PROJECTILE MOTION

6TH FORM PHYSICS

STUDENT NOTES

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CONTENTS

SECTION: 5

STUDENT NOTES

LESSON NUMBER: Covers all lessons.

PROJECTILE MOTION

Here is a strobe photograph of two golf balls, one projected horizontally at the same time that the other was dropped. The strings are 15 cm apart and the time between flashes was $\frac{1}{30}$ second.

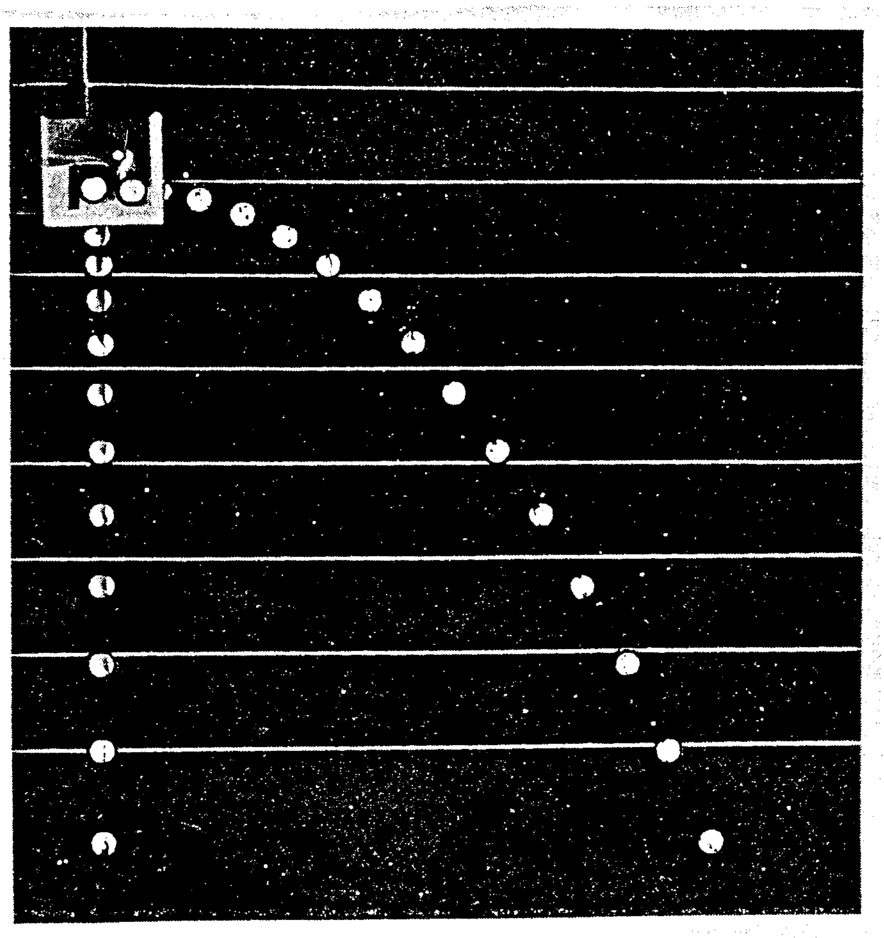
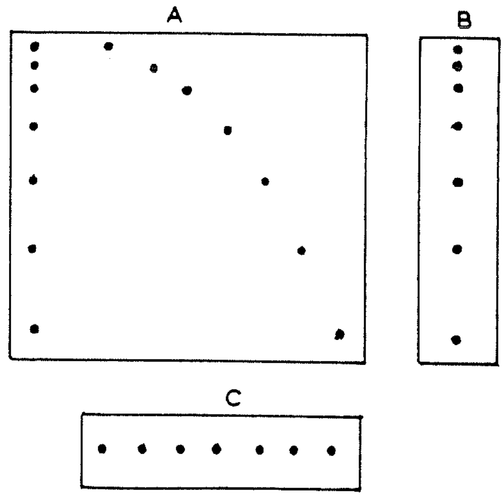


Diagram A is a copy of a multiflash photograph of the motion of two balls; one dropped vertically from rest, and the second projected horizontally. The trajectory of the second ball is seen to be curved. If the motion of the projected ball is photographed end-on, diagram B results.



It shows the vertical component of the projected ball's motion. Diagram C shows the horizontal component of the motion. It comes from a photograph taken from above.

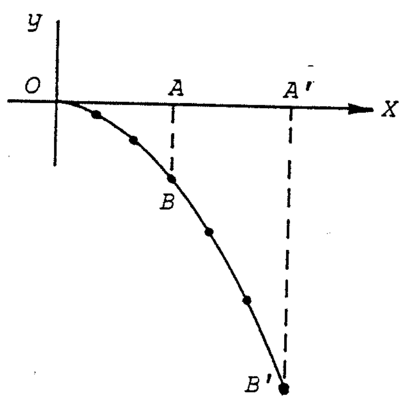
Note how the horizontal component appears to be uniform motion; and the vertical component is the same as the motion of the ball dropped from rest, that is, uniform acceleration. The motion of this projectile is a combination of constant horizontal velocity and constant vertical acceleration.

Examination of the trajectory from Diagram A shows it to be parabolic: the y-coordinate of any point on the trajectory is proportional to the square of its x=coordinate.

For example

$$OA' = 2 \times OA$$

$$A'B' = 4 \times AB$$

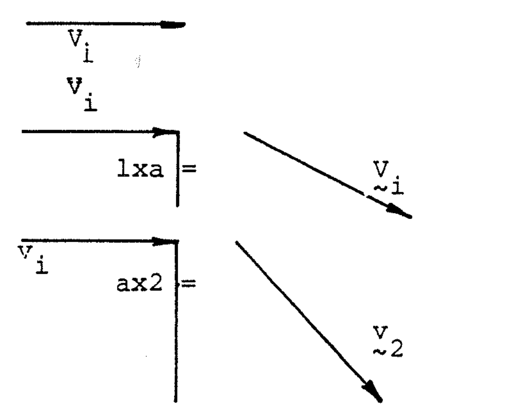
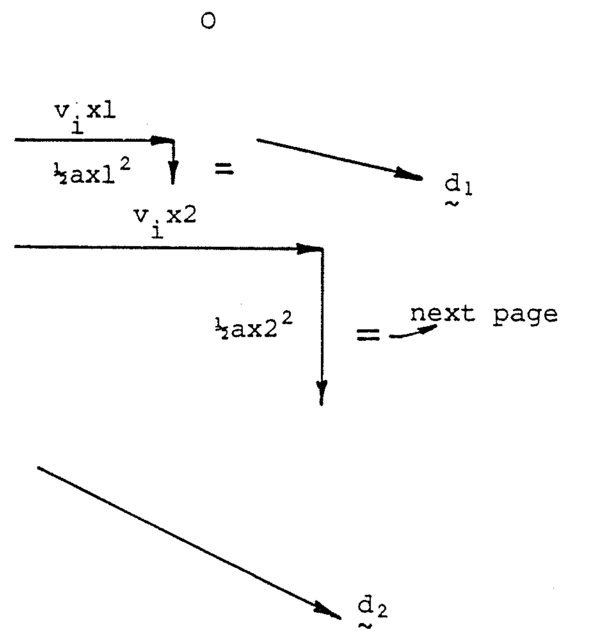


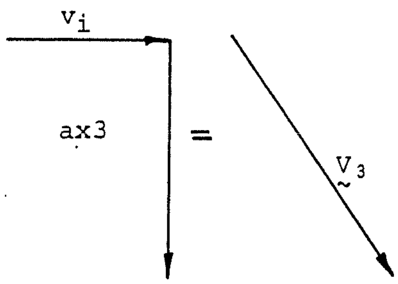
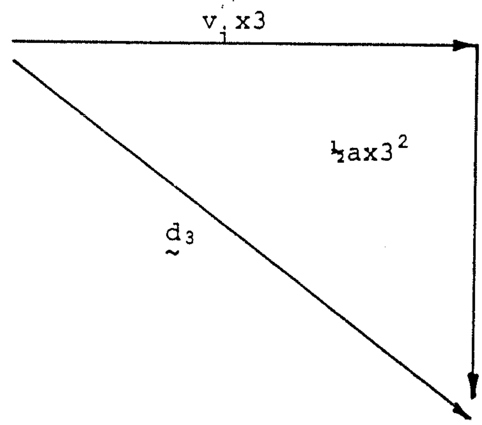
We can use this constant-horizontal-velocity/constant-vertical-acceleration description of this projectile motion to reconstruct its trajectory and to find the velocity at any moment:

For example. Project a ball horizontally from the origin (initial displacement is $\begin{pmatrix}0\\0\end{pmatrix}$ m) with initial velocity $\underset{\sim}{v}_i = \begin{pmatrix}20\\0\end{pmatrix}$ ms^{-1} and acceleration $\underset{\sim}{a} - \begin{pmatrix}0\\-10\end{pmatrix}$ ms^{-2}.

We know how to calculate (a) displacement for uniform motion: $d = vt$, and (b) velocity and displacement for motion with uniform acceleration from rest: $V = at$ and $d = \frac{1}{2}at^2$. Equation (a) yields the horizontal component of the ball's displacement, and equations (b) give the vertical components of its velocity and displacement.

Scales: $accel^n$ 20 $ms^{-2} \equiv 1$ cm; velocity 20 $ms^{-1} \equiv 1$ cm; displacement 20 m $\equiv$ 1 cm

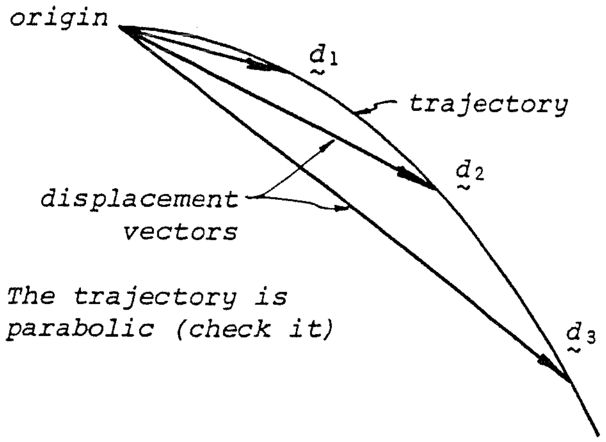
Time	$Accel^n$	Velocity	Displacement
0s	2 ↓		
1s	a ↓		
2s	a ↓		

3s	a 		

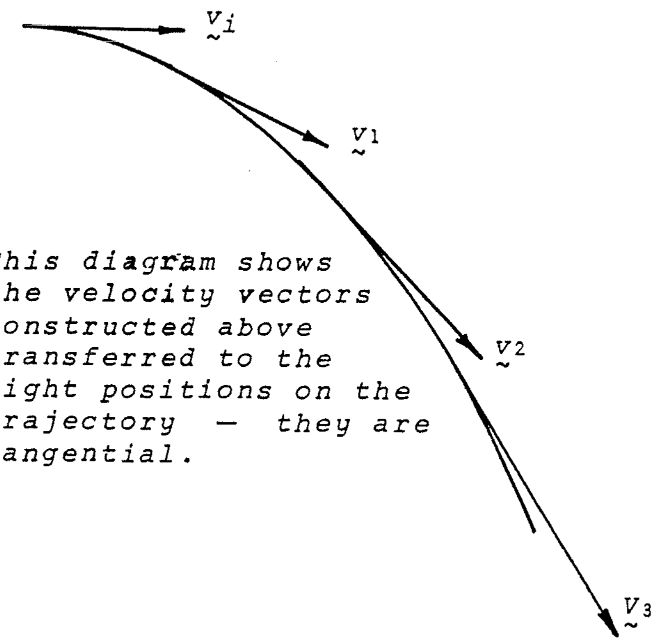
We can write instructions for finding $\underset{\sim}{v}$ and $\underset{\sim}{d}$ at time t by using vector notation:

t	$\underset{\sim}{a}$	$\underset{\sim}{v} = \underset{\sim}{v}_i + \underset{\sim}{a}t$ That is adding t times the acceln vector to the initial velocity vector gives the final velocity.	$d = v_i t + \frac{1}{2}\underset{\sim}{a}t^2$ That is multiplying the acceln vector by $\frac{1}{2}t^2$ and adding it to t times the initial velocity vectors gives the displacement vector
	$\begin{pmatrix} 0 \\ -10 \end{pmatrix}$ ms^{-2}	$\underset{\sim}{v} = \begin{pmatrix} 20 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -10 \end{pmatrix} t$	$\underset{\sim}{d} = \begin{pmatrix} 20 \\ 0 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -10 \end{pmatrix} t^2$

Now we bring all the displacement vectors to a common origin. The trajectory is the curve joining the ends of the vectors. (Imagine the displacement vector stretching and turning as the ball moves along its path.)



The trajectory is parabolic (check it)

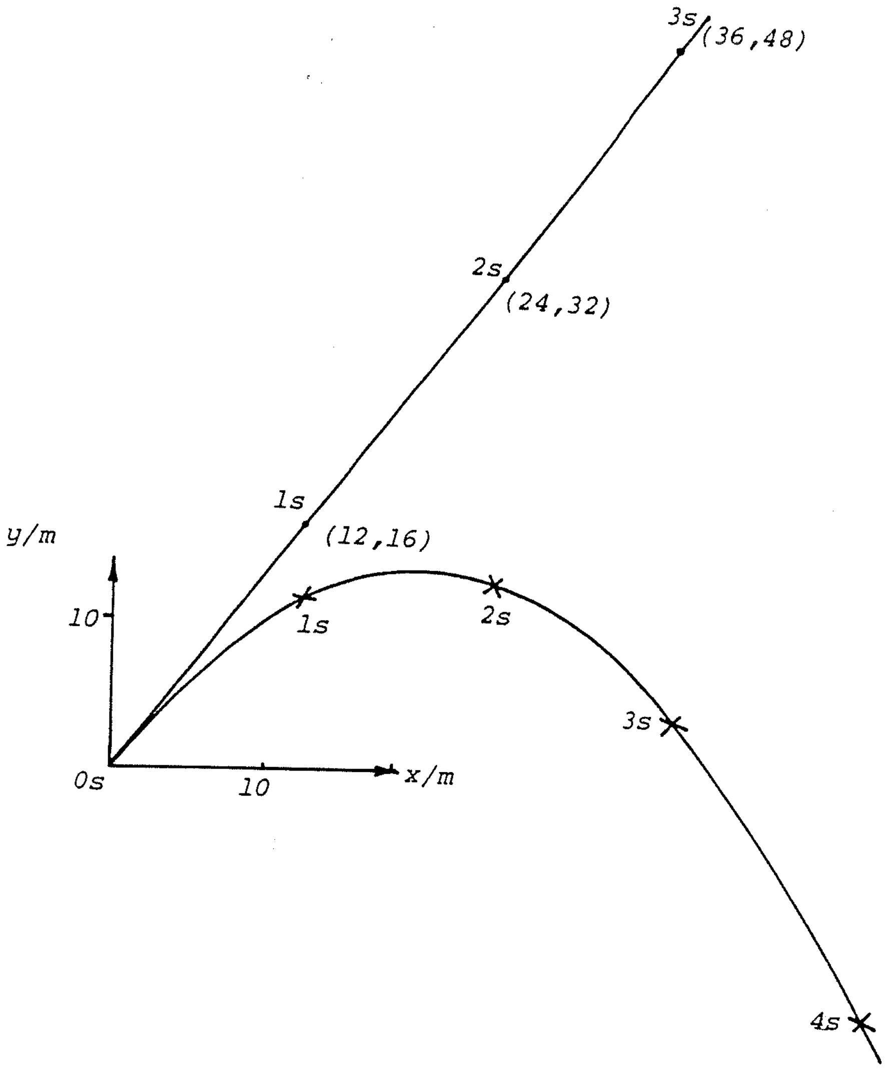


This diagram shows the velocity vectors constructed above transferred to the right positions on the trajectory — they are tangential.

Notice that the equations in the table above are just like the equations of motion for motion in a straight line with constant acceleration discussed in earlier lessons except whereas then d, v, and a were representing integers, now they are vectors. We don't need to learn new equations to describe projectile motion — we just modify the old familiar equations and use our skills with vectors. To illustrate this point, we will predict the trajectory of a ball fired with an initial velocity in a direction other than horizontal.

For example, a ball is fired with an initial velocity of $\begin{pmatrix}12\\16\end{pmatrix}$ ms^{-1}, or 20 ms^{-1} at an angle of 53° to the horizontal.

This straight line shows the path of the ball had it been fired in a region with no gravity. It is the steady downward acceln of gravity which deflects projectiles from uniform motion into 'parabolic' motion.



The displacement from the origin after t seconds is given by

$$\underset{\sim}{d} = \begin{pmatrix}12\\16\end{pmatrix} t + \frac{1}{2}\begin{pmatrix}0\\-10\end{pmatrix} t^2$$

At $t = 0 : \underset{\sim}{d} = \begin{pmatrix}12\\16\end{pmatrix}0 + \frac{1}{2}\begin{pmatrix}0\\-10\end{pmatrix}0 = \begin{pmatrix}0\\0\end{pmatrix}$

$t = 1 : \underset{\sim}{d} = \begin{pmatrix}12\\16\end{pmatrix}1 + \frac{1}{2}\begin{pmatrix}0\\-10\end{pmatrix}1^2 = \begin{pmatrix}12\\11\end{pmatrix}$

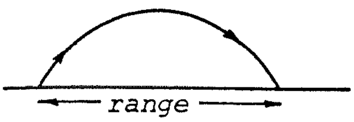
$t = 2 : \underset{\sim}{d} = \begin{pmatrix}12\\16\end{pmatrix}2 + \frac{1}{2}\begin{pmatrix}0\\-10\end{pmatrix}2^2 = \begin{pmatrix}24\\32\end{pmatrix} + \begin{pmatrix}0\\-20\end{pmatrix}$

$= \begin{pmatrix}24\\12\end{pmatrix}$

$$t = 3 : \underset{\sim}{d} = \begin{pmatrix} 12 \\ 16 \end{pmatrix} 3 + \frac{1}{2} \begin{pmatrix} 0 \\ -10 \end{pmatrix} 3^2$$

$$= \begin{pmatrix} 36 \\ 48 \end{pmatrix} + \begin{pmatrix} 0 \\ -45 \end{pmatrix} = \begin{pmatrix} 36 \\ 3 \end{pmatrix} \quad \text{etc.}$$

The trajectory calculated from the vector equation is parabolic — as it should be.



<u>NOTE</u>: 1. Some special terms used when discussing projectile motion are:

range: maximum horizontal displacement on the flat earth's surface.

time of flight: means what it says.

(Remember that much of the early understanding of this topic came from artillerymen figuring out how best to throw rocks and cannonballs around.)

2. The motion's symmetry => time to reach highest position

$= \frac{1}{2} \times$ time of flight

3. The vertical component of the velocity = 0 at the highest position. This fact can be used to find the time needed to reach the greatest height, which then gives us the time of flight (by doubling), which then gives us the range (by multiplying by the horizontal velocity component).

For example in the motion of the previous example, the velocity equation is

$$\underset{\sim}{v} = \underset{\sim}{v}_i + \underset{\sim}{a}t$$

$$\Rightarrow \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 12 \\ 16 \end{pmatrix} + \begin{pmatrix} 0 \\ -10 \end{pmatrix} t$$

$$\Rightarrow v_y = 16 - 10t$$

$$= 0 \text{ when } 16 = 10t$$

That is, t = 1.6 s is the time to reach the greatest height.

=> Time of flight = 3.2 s

=> Range = horizontal velocity component × time of flight

= $12\ ms^{-1} \times 3.2\ s$

= 38.4 m (check this on the trajectory diagram)

4. The greatest height reached can be found by taking the y-component of the displacement equation and putting in the greatest-height time.

For example

$$\tilde{d} = \tilde{v}_i t + \tfrac{1}{2}\tilde{a}t^2$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 16 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -10 \end{pmatrix} t^2$$

$$\Rightarrow y = 16t - 5t^2$$

When $t = 1.6$ s, $y = 16 \times 1.6 - 5 \times 1.6^2$

= 12.8 m - this is the greatest height.

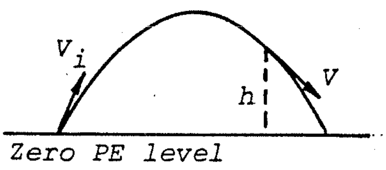
(Check it on the trajectory diagram.)

5. Throughout this discussion, we have ignored the effect of air resistance. Our constant-velocity/constant-vertical-acceln model of projectile motion is valid only for situations where air friction is negligible. The model will predict the trajectory of a steel ball, but not a polystyrene ball or a shuttlecock.

If air friction is negligible, then the net force on the ball is its downward weight force. This is consistent with the vertical acceln being constant, and the horizontal acceln being zero.

6. In the absence of air resistance, the total mechanical energy of the projectile is constant.

That is KE + grav PE = constant = initial KE (taking the PE zero-level to be the level from which the ball was fired).



The energy equation becomes:

$$\tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}mv_i^2$$

$$\Rightarrow \quad v^2 + 2gh = v_i^2 \quad \text{(dividing by } \tfrac{1}{2}m\text{)}$$

$$\therefore \quad \Rightarrow v^2 = v_i^2 - 2gh$$

Once, we recognise -g as being the downward acceln of gravity, we realise this equation is identical to the equation of motion $v_f^2 = v_i^2 + 2ad$.

SECTION: 5

STUDENT NOTES

LESSON NUMBER: 1.2 Activity A: Worksheet 1

Notes:

Plotting the trajectory of a projectile motion Part I

A. Problems

Key Facts

1. Vertical component of projectile motion is uniformly accelerated motion.
2. Horizontal component of projectile motion is uniform motion.
3. Since projectile motion is two-dimensional, vectors must be used to represent position, velocity, and acceleration.

Situation

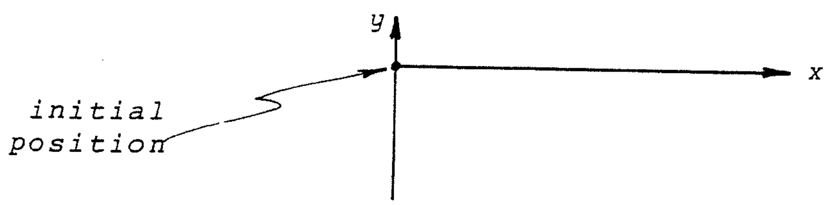
A ball is thrown out horizontally from the origin high up on a clifftop with an initial velocity of 20 ms^{-1}. Its acceleration throughout the motion is 10 ms^{-2} downwards.

Task

1. Draw a scale diagram of the ball's trajectory.
2. Plot on the diagram arrows representing the velocity vectors.

Procedure

1. x and y axes are used to monitor position.



x-coordinate of initial position is .... m

y-coordinate of initial position is .... m

Write a column vector for the initial position:

$$\underset{\sim}{d} = \begin{pmatrix} \dots \\ \dots \end{pmatrix} \text{ m initially}$$

2. x-component of initial velocity is ... ms^{-1}

y-component of initial velocity is ... ms^{-1}.

Write a column vector for the initial velocity:

$$\underset{\sim}{v}_i = \begin{pmatrix} \dots \\ \dots \end{pmatrix} \text{ ms}^{-1}$$

3. x-component of acceleration is ... ms^{-2}

y-component of acceleration is ... ms^{-2}

(Remember: Up is positive)

Write a column vector for the acceleration:

$$\underset{\sim}{a} = \begin{pmatrix} \dots \\ \dots \end{pmatrix} \text{ ms}^{-2}$$

4. Complete this table for x, the horizontal displacement.

time t (s)	0	1	2	3	4	5	t
horizontal displacement x (m)	0	20					

Write an equation for x in terms of t

x = ...

5. Complete this table for y, the vertical displacement.

time t (s)	0	1	2	3	4	5	t
vertical displacement y (m)	0	-5					

(Remember: Up is positive)

If you need help, look at OHP 2

Write an equation for y in terms of t

$$y = \dots$$

6. Complete this table for $\underset{\sim}{d}$, the position vector of the ball. (Remember: $\underset{\sim}{d} = \begin{pmatrix} x \\ y \end{pmatrix}$).

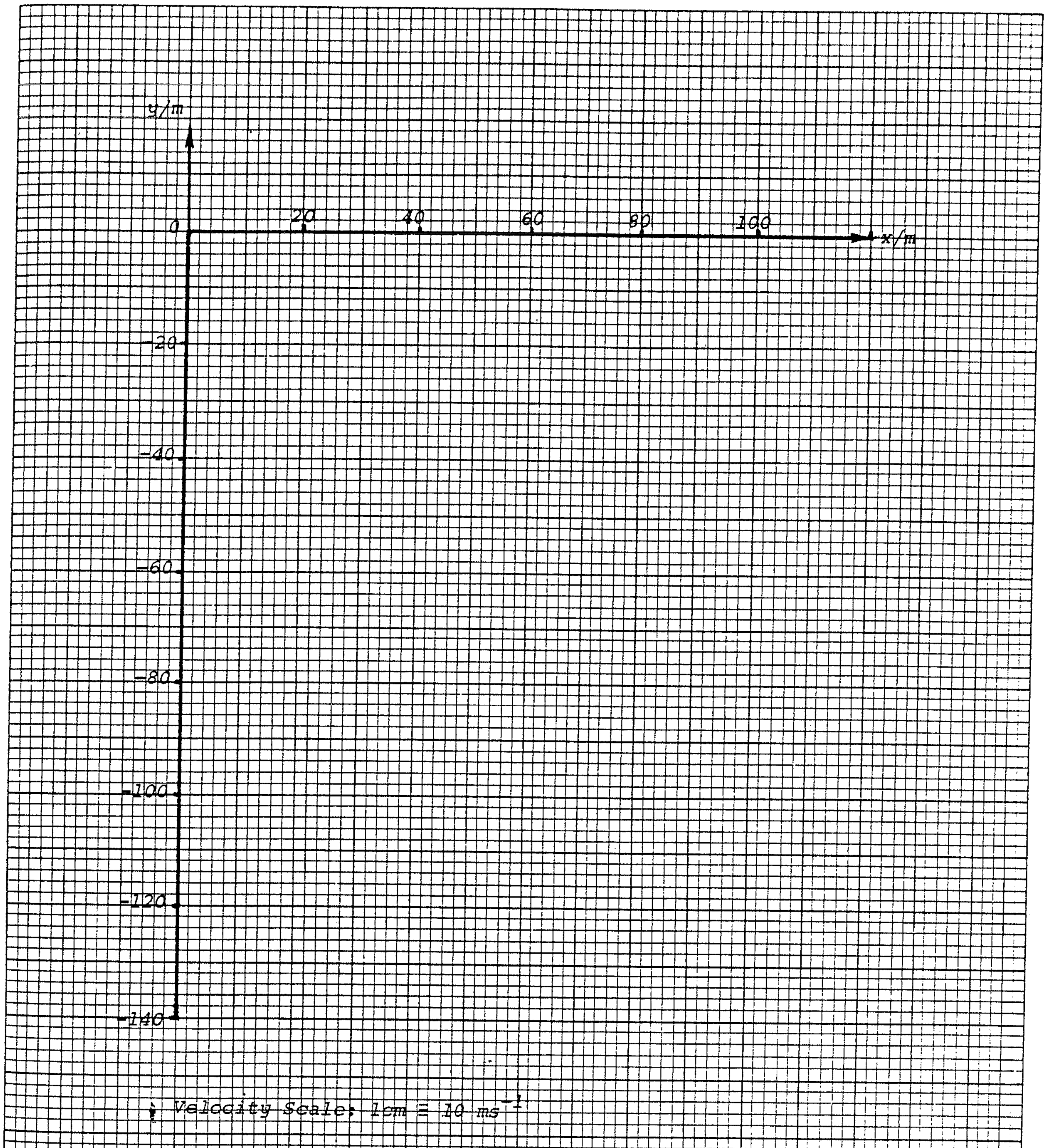
time t (s)	0	1	2	3	4	5
position vector	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 20 \\ -5 \end{pmatrix}$	$\begin{pmatrix} \dots \\ \dots \end{pmatrix}$	$\begin{pmatrix} \dots \\ \dots \end{pmatrix}$	$\begin{pmatrix} \dots \\ \dots \end{pmatrix}$	$\begin{pmatrix} \dots \\ \dots \end{pmatrix}$

7. On the next page of your worksheet, plot each vector listed in step 6 to the scale 1 cm ≡ 10 m. Start each position vector from the origin. Connect the tips with a smooth curve. This is the trajectory.

<u>Check</u>: The point (50, -31) shoud be very close to your curve.

If it is close to your curve, then you are correct.

If not, find and correct your errors.



Position and Velocity of a Thrown Ball

8. x-component of velocity, v_x = 20 ms^{-1} at any time. Complete this table v_y, the y-component of the velocity

time t (s)	0	1	2	3	4	5	t
vertical speed v_y (ms^{-1})	0	-10					

(Remember: up is positive)

Write an equation for v_y in terms of t:

$$v_y = \ldots$$

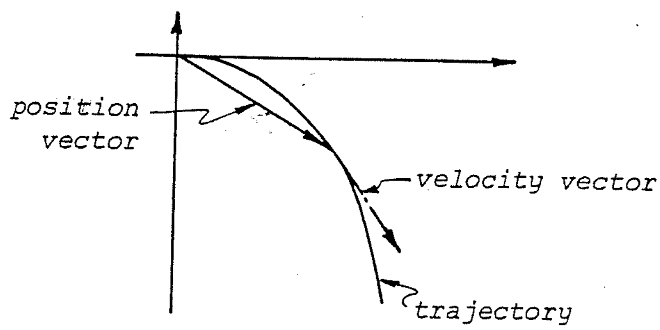
9. Complete this table for $\underset{\sim}{v}$, the velocity of the ball

Remember, $\underset{\sim}{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

	0	1	2	3	4	5
velocity $\underset{\sim}{v}$ (ms^{-1})	$\begin{pmatrix} 20 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 20 \\ -10 \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$

10. On your plot of trajectory from Step 7, plot each of the velocity vectors in Step 9 to a suitable scale (for example, 1 cm ≡ 10 ms^{-1})

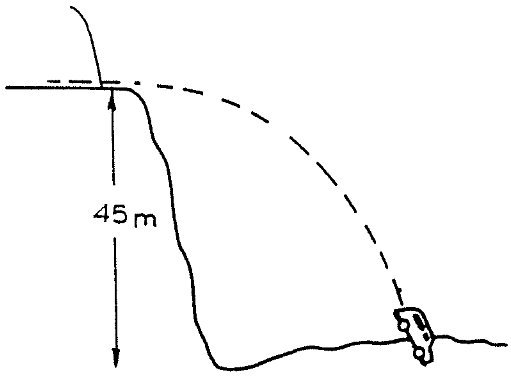
Start each vector from the corresponding position on the trajectory. For example



Check: Each velocity vector should be tangential to the trajectory curve since the vector shows the direction of motion at that instant. If it is, then you are correct. If not, find and correct your errors.

B. Problems

1. A car travelling at 30 ms^{-1} (over 100 km/h!) along a horizontal mountain road failed to take a corner and crashed into a snowdrift 45 m (vertically) below.



(a) How long did the car take to fall?

(b) How far (horizontally) did it land from the place it left the road?

(c) What was its acceleration halfway down?

(d) Calculate the angle of the tunnel that it made in the snowdrift on landing.

(Hint: Find the direction of the velocity vector at the moment of landing.)

2. A man holds a rifle 1.8 m above level ground and aims it horizontally.

(a) How long is it from the moment of firing until the bullet hits the ground?

(b) If the cartridge is ejected horizontally to the side, just as the bullet leaves the barrel, when will the cartridge hit the ground?

(c) Would the man be able to shoot further (with this aim) on the Moon? Give a clear reason for your answer.

SECTION: 5

STUDENT NOTES

LESSON NUMBER: 1.2 Activity B

Practical Guide

TRAJECTORY PLOT

Aim

To obtain and investigate the path of a projectile.

Objectives

At the completion of this experiment you should be able to

(a) Present the path of a horizontally fired projectile as a graph.

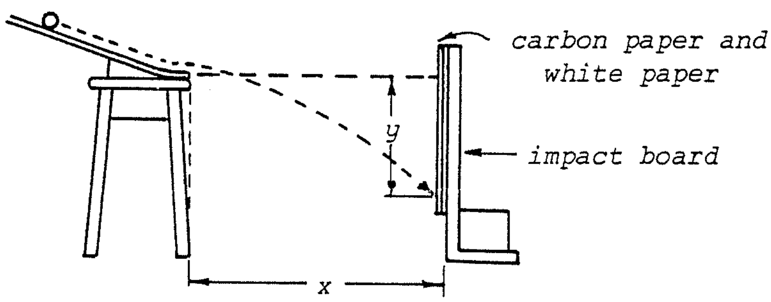
(b) Show that the vertical component of the motion of the projectile is constant acceleration.

Apparatus

Curved ramp (for example, PSSC 2-D collisions kit), lab. stool, board with firm support, steel ball, sheet of thin white paper, carbon paper, metre rule.

Procedure

Experiment A: Set up the apparatus as shown in the diagram:



Ensure that the ramp is set so that the ball leaves it horizontally. The impact board should be firmly held to avoid movement when the ball strikes it.

Tape a sheet of carbon paper to the board with the carbon out and the top of the sheet just above the level of the end of the ramp. Place a sheet of white paper over the carbon paper and tape it at the top only.

Experiment B: Place the impact board at a suitable distance from the end of the ramp (about 25-30 cm) so that when the ball is released from a point on the ramp, it strikes near the bottom of the recording sheet.

Do not hold the ball in your fingers to release it. Instead, hold it back at the release point with a ruler etc. and let it go by moving the ruler quickly away from it down the ramp.

When the ball strikes the recording sheet, the point of impact is marked by the carbon paper, and can be seen through the white paper when the sheet is lifted out from the board. Beside the mark note the horizontal distance x from the end of the ramp to the board.

Release the ball several times (always from the same point) for this value of x.

Q.1 Why should this be repeated?

Experiment C: Move the ramp in equal steps of about 3 cm towards the board and at each stage record the points of impact when the ball is released from the same point on the ramp. Note beside each group of marks the horizontal distance x.

The last position should be with the ramp and impact board virtually touching so that the height of the launching point is recorded.

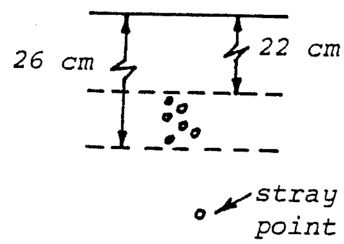
Q.2 At what stage of the balls path is the spread in the impact points.

(a) least?

(b) greatest?

Explain why this is likely to be so.

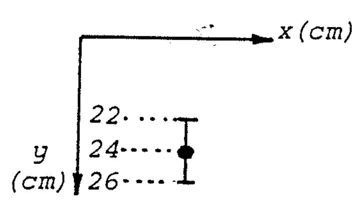
<u>Experiment D</u>: If sufficient trials have been carried out at each stage, any decision to eliminate stray points can easily be made.



Transfer your data from the recording sheet to a table thus:

x (cm)	y (average) (cm)	limits of y (cm)
30	24	22 - 26
.	.	.
.	.	.
.	.	.

<u>Experiment E</u>: From your results plot a graph, labelling the axes as shown:



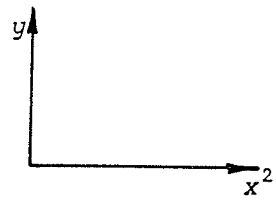
Use the limits of y for each group of points to draw error bars. These indicate the uncertainty in y.

On your graph, draw a smooth curve which bests fits the points.

Q.3 Describe the shape of your graph. This shows the trajectory of the ball.

Experiment F: To test your answer, add a fourth column to the table, for x^2.

Plot a second graph with the axes labelled as shown, and include error bars for y.



Q.4 Allowing for the uncertainty in y, what relationship between y and x is indicated by this graph?

Q.5 At any point on its trajectory, what is the direction of the force acting on the ball? (Ignore air resistance.)

Q.6 Copy and complete: The horizontal velocity of the ball will be ...

Thus the equal horizontal spacing (3 cm) between each group of marks indicates equal time intervals, and x is directly related to the time of flight (measured from when the ball leaves the ramp).

Q.7 For an object released from rest and allowed to fall under gravity with constant acceleration

$$y = \tfrac{1}{2}at^2$$

Explain what each symbol represents in this equation.

Q.8 Describe the shape of the graph produced when (a) y is graphed against t, (b) y is graphed against t^2.

Q.9 Since x corresponds to t, how would you describe the vertical component of the ball's motion?

Q.10 Describe the trajectories that you would expect to obtain if

(a) The ball was released from a higher point on the ramp;

(b) The ramp launches the ball at an angle to the horizontal.

SECTION: 5

STUDENT NOTES

LESSON NUMBER: 2.2 Activity D Worksheet 2

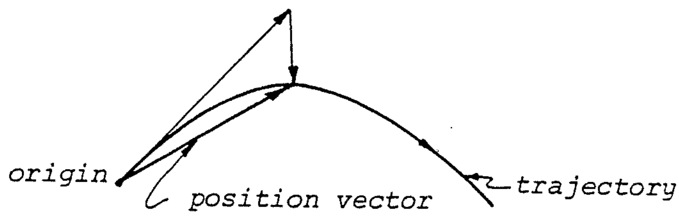
Plotting the trajectory of a projectile motion: Part II

Key Facts

Projectile motion = uniform motion + vertical free free fall from rest

=> at any moment.

position vector = displacementfrom origin due to uniform motion alone + displacement due to vertical free fall from rest alone



Situation

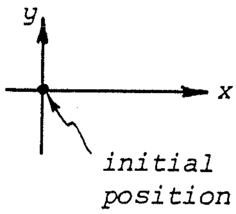
A ball is thrown from the origin at an angle of 53° up from the horizontal with a speed of 25 ms^{-1}. Its acceleration throughout the motion is 10 ms^{-2} downward.

Task

1. Draw a scale diagram of the ball's trajectory
2. Plot arrows representing velocity vectors

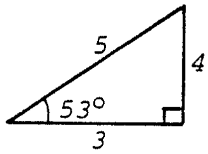
Procedure

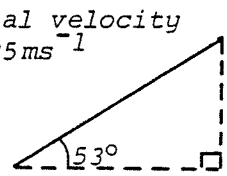
1. Write a column vector for the initial position.



$$\underset{\sim}{d} = \begin{pmatrix} \cdots \\ \cdots \end{pmatrix} \text{ m, initially}$$

2. One angle of a 3-4-5 right-angled triangle is about 53°





x-component of initial velocity is ... ms^{-1}

y-component of initial velocity is ... ms^{-1}

Write a column vector for the initial velocity
(Remember: up is positive)

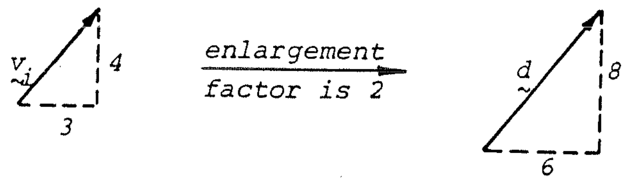
$$\underset{\sim}{v}_i = \begin{pmatrix} \cdots \\ \cdots \end{pmatrix} \text{ ms}^{-1}$$

3. Displacement $\underset{\sim}{d}$ from origin due to uniform motion above is uniform velocity $\underset{\sim}{V}_i$ × time t

$$\underset{\sim}{d} = \underset{\sim}{v}_i t$$

For example suppose $\underset{\sim}{v}_i = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ ms}^{-1}$, then displacement

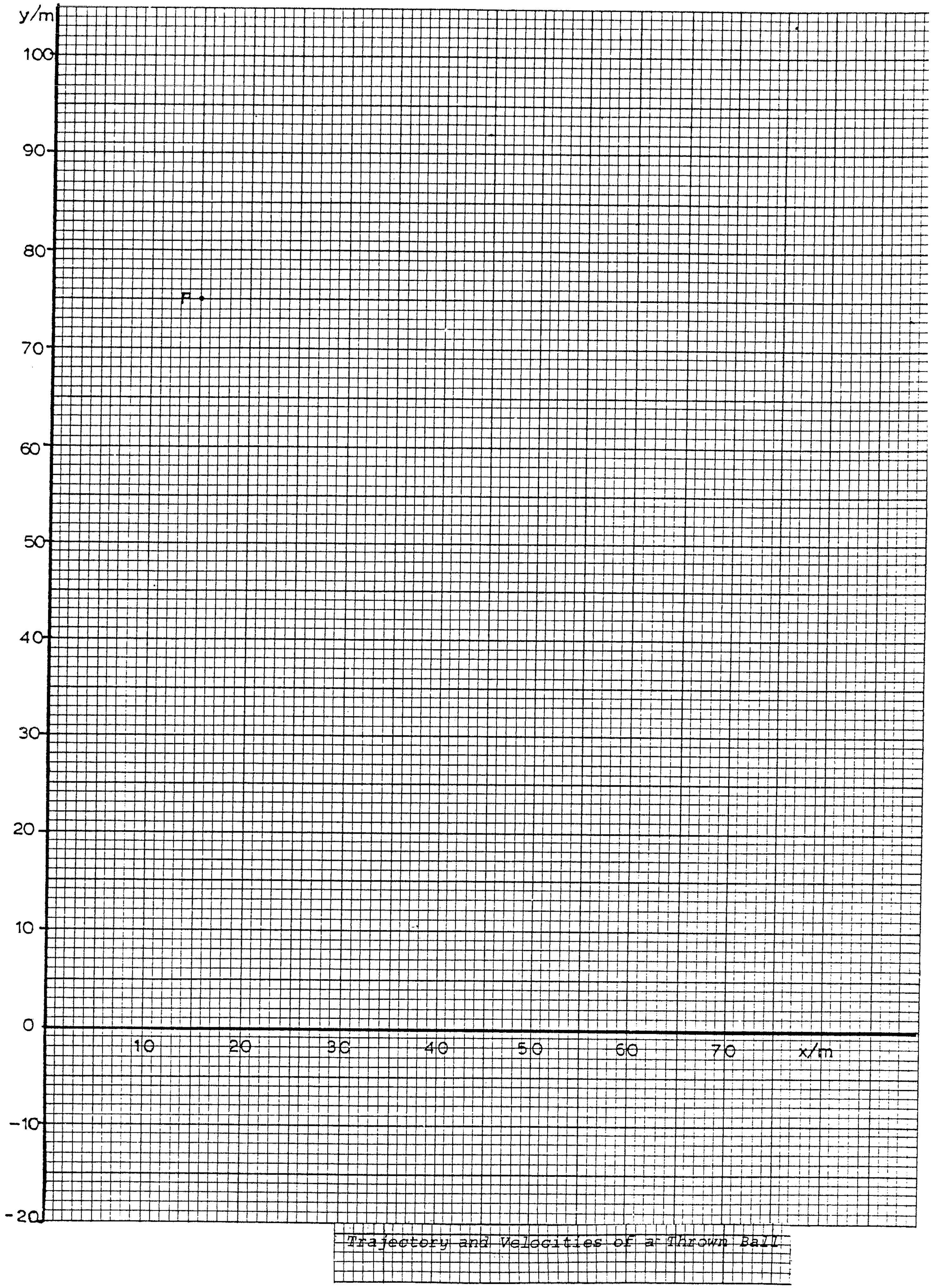
2 seconds later is $\underset{\sim}{d} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ ms}^{-1} \times 2 \text{ s} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \text{ m}$



Complete this table using your column vector for the initial velocity from Step 2

time t (s)	0	1	2	3	4	5
displacement from origin due to uniform motion alone d(m)	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}\dots\\\dots\end{pmatrix}$	$\begin{pmatrix}\dots\\\dots\end{pmatrix}$	$\begin{pmatrix}\dots\\\dots\end{pmatrix}$	$\begin{pmatrix}\dots\\\dots\end{pmatrix}$	$\begin{pmatrix}\dots\\\dots\end{pmatrix}$

4. On the prepared graph page of this worksheet plot the vectors listed in Table 3 using the scale 1 cm ≡ 5 m


y/m
100
90
80
F
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
x/m
-10
-20
Trajectory and Velocities of a Thrown Ball

5. Displacement $\underset{\sim}{D}$ due to vertical free fall alone from rest:

x-component is always zero.

y-component is $\frac{1}{2}at^2$ after t second where a is the acceleration, $-10\ ms^{-2}$, in this case.

Complete this table

time t (s)	0	1	2	3	4	5
displacement due to vertical free fall D(m) from rest	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ \ldots \end{pmatrix}$	$\begin{pmatrix} 0 \\ \ldots \end{pmatrix}$	$\begin{pmatrix} 0 \\ \ldots \end{pmatrix}$	$\begin{pmatrix} 0 \\ \ldots \end{pmatrix}$	$\begin{pmatrix} 0 \\ \ldots \end{pmatrix}$

6. Now find the position of the ball at each moment by adding the displacement due to vertical free fall above from rest to the displacement from the origin due to uniform motion alone, that is, $\underset{\sim}{d} + \underset{\sim}{D}$.

Do this on the graph page by plotting the vectors listed in Steps 5 on the end of the corresponding vector drawn in Step 4. Use the same scale 1 cm ≡ 5 m.

Draw a smooth curve through the endpoints of the combined vectors. This is the trajectory of the thrown ball.

Check: Your trajectory should look parabolic and be close to the point (40,18).

If it is, then you are correct.

If not, find and correct your error.

7. Complete this table by reading components of your trajectory plot.

time (s)	0	1	2	3	4	5
position (m)	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$	$\begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$

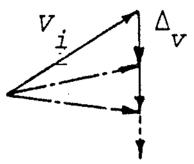
Check: Each column vector listed above should equal the sum of the corresponding column vectors from Step 3 and Step 5.

8. Now to plot velocity vectors.

Plot the initial velocity vector starting from the point P near the top of your graph paper. Use a scale 1 cm ≡ 5 ms^{-1}. An acceleration of 10 ms^{-2} downwards means the velocity changes by 10 ms^{-1} downwards each second.

That is, $\underset{\sim}{\Delta v} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$ ms^{-1} in each second.

Draw a $\underset{\sim}{\Delta v}$ arrow to the same scale from the end of the initial velocity vector. The resultant vector gives the velocity 1 second after the throw.



Repeat this for the next second etc. up to 4 seconds. Redraw these vectors to the same scale but each velocity vector now starts from the corresponding position on the trajectory.

<u>Check</u>: Your velocity vector should be tangential to the curve. If they are, then you are correct. If not, find and correct your error.

9. Complete this table for the velocity.

time t(s)	0	1	2	3	4
velocity $\underset{\sim}{v}$ (ms^{-1})	$\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$ $\underset{\sim}{v_i}$	$\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$	$\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$	$\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$	$\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$

(a) What do you notice about the horizontal speed, that is, the x-component of the velocity?

....

(b) When is the vertical speed zero?

At time ....

(c) Look at Step 7. When is the height (that is, the y-component of the position) greatest?

The height is greatest at time ....

(d) A projectile reaches its greatest height when its vertical speed is ....

(e) When does the ball return to the same horizontal level from which it was thrown?

At time ....

(f) Time to return to same level = ... × time to reach greatest height.

SECTION: 5

STUDENT NOTES

LESSON NUMBER: 2.2 Activity E

Practical Guide

RANGE PREDICTOR

Aim

To apply the equations of projectile motion to predict the range of a projectile and to test this prediction by experiment.

Objective

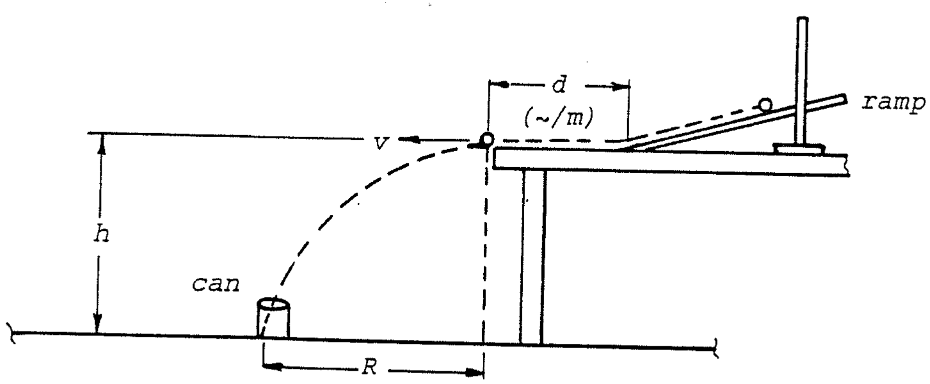
On completion of this experiment you should be able to place a container in the correct position to catch a projectile.

Apparatus

Steel ball, metre rule, ramp, support stand, small can, stopwatch.

Procedure

Experiment A: Set up the apparatus as shown in the diagram.



Q.1 A distance d of approximately 1 metre has been suggested. Explain why it is inadvisable to use a value which is (a) much larger, (b) much smaller than 1 metre.

Experiment B: Measure the height h of the bench above the floor. Hence calculate the time of flight t_2 of the projectile, stating the formula that you have used.

Experiment C: Release the ball from the ramp, measure the time t_1 it takes to travel the distance d and catch the ball as it leaves the bench.

Repeat the measurement several times, always releasing the ball from the same point on the ramp. Record each measurement.

From the average value of t_1 calculate the horizontal velocity v.

Q.2 Explain why it is advisable to repeat the measurement of t_1 several times.

Experiment D: Using your values of v and t_2 calculate the range R of the projectile. Test your answer by placing the can at the predicted landing point and allow the ball to complete its flight.

Q.3 Did you succeed in landing the ball in the can on the first attempt? If not, describe what you did to make success more likely on the next attempt.

Q.4 The inaccuracy in measuring t_1 introduces the major uncertainty in this experiment. Suggest ways in which this inaccuracy might be reduced.

Experiment E: An an extension, take the largest and smallest values that you measured for t_1, use these to calculate the smallest and largest possible values of v and hence determine the limits to the possible values of R.

These limits indicate the degree of uncertainty possible in the range R.

Q.5 In view of this uncertainty, comment on the agreement between the predicted and actual ranges of the ball.

Q.6 How successful do you think this experiment would be if the steel ball was launched from a cliff 1.00 metres high?

Q.7 On what essential properties of projectile motion does success in this experiment depend?

SECTION: 5

STUDENT NOTES

LESSON NUMBER: Assignment Sheet

NOTES:

1. A ball is tossed from the top of a 125 m building with a horizontal speed of 20 m/s. If $g = 10\ m/s^2$, how long will it take to fall and how far from the building will it land?

2. A ball was tossed horizontally from the top of a 320 m building and landed 12 m from its base. If $g = 10\ m/s^2$, what was its initial horizontal velocity?

3. A ball is tossed horizontally from the top of a building with a speed of $7\ ms^{-1}$ and lands 84 m from the base of the building. What was the height of the building given $g = 10\ ms^{-2}$?

4. A boy stands on a railway flat wagon which is moving at 4 m/s. He throws a ball vertically upwards at a speed of $30\ ms^{-1}$. How long will it take for the ball to come down and how far forward will the railway wagon be when the ball does land? ($g = 10\ m/s^2$)

5. A plane flying at a height of 2000 m with a speed of 150 m/s drops a package. How long does it take for the package to reach the ground and how far away from the point over which the package was dropped does it land? ($g = 10\ m/s^2$)

6. A stone is thrown horizontally with a speed of 10 m/s from a balloon which is ascending with a velocity of 5 m/s. The stone reaches the ground in 8 seconds. How high is the balloon above the ground at the moment the stone lands? How far does the stone move horizontally before striking the ground? ($g = 10\ m/s^2$)

7. A ball is thrown so that its horizontal component of velocity is 15 m/s and its vertical component of velocity is 20 m/s. ($g = 10\ m/s^2$)

   (a) When does the ball reach its maximum height and how high is that?

   (b) What is the range of the ball?

   (c) When, during the flight, does the ball have a mimimum speed? How much is it?

   (d) After 1 second what are the horizontal and vertical components of its velocity?

8. A ball is tossed at angle of 30° above the horizontal with a speed of 60 m/s. ($g = 10\ m/s^2$)

   (a) Find its vertical component of velocity at the time the ball is tossed.

   (b) Calculate its flight time.

   (c) Calculate its range.

   (d) Find the maximum height reached.

9. A boy throws a stone at a building 200 m away. The stone is tossed with a horizontal velocity component of 25 m/s and a vertical velocity component of 60 m/s. ($g = 10\ m/s^2$)

   (a) When will the stone hit the building?

   (b) How high up the building will the stone hit?

10. A missile was launched and travelled 6000 m horizontally. The maximum height attained was 1125 m. ($g = 10\ m/s^2$)

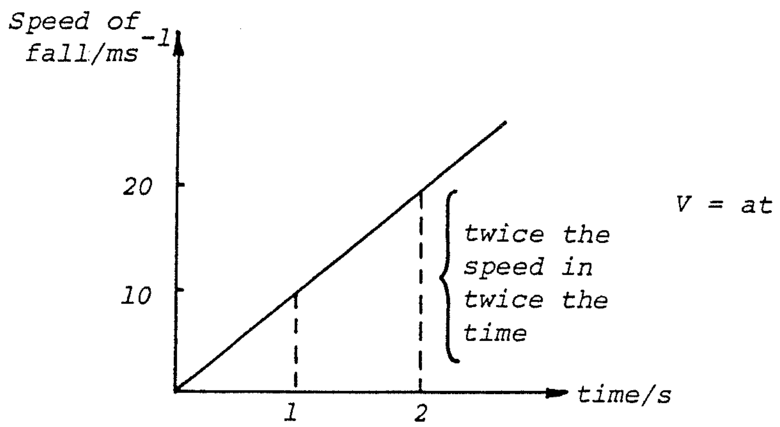
   (a) Find the time of flight.

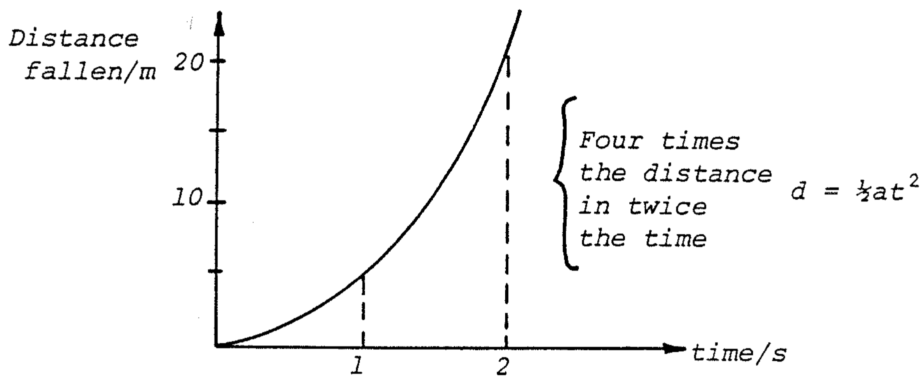
   (b) Find the horizontal and vertical components of velocity at the time the missile was fired.

   (c) At what angle with the horizontal was the missile fired?

OHP 1

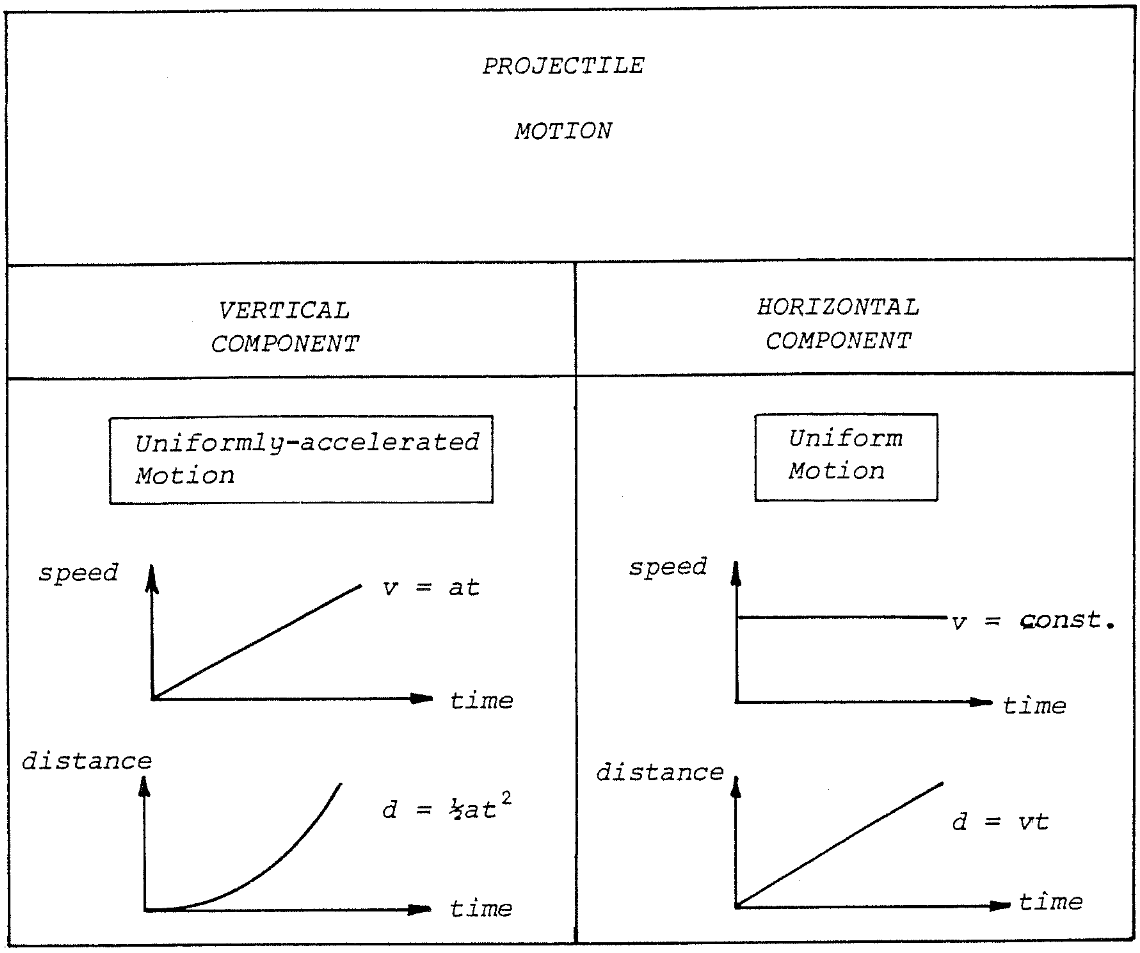
Motion Graphs for a Dropped Ball

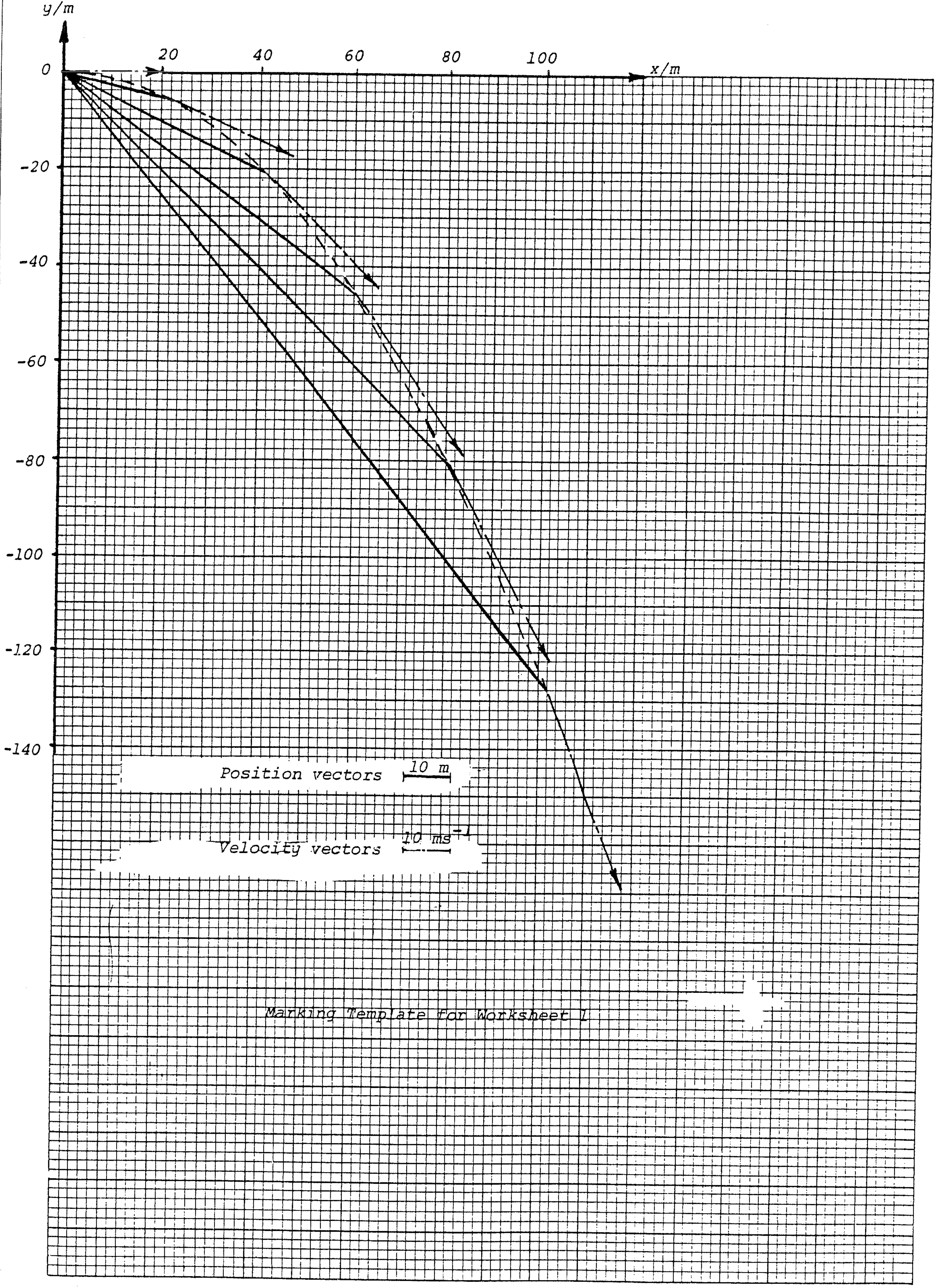




OHP 2

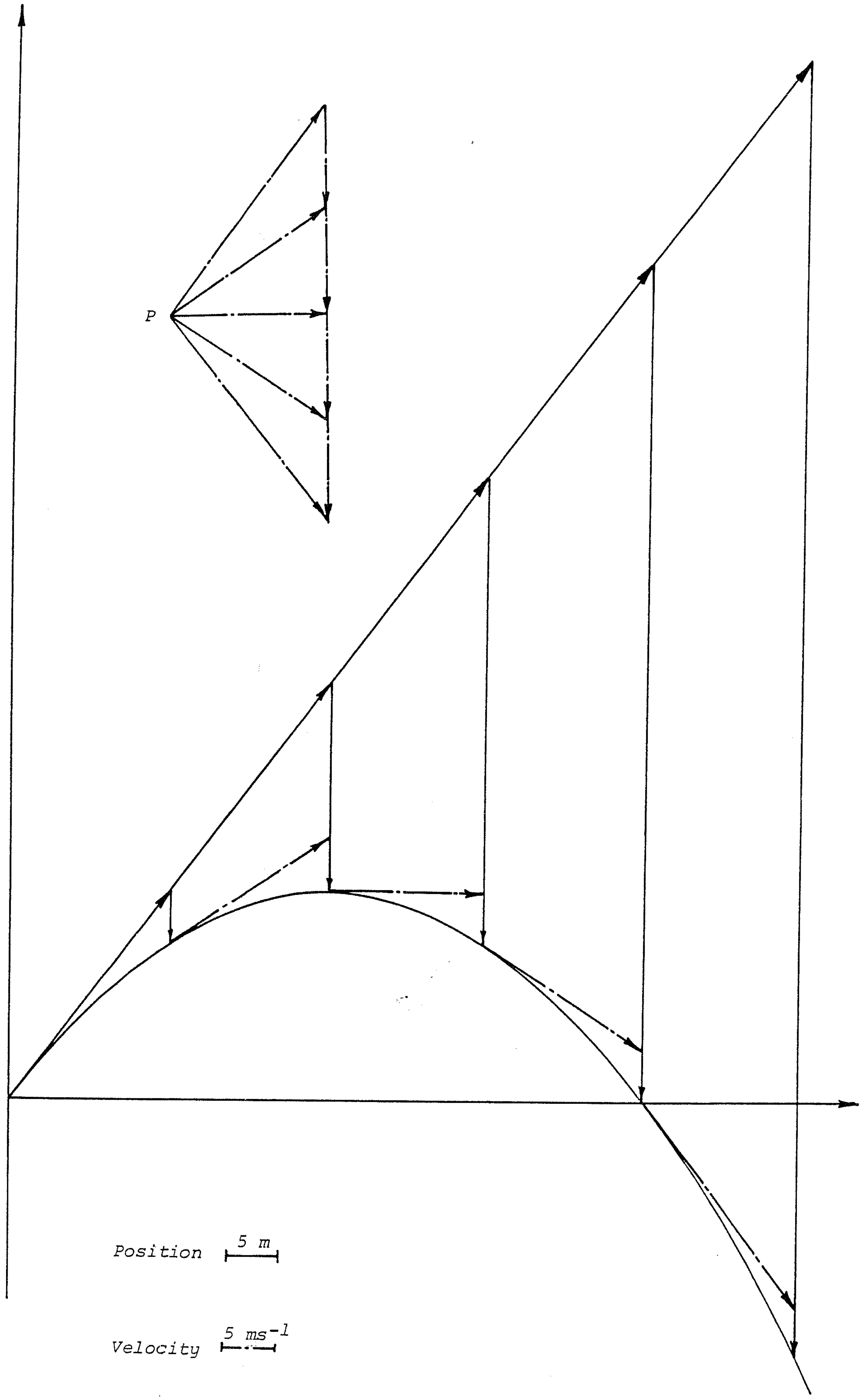
Computer Modules 1 and 2: SUMMARY





Marking Template for Worksheet 1

OHP 4



STOP PRESS

SHUTTTLECOCK TRAJECTORY

This program simulates the flight of a shuttlecock with significant air resistance.

Reference: American Journal of Physics 48(7) July 1980 p511
Terminal Velocity of Shuttlecock in Vertical Fall
Peastrel, Lynch and Armenti

Article showed: drag $\propto$ speed^2

drag force = $-kmv^2$

drag coefficient $k = \frac{g}{V_T^2}$ where V_T is terminal velocity measured to be 6.8 ms^{-1}

$\Rightarrow \quad k = 0.21\ m^{-1}$

Program assumes same drag coefficient in projectile motion as in free fall.

Program

```
10  X = 0 : Y = 0 : T = 0 : G = -9.8
20  INPUT "INITIAL VX, VY";  VX, VY
30  INPUT "TIME STEP";  DT
50  INPUT "DRAG COEFFICIENT";  K1
60  INPUT "SHUTTLECOCK MASS";  M
70  FOR I = 1 TO 5
80  V = SQR (VX * VX + VY * VY)
90  K = K1 * V/M
100 AX = -K * VX
```

```
110  AY  =  G - K * VY
120  IF T > 0 THEN 160
130  VX = VX + AX * DT/2
140  VY = VY + AY * DT/2
150  GOTO 180
160  VX = VX + AX * DT
170  VY = VY + AY * DT
180   X = X + VX * DT
190   Y = Y + VY * DT
200   T = T + DT : NEXT I
210  PRINT T, X, Y
220  IF T => 10 OR IF Y <= 0
     THEN 230 ELSE 70
230  END
 65  PRINT "T", "X", "Y"
```