{p}^2 = x^2 + y^2 + z^2 = 1$$

Ray-Sphere Intersections

We have the equations:

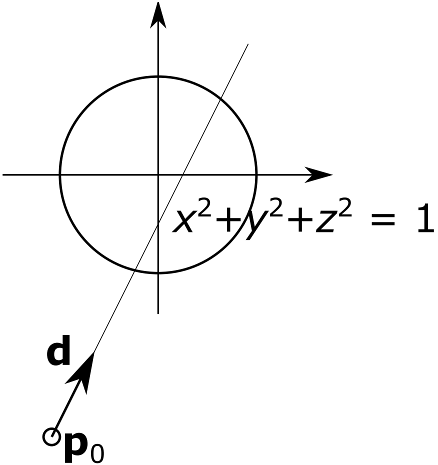
$$\mathbf{p} = \mathbf{p}_0 + \lambda \mathbf{d}$$

and

$$\mathbf{p} \cdot \mathbf{p} = 1$$

So

$$(\mathbf{p}_0 + \lambda \mathbf{d}) \cdot (\mathbf{p}_0 + \lambda \mathbf{d}) = 1$$
$$\mathbf{p}_0 \cdot \mathbf{p}_0 + 2\lambda \mathbf{p}_0 \cdot \mathbf{d} + \lambda^2 \mathbf{d} \cdot \mathbf{d} = 1$$



Ray-Sphere Intersections

This is a quadratic equation

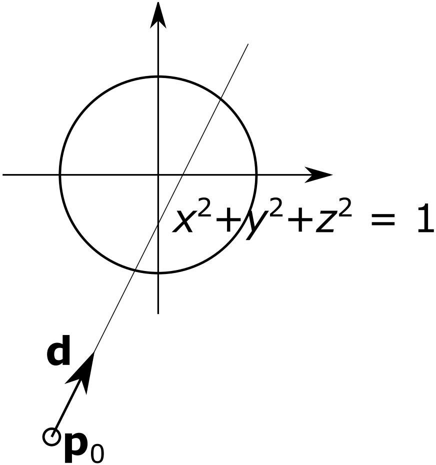
$$a\lambda^2 + b\lambda + c = 0$$

So the solutions are

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

0, 1, or 2 real solutions

- Why not always 2 solutions?
- What do these 3 cases mean?
- What is the first hit?



Ray-Sphere Intersections

What if the sphere is not at the origin?

- We can represent this by a translation
- More generally we can have any transformation, T
- This is represented as a 4×4 homogeneous matrix

Easier to apply the transform to the ray

- Not the same transform though
- If we moved the sphere left, we'd move the ray right
- Apply the inverse transform, T^{-1}, to the ray

Transforming Rays

- Rays have two parts – a starting point and a direction
- In homogeneous form this is

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ 0 \end{bmatrix}$$

- Points (locations) and directions are affected differently
- In particular, translation doesn't affect directions

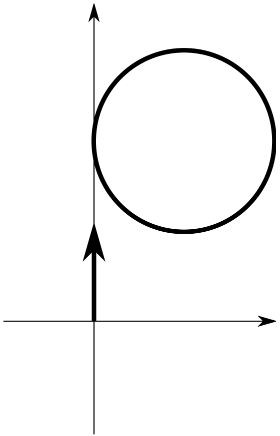
A Simple Example

Where does the primary ray hit with:

- Unit sphere translated by $\begin{bmatrix} 1 & 0 & 2 \end{bmatrix}^T$
- A 5×5 image
- Ray through the pixel (2,2) of the image
- Focal length of 1

We can draw a picture:

- Ray is through the middle of the picture
- This means it goes along the Z axis
- We expect just one intersection
- This is at the 'left' of the sphere



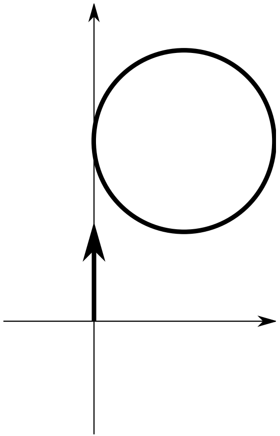
A Simple Example

First form the primary ray:

$$\begin{bmatrix}0\\0\\0\\1\end{bmatrix} + \lambda \begin{bmatrix}-1 + \left(u + \frac{1}{2}\right)\frac{2}{w}\\ \frac{-h}{w} + \left(v + \frac{1}{2}\right)\frac{2}{w}\\ f\\ 0\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\0\\1\end{bmatrix} + \lambda \begin{bmatrix}-1 + \left(2 + \frac{1}{2}\right)\frac{2}{5}\\ \frac{-5}{5} + \left(2 + \frac{1}{2}\right)\frac{2}{5}\\ 1\\ 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^{\mathsf{T}} + \lambda \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}^{\mathsf{T}}$$



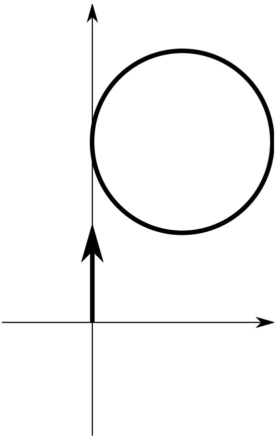
A Simple Example

The transformation of the sphere is

$$T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So the ray is transformed by

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

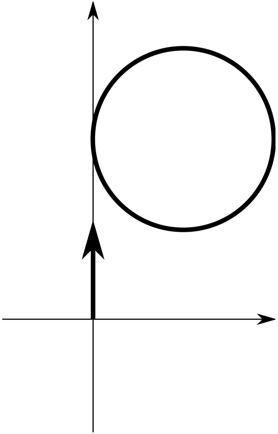


A Simple Example

The transformed ray is then

$$\begin{bmatrix}1 & 0 & 0 & -1\\0 & 1 & 0 & 0\\0 & 0 & 1 & -2\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} + \lambda\begin{bmatrix}1 & 0 & 0 & -1\\0 & 1 & 0 & 0\\0 & 0 & 1 & -2\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$$

$$\begin{bmatrix}-1\\0\\-2\\1\end{bmatrix} + \lambda\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$$



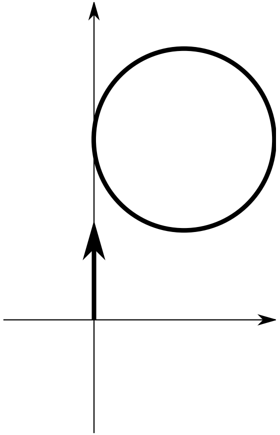
A Simple Example

This transformed ray intersects the unit sphere where

$$
\begin{aligned}
(-1)^2 + 0^2 + (-2+\lambda)^2 &= 1 \\
(\lambda - 2)^2 &= 0 \\
\lambda &= 2
\end{aligned}
$$

So intersection with the unit sphere is at

$$
\begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$



Ray-Cube Intersections

A unit 'radius' cube

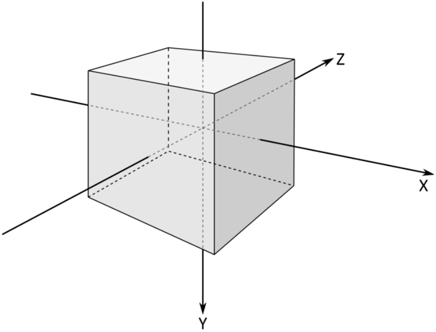
- From -1 to +1 on each axis
- Intersect ray with each face
- We will consider $X = +1$

The X co-ordinate of a ray is

$$x_0 + \lambda \Delta x$$

So at the plane $X = 1$ we have

$$1 = x_0 + \lambda \Delta x$$
$$\lambda = \frac{1 - x_0}{\Delta x}$$



Ray-Cube Intersections

Check whether we are in the face:

- $y_0 + \lambda\Delta y \in [-1, 1]$
- $z_0 + \lambda\Delta z \in [-1, 1]$

Problems if $\Delta x = 0$:

- What does this indicate?
- Exclude very small Δxs

Then repeat for other 5 faces