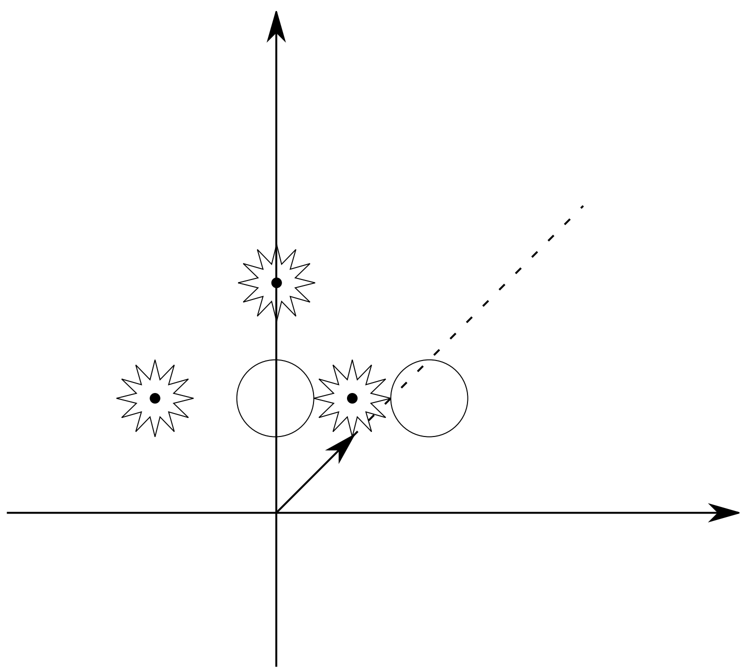
COSC342: Lab 07

Ray Tracing and Shadows

1 Introduction

In this tutorial we will be considering a simple scene with two unit spheres and three point light sources. The spheres are centred at (0, 0, 3) and (4, 0, 3). The light sources are located at (2, 0 3), (-3, 0, 3) and (0, 0, 6), as shown below. Note that the Y-axis is not drawn.



We will be considering the initial ray, $(0, 0, 0) + \lambda(1, 0, 1)$. From the diagram, it is clear that the ray misses the first sphere, and intersects the second sphere twice. However, this is not obvious just from the numbers that the computer can see.

2 Object Intersections

We will begin with the intersection between our ray and the first sphere.

1. What transformation would applied to a unit sphere centred at the origin in order to make the first sphere in the scene?
2. What matrix represents this transformation in 3D homogeneous co-ordinates?
3. What transformation is applied to the *ray* to turn the problem of intersecting the ray with the first sphere into an intersection of a ray and a unit sphere at the origin?
4. What equation represents the transformed ray?
5. What equation represents the unit sphere at the origin?
6. What is the quadratic equation that arises from solving these two equations simultaneously?
7. How do we know there are no (real) solutions to this equation? *This is less than zero, so no real solutions*

Next, we consider the second sphere:

1. What transformation would applied to a unit sphere centred at the origin in order to make the first sphere in the scene?
2. What matrix represents this transformation in 3D homogeneous co-ordinates?
3. What transformation is applied to the *ray* to turn the problem of intersecting the ray with the first sphere into an intersection of a ray and a unit sphere at the origin?
4. What equation represents the transformed ray?
5. What equation represents the unit sphere at the origin?
6. What is the quadratic equation that arises from solving these two equations simultaneously?
7. What are the two solutions to this equation?
8. How do we know which solution represents the *first* intersection of the ray and the sphere?
9. Where is this intersection?

3 Shadow Rays

In order to determine what the sphere looks like at the hit point, we need to know which lights illuminate the sphere at that point. To do this we cast *shadow rays* from the hit point to each light source in the scene. If these rays hit any object before they get to the light, the hit point is in shadow with respect to that light source.

Recall that given two points, $\mathbf{p}_1$ and $\mathbf{p}_2$, the ray from $\mathbf{p}_1$ to $\mathbf{p}_2$ is given by:

$$\mathbf{p}_1 + \lambda(\mathbf{p}_2 - \mathbf{p}_1).$$

1. What is the value of λ that gives $\mathbf{p}_1$?
2. What is the value of λ that gives $\mathbf{p}_2$?
3. What does a value of λ in the range $(0, 1)$ represent?
4. What does a value of λ greater than 1 represent?
5. What does a value of λ less than 0 represent?

Our initial ray first hits an object (the second sphere) at (3, 0, 3), and we have light sources at (2, 0, 3), (-3, 0, 3), and (0, 0, 6).

1. What are the rays from our hit point to each of the three light sources?
2. What transform do we apply to these rays to intersect them with the first sphere (centred at (0, 0, 3))?
3. What are the transformed rays?
4. What are the λ value(s) for the intersection of these rays with the first sphere?
5. What do these values tell us about the light at our hit point?